

PARTICLE SHAPE EFFECT OF GRANULAR COLUMN COLLAPSE ON AN ERODIBLE BED: A SUPERQUADRIC DEM STUDY

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Abstract. Granular flows are common phenomena observed in natural disasters such as landslides and rock avalanches. These geophysical mass flows commonly exhibit inherently complex behavior due to the interplay of multiple impact factors, especially the particle shape effect and the underlying erodible bed. In this work, we systematically study the effects of particle shape on granular column collapse on an erodible bed using the superquadric discrete element simulations. The non-spherical particle shapes are accurately defined utilizing the aspect ratio A and blockiness B , resulting in a change from elongated to platy and cubic shapes. The granular column is composed of different superquadric grains; meanwhile, the erodible bed is kept constant in a weak binary-size mixture of spherical grains for simply describing and quantifying the erosion behavior of erodible grains. The results show that the degrees of erosion slightly increase with decreasing elongation of superquadric grains. The runout distance of the granular columns changes complicatedly with the increase of the particle aspect ratio, while it declines significantly with increasing blockiness. The blockiness of superquadric grains also reveals significant impacts on the erosion behavior of granular materials. These findings may improve the understanding of more general scenarios of granular mass flows on complex surfaces such as an erodible bed.

Keywords: erosion mechanism, erodible bed, granular collapse, particle shape, runout distance.

1. INTRODUCTION

Gravity-driven granular flows are prevalent in natural events such as rock avalanches, landslides, and debris flows (He et al., 2024; Ouyang et al., 2015). These geophysical processes often exhibit complex phenomena and dynamic characteristics (Pudasaini & Krautblatter, 2021). In nature, these gravity-driven flows often occur over erodible beds composed of different materials and surface conditions. These processes may lead to the erosion and entrainment of erodible grains (Ligneau et al., 2024). Once erosion and entrainment occur, a granular flow may experience enhanced kinetic energy, extended runout distance, increased accumulation mass of material, and increased impact forces on downstream infrastructure (Chou et al., 2023; Crosta et al., 2009; Mangeney et al., 2010; Vo et al., 2024). Therefore, a comprehensive understanding of the collapse dynamics as well as the mechanisms of erosion and entrainment plays a crucial role in mitigating or controlling the natural hazard risks and proposing effective strategies for landslide disaster prevention.

To comprehensively and deeply understand the collapse dynamics of granular materials, different axisymmetric configurations and dam-break models of granular columns collapse on horizontal and inclined rigid planes have been studied extensively in theoretical models, experiments, and numerical simulations (Lajeunesse et al., 2005; Lube et al., 2005; Man et al., 2023; Mangeney et al., 2010). Both theoretical models and experiments have provided fundamental understanding of granular column collapse on a horizontal plane, offering valuable insights into the deposition height and runout distance. These approaches also provide a valuable foundation for validating the numerical simulations. However, while theoretical models are often limited by idealized assumptions such as uniform particle size and friction effects or frictionless conditions, experimental works face challenges in measuring the microscopic responses and considering experimental scale effects such as field scale. Overcoming the limitations above, numerical simulations offer valuable advantages in describing dynamic collapses of granular columns (Man et al., 2021; N. H. T. Nguyen et al., 2020; Vo & Nguyen, 2024). In fact, numerical simulations have provided fundamental understanding of the morphology evolution, runout distance, collapse mobility, as well as the linking behavior between microscopic properties and macroscopic responses of granular column collapse (T.-H. Nguyen & Vo, 2025; Vo & Nguyen, 2025).

However, the previous works have only focused on granular columns collapse on horizontal and inclined planes. As mentioned above, granular materials often collapse and flow on erodible beds in practice, leading to the erosion and entrainment of underlying materials. Therefore, some recent works have raised attention to the model of granular column collapse on erodible surfaces. Crosta et al. (2009) first considered the erosion, entrainment, sinking, ploughing behavior of erodible beds under the collapse of granular columns. Wu et al. (2018) and Wu et al. (2021) further investigated the granular columns collapse in erodible beds in experiments and simulations. They found that three different erosion-entrainment mechanisms are observed during the collapse process of granular materials over erodible beds. The velocity evolution and migration of interface between mobilized and immobilized particles were also discussed in detail. Recently, we comprehensively studied the effects of different principal parameters such as initial aspect ratio, slope angle and friction coefficient on the collapse dynamics and erosion-entrainment mechanisms of granular columns collapsing on an erodible-inclined bed. Remarkably, both accumulation mass and runout distance of granular materials are nicely described by a dimensionless number incorporating all principal parameters above. Although extensive investigations have provided fundamental understanding of the collapse dynamics and erosion-entrainment mechanisms of granular materials, they mainly considered spherical grains, which are strongly influenced by the rolling resistance of particles. Especially, the interaction between granular columns composed of different superquadric grains and erodible-spherical particles remains limited. Thus, the current work aims to bridge these gaps by using the discrete element method. As we shall see the aspect ratio and blockiness of superquadric grains differently affect the collapse dynamics of granular columns and erosion-entrainment mechanisms of erodible particles.

The remainder of this paper is organized as follows: We briefly introduce the DEM method for defining superquadric grains and their interactions as well as the model setting of a granular column collapse on an erodible bed in Section 2. We then illustrate the collapse dynamics and erosion-entrainment mechanisms in Section 3. The runout distance and kinetic energy evolution are analyzed and discussed in Section 4. Finally, Section 5 summarizes salient results drawn from this work and further research directions.

2. NUMERICAL METHOD AND MODEL SETTING

As mentioned previously, the current work employs superquadric grains using the discrete element method (DEM) (Banerjee et al., 2024; Hoang & Nguyen, 2023). This approach has been constructed in the open-source LIGGGHTS code (Kloss & Goniva, 2011). Each superquadric

particle is modeled as a rigid body and interacts with each other via the soft-particle law, implying allowing a small overlap between grains at their contact points. This contact law not only allows for easy determination of the interaction between complex-shaped particles but also guarantees the stable state of granular materials. The DEM LIGGGHTS code uses the Hertzian normal contact law and Coulomb friction law for fully considering the interaction forces between grains. These contact laws require principal input parameters, including Young's Modulus, Poisson's ratio, coefficient of restitution, interparticle friction coefficient, and particle-wall friction coefficient.

In this paper, we select the superquadric grain shape, which requires less computational cost compared to other complex shapes. Each superquadric grain shape is defined by the unified superquadric function (Barr, 1981):

$$f(x, y, z) = \left(\left| \frac{x}{a} \right|^{b_2} + \left| \frac{y}{b} \right|^{b_2} \right)^{\frac{b_1}{b_2}} + \left| \frac{z}{c} \right|^{b_1} - 1 = 0, \quad (1)$$

where a , b , c denote the half lengths of a particle in the x , y , and z directions, respectively. $a_1 = b/a$ and $a_2 = c/a$ are the aspect ratio of superquadric particles. In this work, we choose $A = a_1 = a_2$ to simply consider the particle shape. b_1 and b_2 denote the blockiness of superquadric grains. Similarly, we considered $B = b_1 = b_2$ in the current work for the simplicity of the edges of each grain. In our simulations, we systematically change the particle aspect ratio A from 0.25 to 2, while the particle blockiness B varies from 2 to 10 for comprehensively considering the superquadric grains from elongated to platy particles and from round to sharper edges. All particle shape geometries used in our current work are listed in Fig. 1.

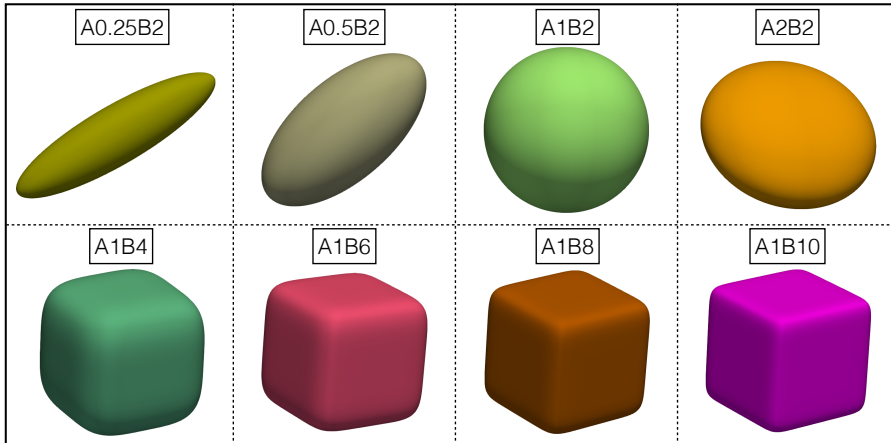


Fig. 1. Different particle shapes with various aspect ratios A and blockiness B

The numerical model considers a granular column with an aspect ratio $h_0/l_0 = 3$ composed of different superquadric particles collapsing on a horizontal-erodible bed consisting of spherical grains. The granular columns, 0.1 m in length, 0.3 m in height, and 0.1 m in width, composed of granular materials are initially introduced on a rigid block. Each granular column is composed of one superquadric particle shape. In order to prevent the crystallization effects of material and order arrangement of superquadric grains, we use two particle size groups and set the random orientations of particles. To guarantee a similar inertial effect of granular materials, we use a constant volume for each superquadric grain (equivalent to $8.09 \times 10^{-8} \text{ m}^3$ for large grains and $6.736 \times 10^{-8} \text{ m}^3$ for small grains). The erodible bed is composed of a binary-size mixture of spherical grains (equivalent $4.91 \times 10^{-8} \text{ m}^3$ for large grains and $3.58 \times 10^{-8} \text{ m}^3$ for small grains), randomly filled in a rectangular domain with 0.1 m in height, 0.7 m in length, and 0.1 m in width.

This erodible domain is large enough for the maximum runout distance of granular materials within the columns. The inter-particle friction coefficient is set constant to 0.3, while wall-particle friction coefficient is set to 0.1. Both these friction coefficients are considered as a Coulomb static friction limit, implying that the sliding between grains or between grains and wall is handled automatically once the Coulomb limit is exceeded, but the friction coefficient remains the same value. In the current work, Young's Modulus of all grains is set to 5.84 MPa, Poisson's ratio equals 0.3, and the coefficient of restitution is set to 0.3. The choice of the value of the Young's Modulus helps to improve the balance between the numerical stability and computational cost. By immediately removing the front gate of the granular column, superquadric grains start to collapse and spread on the erodible bed. During this process, superquadric grains plough the erodible surface, leading to the occurrence of erosion and the entrainment of underlying grains. These collapse dynamics and erosion-entrainment mechanisms will be deeply analyzed and discussed in the following sections.

3. COLLAPSE PROCESS AND EROSION MECHANISMS

Figs. 2 and 3 illustrate the degree of erosion of the erodible bed and final deposition morphology of granular columns composed of different superquadric grains by changing the particle aspect ratio A in the case of keeping blockiness $B = 2$ and increasing blockiness B from 4 to 10 in the case of keeping $A = 1.0$, respectively. It is interesting to see that the runout distance of granular materials and erosion of erodible grains for the case of the longest superquadric grains ($A = 0.25$) reveal a significant difference compared to other cases, which only show a slight difference in these physical properties, as shown in Fig. 2. These observations imply that the elongated grains mainly slide on the surface of erodible bed, while spherical, elliptical, and platy grains plough on this surface, leading to the erosion and entrainment of erodible particles. In contrast to the effects of the particle aspect ratio A , the particle blockiness B clearly affects the collapse morphology and erosion property of erodible bed, as shown in Fig. 3. Indeed, the granular columns composed of superquadric grains with sharper edges lead to an improvement of stacking behavior of granular materials and a reduction of runout distance. Furthermore, the increase of particle blockiness tends to increase the erosion degree of erodible grains.

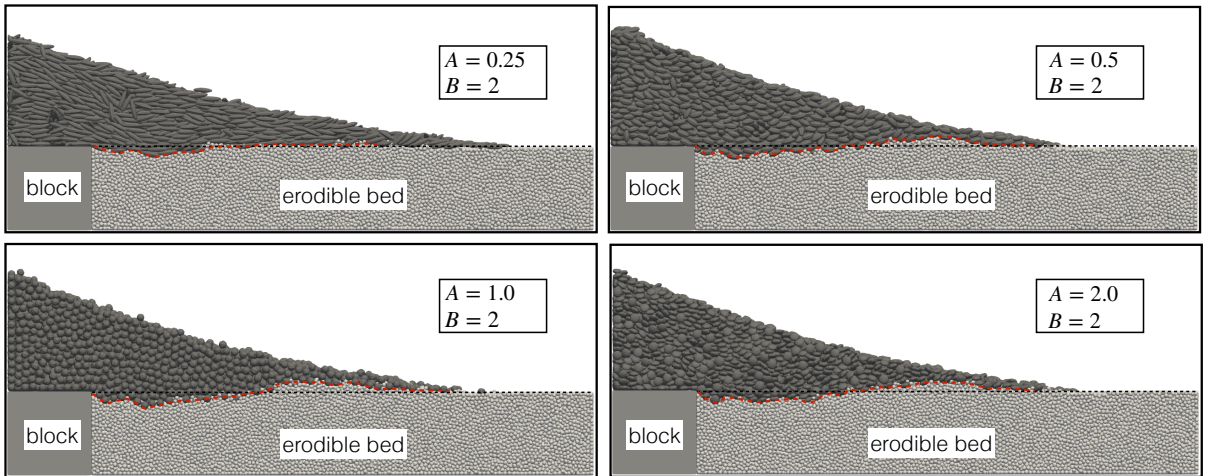


Fig. 2. Final deposition morphology of granular columns and erosion behavior of erodible bed for different values of particle aspect ratios A by keeping the particle's blockiness $B = 2$

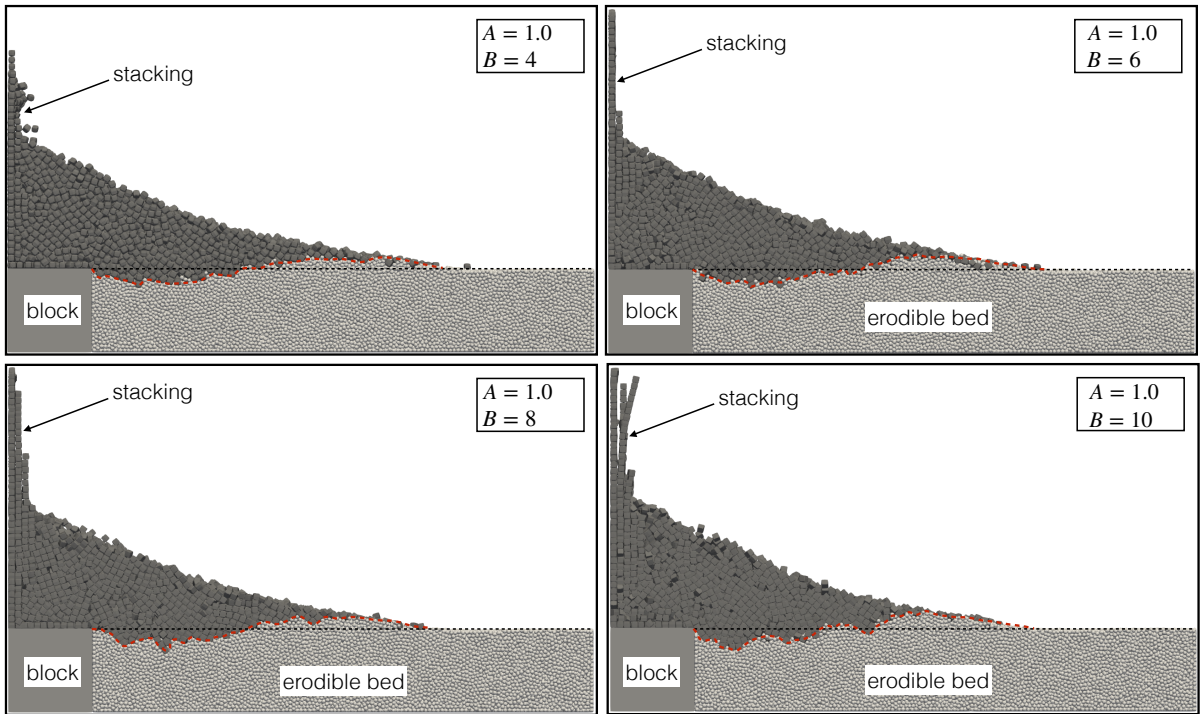


Fig. 3. Final deposition morphology of granular columns and erosion behavior of erodible bed for different values of particle blockiness B by keeping the particle aspect ratio $A = 1.0$

To deeply explain the erosion behavior of erodible bed under the collapse dynamics of granular columns, the morphology evolution of the columns and evolution of the interface between the static and flowing layers of the erodible bed are considered. Fig. 4 shows the morphology evolution and interface evolution over time for the granular columns composed of elongated (a), elliptical (b), spherical (c), and platy (d) grains. For the column of elongated grains, after releasing from the static state ($t = 0.0$ s), the granular column starts to vertically collapse. This leads to the ploughing behavior of erodible grains on the top of the erodible bed. However, this ploughing process only slightly occurs due to the slight penetration of elongated grains into the erodible bed. After ploughing erodible grains, the granular flows in front part of the column lead to the erosion and entrainment of erodible particles before reaching the rest. Continuously increasing the particle aspect ratio, the ploughing process occurs more strongly as a consequence of strong penetration of superquadric grains. However, due to the low blockiness of superquadric grains, there is no interlocking behavior observed during the process.

Similarly, Fig. 5 displays the evolution of morphology profiles of granular columns composed of cubic grains with different particle blockiness over time. It is interesting to note that the stacking behavior of superquadric particles close to the behind wall as a consequence of their order arrangement. This stacking phenomenon is enhanced with increasing the blockiness B from 2 to 10, leading to observe a shorter runout distance of granular materials on the erodible bed. Furthermore, the stacking phenomenon of superquadric particles within the columns also leads to a reduction in the ploughing and penetration degrees into the erodible bed. As a result, the change of the interface between static and flowing area of erodible bed decreases with increasing blockiness B .

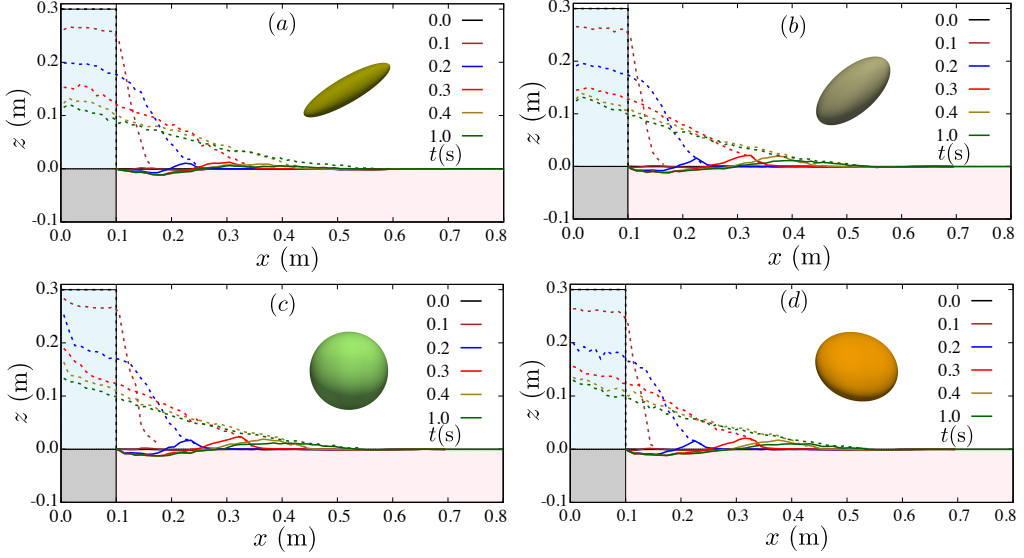


Fig. 4. Evolution of deposition profiles of granular columns (dash lines) composed of superquadric grains in four different aspect ratios and interface evolution of erodible bed (solid lines)

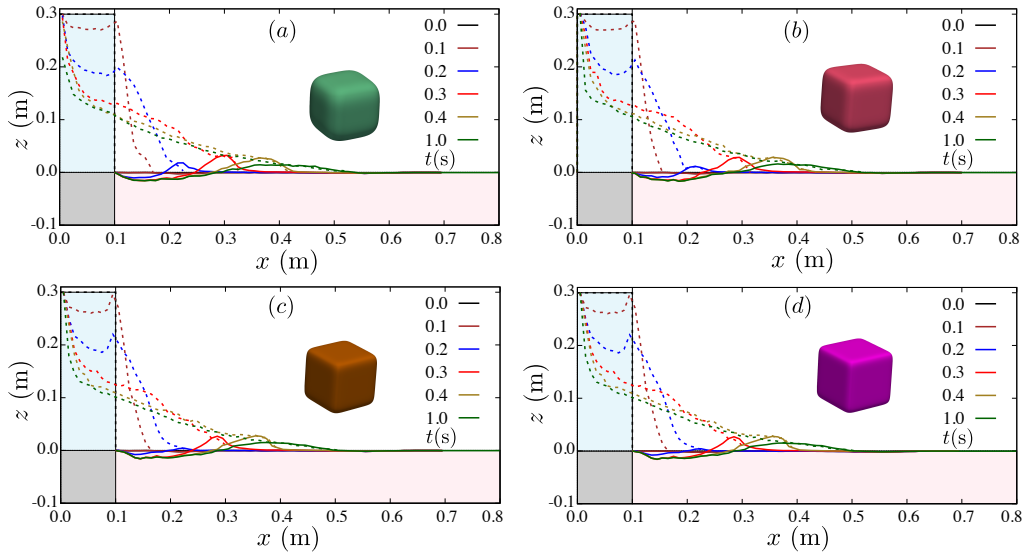


Fig. 5. Evolution of deposition profiles of granular columns (dash lines) composed of superquadric grains in four different values of blockiness and interface evolution of erodible bed (solid lines)

4. RUNOUT DISTANCE AND KINETIC ENERGY

Although the above observations comprehensively display the collapse dynamics and erosion mechanisms as well as the evolution of the interface between the static and flowing region of erodible bed, it is necessary to quantify the kinetic energy evolution of the superquadric and erodible grains as well as the runout distance of granular materials. The runout distance is defined as a ratio between the deposition length $x - l_0$ and the initial length l_0 . The normalized energy is defined as a ratio between the kinetic energy $E_k = \frac{1}{2} \sum m_i v_i^2$ and the initial potential

energy $E_{p0} = mgh$, where m_i and v_i are the mass and translational velocity of particle i (superquadric grains in the columns or erodible particles), m and h denote the mass and height of the center of mass of the granular columns.

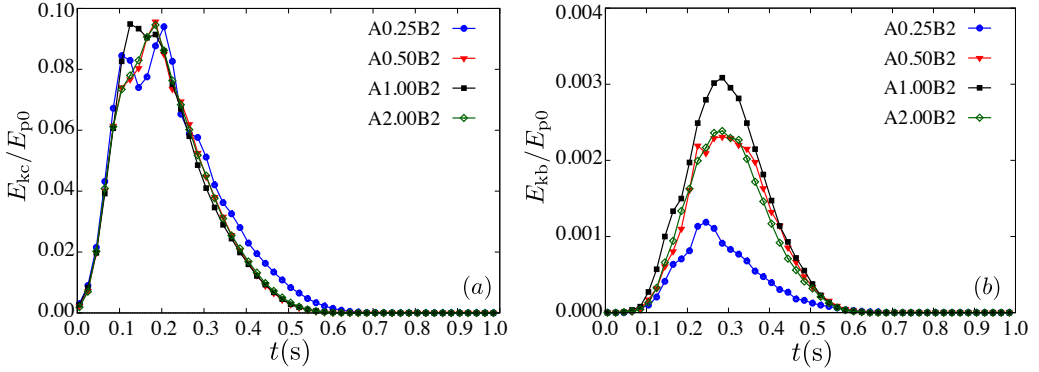


Fig. 6. Normalized kinetic energy of granular columns (a) and of the erodible bed (b) during the collapse process with four different particle aspect ratios

Fig. 6 illustrates the evolution of the normalized kinetic energy of the granular columns E_{kc}/E_{p0} (a) and of the erodible bed E_{kb}/E_{p0} (b) during the test for different particle aspect ratios A by keeping $B = 2$. Considering the same column aspect ratio, the kinetic energy evolution and its peak are similar for all cases of particle aspect ratio except for the elongated grains. Indeed, instead of continuously increasing from the rest before reaching the peak and then declining the kinetic energy, the mobility of elongated grains can re-increase as a consequence of the sliding behavior of the longest particle shape on the erodible bed, as shown in the blue line in Fig. 6(a). In contrast to the similar evolution of kinetic energy of the granular columns during the collapse process, the normalized kinetic energy of erodible bed E_{kb}/E_{p0} is strongly affected by the superquadric shapes constructed in the granular columns. Due mainly to sliding on the underlying surface, the movement of erodible grains observed for the collapse of elongated particle is significantly less than that for another superquadric shape, as shown in Fig. 6(b). Contrarily, the erodible grains are much more eroded and move faster than other cases of superquadric shapes. These observations are consistent with previous findings for the collapse and movement of superquadric grains on a rigid plane (Hoang & Nguyen, 2023).

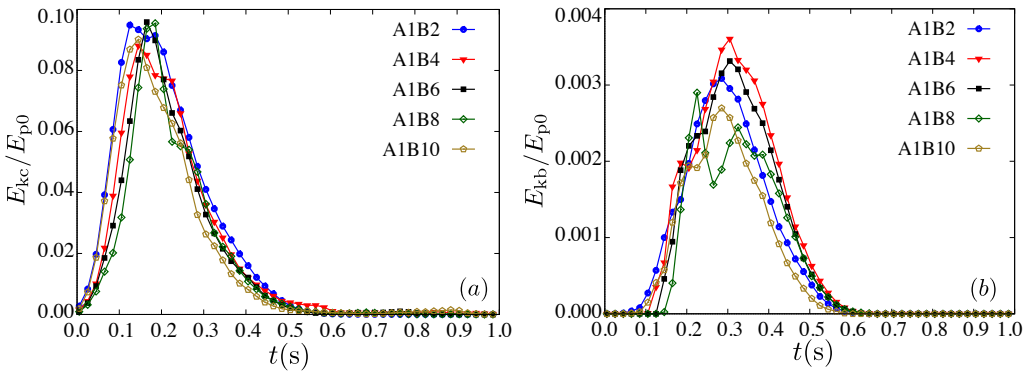


Fig. 7. Normalized kinetic energy of granular columns (a) and of the erodible bed (b) during the collapse process with five different values of particle blockiness

Figs. 7(a) and 7(b) display the evolution of E_{kc}/E_{p0} and E_{kb}/E_{p0} over time for different values of particle blockiness B , respectively. It is remarkable to note that although the peak

value of E_{kc}/E_{p0} is nearly independent of the blockiness of superquadric grains constructed in granular columns, the evolution rate of E_{kc}/E_{p0} decreases with increasing the blockiness as a consequence of improving the shape effects of the edges of superquadric grains. In contrast to the near independence of blockiness on the peak values of the normalized kinetic energy of the columns, the peak values of E_{kb}/E_{p0} are strongly affected by the particle blockiness. In particular, these values nearly decrease with increasing the shape effect of the edges of particles.

As is clearly known, the kinetic energy strongly governs the runout distance of granular materials. Figs. 8(a) and 8(b) illustrate the evolution of the runout distance $(x - l_0)/l_0$ as a function of the collapse time t for different particle aspect ratios and different values of blockiness, respectively. It is remarkable to note that the largest runout distance has similar influences on the particle aspect ratio and blockiness. Indeed, while the elongated and platy grains show a dominance in the displacement of granular materials compared to the spherical and elliptical grains constructed in the columns, the runout distance significantly decreases with increasing the blockiness. These findings are consistent with previous observations for superquadric granular columns collapse on a rigid plane.

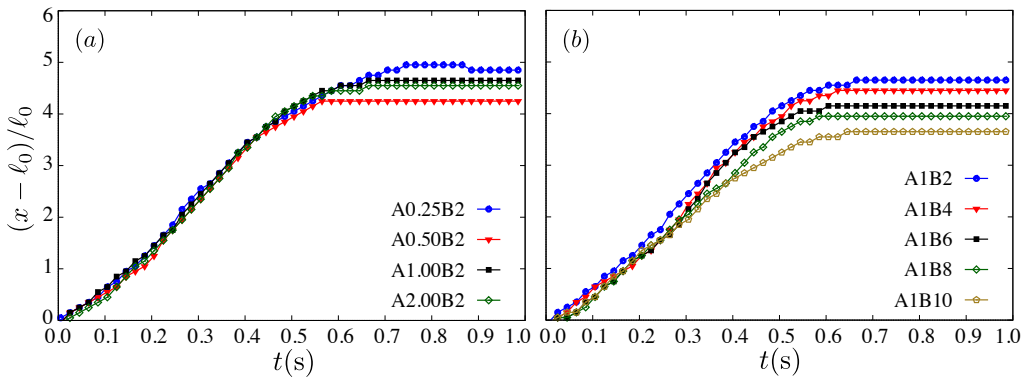


Fig. 8. Evolution of runout distance of granular materials for different particle aspect ratios (a) and for five different values of blockiness

5. CONCLUSIONS

In this paper, we used superquadric DEM simulations to investigate the collapse dynamics and erosion behavior of a granular column composed of different grain shapes collapsing on an erodible bed consisting of spherical particles. The superquadric DEM approach is constructed in the open-source LIGGGHTS code. The superquadric grains are defined by two principal parameters: the particle aspect ratio and blockiness. While the particle aspect ratio describes the elongation of a particle in two directions compared to one remaining direction; the particle blockiness is characterized by the shape effect of the edges of superquadric grains.

The results show that the particle aspect ratio and blockiness differently affect the collapse dynamics of the granular columns and erosion mechanisms of erodible grains. In particular, while the particle aspect ratio slightly affects the kinetic energy and runout distance of granular materials, except for the elongated and platy grains; the blockiness strongly governs the erosion and runout distance of the material. The erosion mechanisms and movement of erodible grains are proportional to the runout distance of granular materials by increasing the blockiness of the particles. These observations are consistent with previous works for granular columns composed of different superquadric particle shapes that collapse on a rigid plane. These findings may provide a better and deeper understanding of granular flows on erodible layers as they commonly occur in natural events such as rock avalanches, landslides, and debris flows.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CREDIT AUTHOR STATEMENT

Thanh-Hai Nguyen: *Methodology, Validation, Investigation, Visualization, Writing – original draft, Writing – review & editing.* Thanh-Trung Vo: *Conceptualization, Methodology, Formal analysis, Validation, Investigation, Supervision, Visualization, Writing – original draft, Writing – review & editing.* Duc Chung Vu: *Methodology, Investigation, Visualization, Writing – review & editing.* Nguyen Hoang Phuong Luong: *Investigation, Visualization, Writing – review & editing.*

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