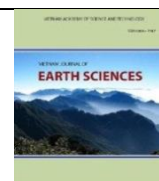




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Estimation of friction capacity of driven piles in clay using artificial Neural Network

Thuy-Anh Nguyen¹, Hai-Bang Ly^{1*}, Abolfazl Jaafari², Binh Thai Pham¹

¹University of Transport Technology, Hanoi 100000, Vietnam

²Research Institute of Forests and Rangelands, Agricultural Research, Education and Extension Organization (AREEO), P.O. Box 64414-356, Tehran, Iran

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ABSTRACT

The load capacity of driven piles is a crucial mechanical property, and correctly determine the corresponding value is important in geotechnical engineering. Concerning piles driven in clay, the load capacity is mainly associated with the side resistance of the pile. The soil load capacity of conventional piles is determined by different methods and then reassessed by the static load test. Nonetheless, this method is time-consuming and costly. Therefore, the development of an alternative approach using machine learning techniques to solve this problem has been investigated recently. In this work, the backpropagation network model (ANN) with a 4-layer structure [4-8-6-1] was introduced to predict the frictional resistance of pile driven in clay. The dataset for the development of the ANN model consisted of 65 instances, extracted from the available literature. The performance of the proposed ANN algorithm was assessed by two statistical measurements, such as the Pearson correlation coefficient (denoted as R), and Root Mean Square Error (RMSE). In addition to the original contribution, the present work conducted a step further toward a better knowledge of the role of inputs used in the prediction phase. Using partial independence plots (PDP), the results of this study showed that the effective vertical stress and the undrained shear strength were the prediction variables that had a significant influence on the friction capacity of driven piles.

Keywords: Artificial Intelligence (AI); Artificial Neural Network (ANN); Levenberg Marquart algorithm; friction capacity of driven piles.

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1. Introduction

Currently, pile foundations are the most commonly used foundations for constructions on soft ground or structures with relatively large loads, of which pile is the primary bearing part (Tan et al., 2011). Therefore, the determination of piles capacity is vital in

construction because it is a factor that measures the quality of the work, ensuring the reliability and durability of the foundation (Randolph et al., 1979). The pile load capacity could be defined as the maximum load that a pile can receive and ensure that the building still works under normal conditions. In soil, the load-bearing capacity of the pile is created by the friction on the surface around the pile

*Corresponding author, Email: banglh@utt.edu.vn

(f_s) and the reaction of the ground on the tip of the pile (f_b) (Randolph, 2003). Meanwhile, the pile bearing capacity depends on other factors, such as pile geometric characteristics (the length and size of the pile), the load condition, the nature of the ground, the method of lowering the pile (Samui, 2008). Up to date, various approaches have been introduced to determine the piles bearing capacity. Conventional approaches mainly based on the limit-equilibrium theory, the background model, or empirical formulations derived from experiments such as static, dynamic load tests (Wrana, 2016). When piles are driven in clay, the pile bearing capacity generally associated with the friction between the pile and the surrounding soil (Prayogo and Susanto, 2018). However, these methods have been used with different assumptions and could not correctly determine the frictional resistance of piles, especially in cohesive soil (Samui, 2008). Therefore, the primary objective of the present work is to propose a general approach to predict the friction capacity of driven piles.

Over the past decades, an alternative manner to estimate the results from experiments has been the subject of intense researches (Dao et al., 2020). Besides conventional approaches such as regression analysis or simulations, Artificial Intelligence (AI) algorithms have gained increasing interest due to many advantages (Le et al., 2020; Nguyen et al., 2019; Pham et al., 2020; Phong et al., 2019). Among AI algorithms, artificial neural network (ANN) has been effectively used to deal with many complex engineering problems (H.-B. Ly et al., 2019; H.B. Ly et al., 2019). The artificial neural network algorithm is known for its capability in solving nonlinear and particularly problems where a direct relationship of inputs and output(s) can hardly be found. An outstanding advantage of an artificial neural network algorithm is the ability to self-study and adjust the weights. Thus, the results of the calculation are consistent without

depending on the mechanical equations, physicochemical, or subjective opinion. Many complex issues related to structural engineering (Le et al., 2019), materials science (Van Dao et al., 2019), structural analysis, and design (VANLUCHENE and SUN, 2008) have been solved with excellent performance. In the geotechnical engineering field, there have been many studies using ANN model to predict the pile bearing capacity (Chan et al., 1995; Chen et al., 2020; Chow et al., 1995; Goh, 1995a; Harandizadeh et al., 2019; Jahed Armaghani et al., 2017; Momeni et al., 2015; Yong et al., 2020). In particular, in Goh's study, the ANN model was successfully built to predict the frictional bearing capacity of piles in clay (Goh, 1995b). However, the importance of these inputs is not mentioned in this document. Therefore, the present study contributes to the improvement of the pile frictional load prediction as well as analyzing the influence of input factors in the simulation process.

2. Database construction

In this study, the experimental data of driven pile friction capacity were extracted from the work of Goh (Goh, 1995a). Some 65 experimental data are used for the ANN model. The ANN model used four input parameters: pile diameter (denoted as D , cm), pile length (denoted as L , m), effective vertical stress (denoted as σ'_v , kPa), undrained shear strength (denoted as S_u , kPa). The output parameter was friction capacity (kPa). From a statistical point of view, the value of pile length varied in the range of 4.6–96 m, the pile diameter was in the range 11.4–76.7 cm, the effective vertical stress varied from 19–718 kPa, and the undrained shear strength ranged between 9 and 1205 kPa. Besides, the friction capacity values were in the range of 8 to 192.1 kPa. The histograms of the corresponding variables are presented in Fig. 1.

The dataset in this study was divided into two sub-datasets: The first one (included 70%

of the data) was used for ANN network training, called the training part. The second dataset (30% of the remaining data) used to verify the model, referred to the testing part. The 70/30 ratio for generating the dataset was chosen, as suggested in the contribution of (Khorsheed and Al-Thubaity, 2013) or (Leema et al., 2016). With the above division, the data set of 65 data had 45 samples for the training dataset and 20 samples that used to estimate the prediction performance of the ANN network. The dataset in this study,

including input and output variables, was normalized in the 0-1 range, following Eq. (7). This technique has mainly been used in artificial intelligence problems to reduce numerical errors.

$$x' = \frac{x - \min(X)}{\max(X) - \min(X)} \quad (1)$$

where $\max(X)$ is the max value of variable X , and $\min(X)$ is the min value of the corresponding variables used to normalize, respectively.

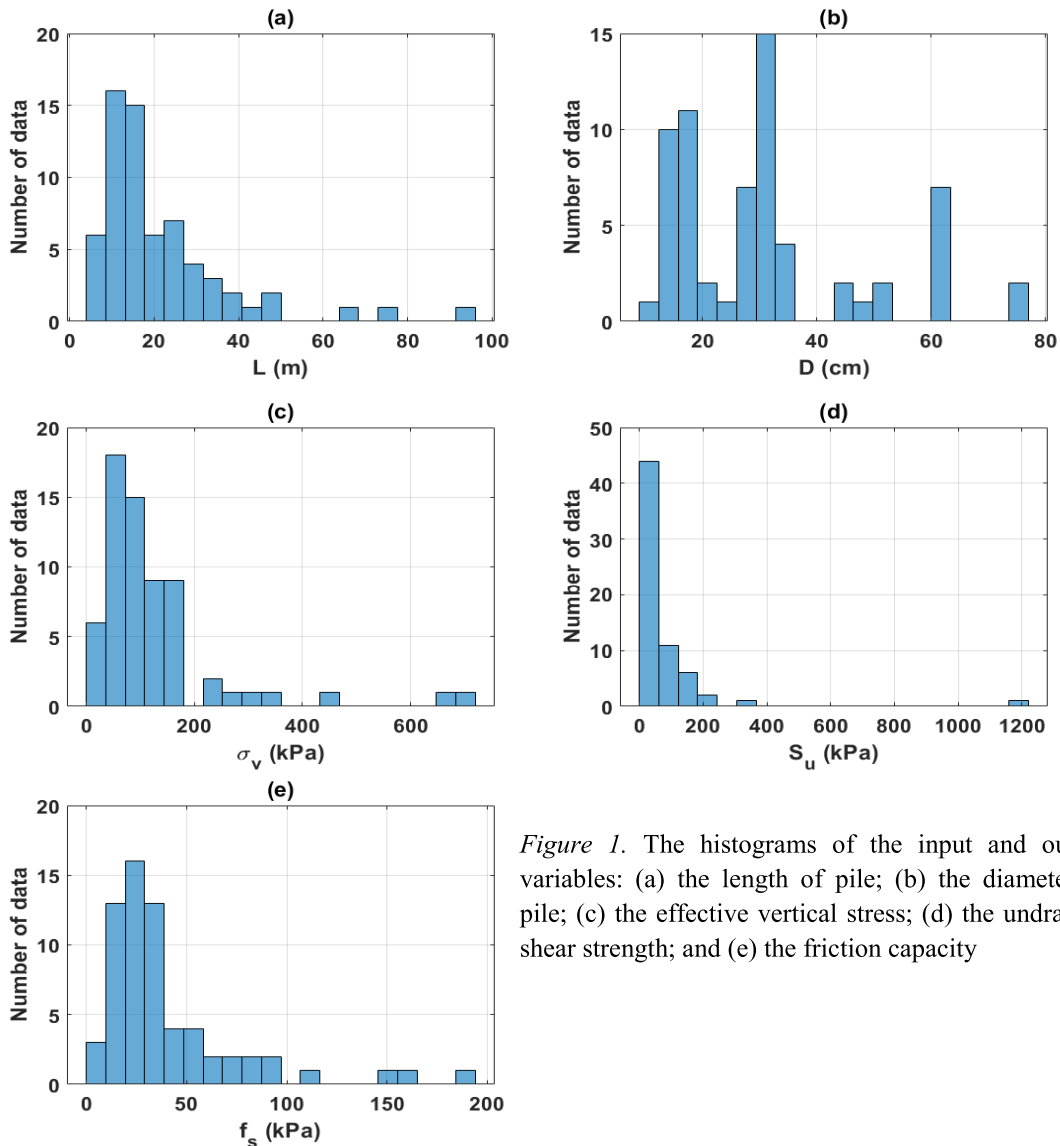


Figure 1. The histograms of the input and output variables: (a) the length of pile; (b) the diameter of pile; (c) the effective vertical stress; (d) the undrained shear strength; and (e) the friction capacity

3. Methods

3.1. Artificial Neural Network

Artificial neural network (generally denoted as ANN) is an abstraction of the function of the biological neural structures of the human brain (Armaghani et al., 2020; Du et al., 2017). This is a practical soft computing approach to solve overly complex problems compared with classical mathematics and traditional methods (Jegadesh and Jayalekshmi, 2015). Besides, reverse propagation neural networks (BPNN) are commonly used in practical applications or regression problems (Singh, 2012). The structure of the backpropagation network consists of three different layers: (i) the input layer, (ii) the output layer, and (iii) the hidden layer that plays a role as the connection of the input and output layers. The hidden layers contain one or many elements, called neurons, responsible for the transmission and process information from the input layer to the output (Goh, 1995a). The number of hidden layers and the number of neurons depend on the complexity of the given problem. During the training process, an input set is put into a particular presumptive system to calculate the output value, and then the output value is compared with the actual measured value. If there is no difference, then there is no need to perform a test. Otherwise, the weights will be changed during backpropagation in the neural network to reduce the difference. The backpropagation network usually has one or many hidden layers with sigmoid neurons, and the output layer is neurons with linear transfer function. Multilayer networks using backpropagation algorithms are the most widely used in the field of neurons. Basically, the backpropagation algorithm is a general form of the Least Means Square (LMS) algorithm because it uses the same reduction technique in the direction of the gradient vector but with the complexity of the error

function more prominent. This algorithm is an approximation algorithm to find the points at which the network performance is optimal. The performance index is usually determined by a function of the weight matrix and specific inputs that are in the process of understanding the problem.

However, the basic BPNN algorithm has some weaknesses, such as the convergence rate and problem of the local minimum (Zhou et al., 2018). The study of faster algorithms is divided into two groups. The first group develops heuristic techniques. These heuristic techniques offer ideas such as variable arithmetic, using momentum and elasticities. The second group develops in the direction of numerical optimization techniques. Some techniques of digital optimization that have been successfully applied to multilayer neural networks are the conjugate gradient algorithm and the Levenberg-Marquardt algorithm (LM - another version of the Newton method) (Dahou et al., 2009).

The Levenberg Marquart (LM) algorithm is a combination of the slope attenuation optimization algorithm with the Newton one, capable of increasing the convergence rate by optimizing the iteration process (Ranganathan, 2004). Compared to the original BPNN, the convergence rate of the LM algorithm is faster than any traditional or improved algorithm. The improvement of BPNN using the LM algorithm has gained many achievements in the literature. For these reasons, the LM algorithm is used in this study.

The effectiveness of ANN models depends on the structure of the network, which is represented by the number of hidden layers and the neurons. After several trial-and-error tests, the ANN structure chosen in this study included four layers. The input layer consisted of 4 neurons, corresponding to 4 input variables (L , D , σ'_v , S_v). The first hidden layer consisted of 8 neurons, the second hidden layer consisted of 6 neurons. Finally, the

output layer included one neuron, representing the value of the friction capacity (Fig. 2). Regarding the simulation process, a code was

constructed, mainly based on the in-built ANN program using Matlab software with several little modifications.

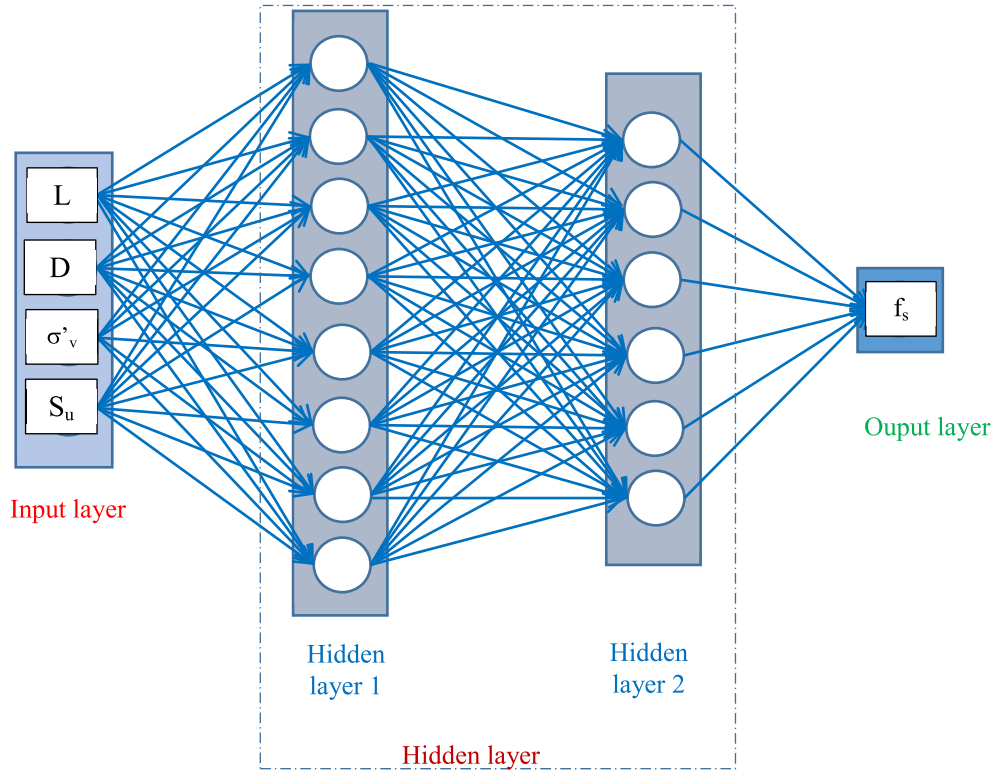


Figure 2. The structure of the ANN network in this study

3.2. Performance Evaluation

In this study, the two statistical measurements were used to evaluate the accuracy of the ANN model, namely the Pearson correlation coefficient (R), and Root Mean Square Error (RMSE). Criterion R is widely used in regression problems (Menard, 2000) to estimate the correlation between the target and predicted outputs (Le et al., 2019). The value of R is in the range [-1; 1]. Besides, RMSE measures the average magnitude of error between the target and predicted outputs (Chai and Draxler, 2014; Dao et al., 2019). Quantitatively, the RMSE value closes to 0, and the absolute value of R closes to 1 represent the higher accuracy of the machine learning model. These values are expressed by

the following equations (Han et al., 2020; Murlidhar et al., 2020; Pham et al., 2019; Sadeghi et al., 2020; Sun et al., 2020):

$$RMSE = \sqrt{\frac{1}{N} \sum_{j=1}^N (p_{0,j} - p_{t,j})^2} \quad (2)$$

$$R = \frac{\sum_{j=1}^N (p_{0,j} - \bar{p}_0)(p_{t,j} - \bar{p}_t)}{\sqrt{\sum_{j=1}^N (p_{0,j} - \bar{p}_0)^2 \sum_{j=1}^N (p_{t,j} - \bar{p}_t)^2}} \quad (3)$$

where N is the number of the samples, p_0 , and \bar{p}_0 is the actual value and the average real experimental value, p_t , and \bar{p}_t is the predicted value and the average predicted value, calculated according to the forecast model.

4. Results and discussions

As described above, the ANN model with a four-layer structure [4-8-6-1] was applied to predict friction capacity. Prior to the simulation process, a trial-and-error test was first conducted in varying the hidden layer from 1 to 2, combining with the number of neurons from 1 to 10 neurons. The selected ANN structure exhibited optimal performance. Of the collected data, 70% of the data corresponds to 45 data randomly assigned to the network training phase, and the remaining 30% corresponds to the 20 data assigned to the testing phase. The essence of this is to separate the testing and training parts. This means that the data of the testing (30%) is entirely unknown to the ANN model (learned) before. For that reason, the forecasting capacity of the ANN model can be evaluated objectively and accurately. In the general forecasting problem, the forecasting capacity of the model is the most important. It is expressed through the error evaluation criteria, as previously mentioned. On the other hand, the influence of the inputs on the driven pile's friction capacity in clay is analyzed in this section.

4.1. Performance of Backpropagation ANN

Figure 3 shows the friction capacity of

driven piles predicted by the ANN model (a) training; (b) testing comparing with experimental data. The results demonstrated that the proposed algorithm could correctly predict the friction capacity in comparison with experimental data. The error of the model for the training and testing datasets were small compared to the experimental data (cf. Figs. 4a, b). Figure 4a shows the frequency versus error value of the training phase, whereas Figure 4b shows that of the testing phase. During the training period, the errors were small, and some errors were found in the range [-20;40] (kPa). In the testing phase, the errors were slightly higher than the errors of the training phase with range [-45;55] (kPa). However, the errors were closely concentrated around 0 for both the training and testing parts. These errors showed that the predictive ability of the ANN model proposed is good with the lowest error. The mean errors of the training, testing parts were estimated as 2.9181, and 2.6481, respectively. Besides, the standard deviation values were 12.6476 and 26.5719, and the RMSE values were 12.8422 and 25.9984 for the training and testing datasets, respectively.

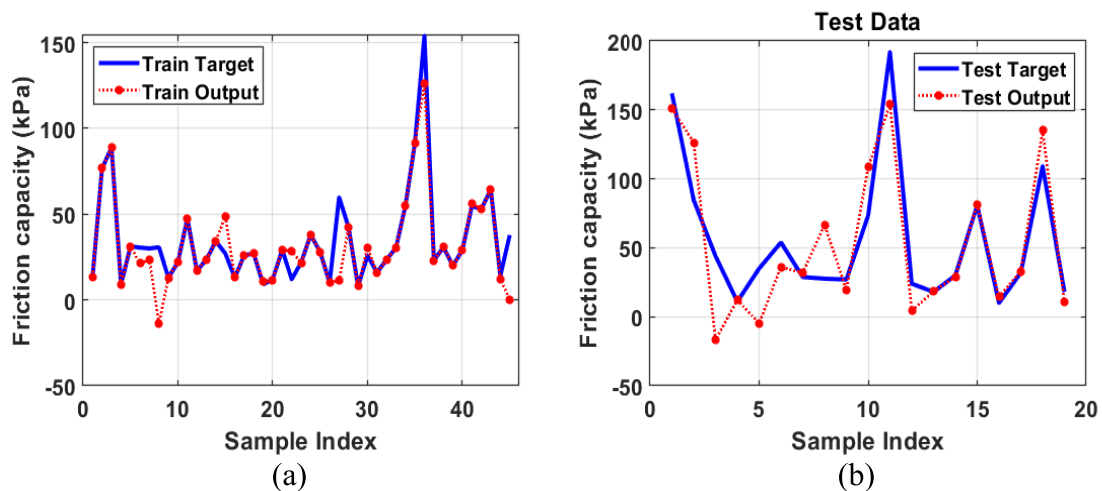


Figure 3. Friction capacity of driven piles by ANN model (a) Training; (b) Testing

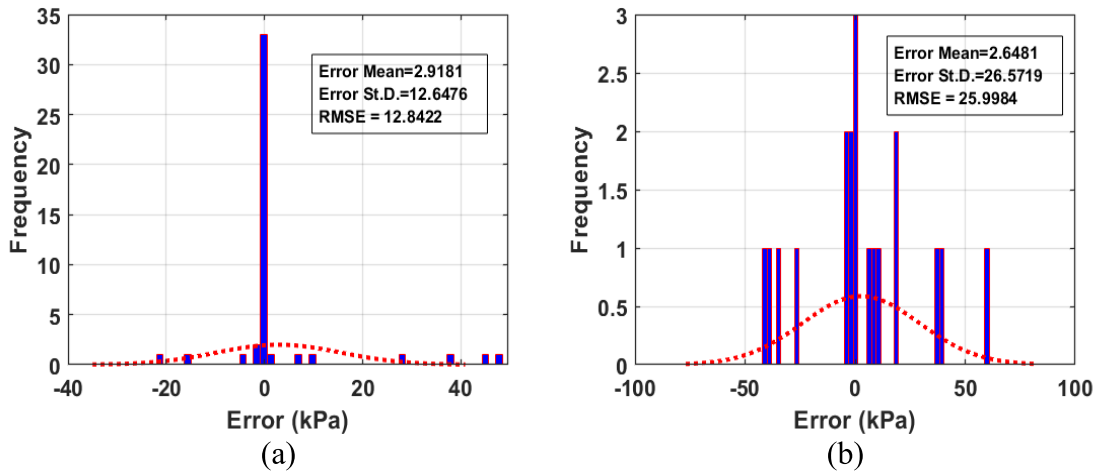


Figure 4. Errors of the ANN algorithm for (a) training dataset; (b) testing dataset

The regression model for the training and testing phases is shown in Figs. 5a and 5b, respectively. It could be observed that the predictive ability of the ANN model is relatively high. The correlation value obtained for training was $R = 0.88814$, and that of the

testing phase was $R = 0.87895$. It could be concluded that applying the ANN model for predicting friction capacity of driven strength is possible with high accuracy and low errors. Therefore, the ability ANN model for predicting was relatively high.

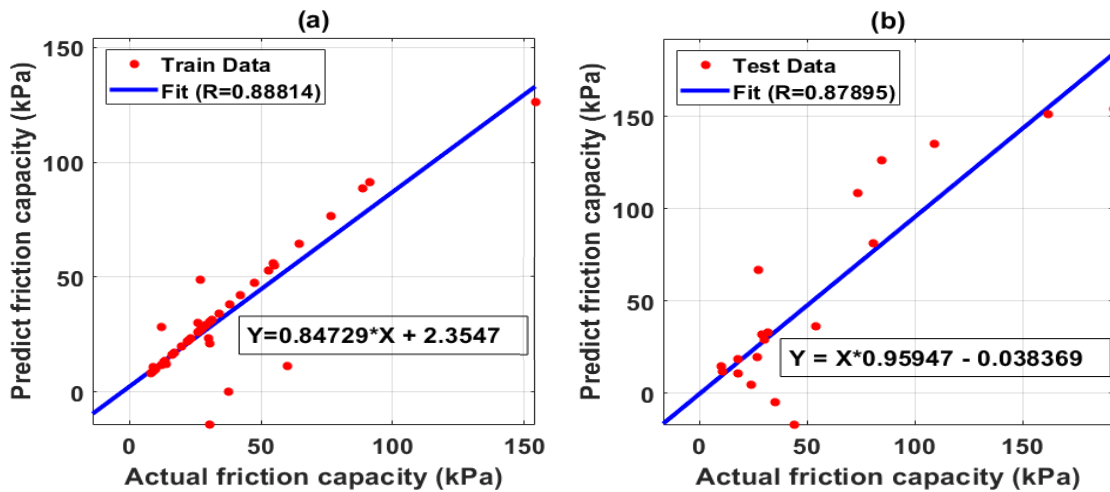


Figure 5. ANN regression results for (a) training; (b) testing

4.2. Significance of input variables

The numerical value of friction capacity of driven piles predicted by the ANN model and each selected input variable is dependent, which is effectively estimated by Partial Dependence Plots (PDP). The PDP of 4 input

variables is shown in Fig. 6. For the pile length, the latter value varied from [85 to 0]. The latter value is from [75 to 0] for the pile diameter. The latter value varied from [180 to 25] for effective vertical stress and from [120 to 25] for undrained shear strength. The latter

value of 6 inputs strongly fluctuates. However, based on the variation amplitude of each range, the influence on the prediction output is in order of effective vertical stress, undrained shear strength, pile length, and pile diameter, respectively. In fact, the undrained shear strength, depending on water, is found as the crucial variable for the soil shear

strength. In general, the presence of water reduces the angle of friction and connection among the particles of soil. This increases the friction capacity of the pile by increasing the undrained shear strength. The geometrical variables such as pile length, pile diameter have less influence on the friction capacity of driven piles.

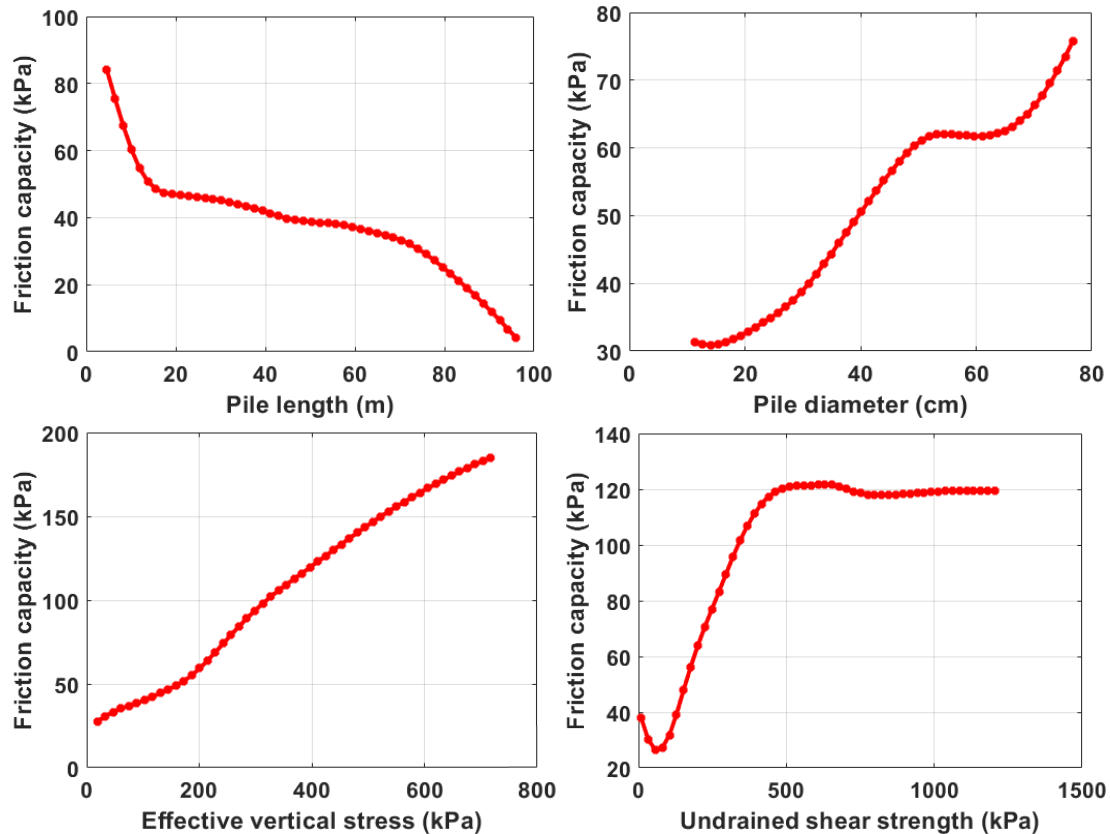


Figure 6. Partial dependence plots (PDP) of variables used in this study

5. Conclusions

This study shows a simple but effective approach using the ANN algorithm to predict the frictional resistance of piles in clay. An optimal ANN structure [4-8-6-1] has been proposed, statistically evaluated to verify the reliability of the results and confirm the excellent performance of the proposed ANN model. Two criteria, the correlation coefficient (R) and the Root Mean Square

Error (RMSE) were used to assess the correlation between the predicted values and the actual experimental ones. The proposed ANN model showed high reliability ($R = 0.88814$). Besides, a sensitivity analysis was performed using PDP to assess the significance of input variables. The obtained results showed that effective vertical stresses and undrained shear strength were the factors that significantly affected the friction

resistance. The results of this study might be useful for quickly and accurately predicting the friction capacity of driven piles practice.

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