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THE BALANCING OF ROTATING MACHINERY AS NONLINEAR SYSTEM

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ABSTRACT. In this paper a method for dynamical balancing of rotating machines as a nonlinear system is proposed. After analysing and identifying the nonlinear system, a procedure of dynamical balance including measurement and processing of vibration signals, for calculating magnitude and location of imbalance mass is presented. An example of simulation and calculation is investigated for illustration of the method.

1. Introduction

One of the main causes of machinery vibration is imbalance of rotational parts. The dynamical balancing of rotating machines has been developing for new kinds of machines with high speed and improving balancing results more exactly and quickly. The system of machines with its rotational part is usually regarded as a linear system and there are some methods for balancing it. But sometimes it is difficult to find the location and magnitude of imbalance mass of the rotational part. Therefore, the balancing time is lasted so long and makes influence on production time. One of the reasons of this situation is the nonlinearity of the system. In this paper an effort is made for balancing a simple rotational part of machine with non-linear stiffness of the system.

2. The method for balancing rotational part of machine in linear case

The brief of a dynamical balancing method for linear system is explained in the Fig. 1, where m_3 is rotational mass, m_4 and m_t are eccentric and trial ones respectively. In this case the centrifugal force is proportional to according amplitude of vibration which can be measured. Therefore with three times of operating machine, it is able to find the location and magnitude of imbalance mass for balancing the rotational part of machine on the basis of triangle OAB (Fig. 1), here A_0 is vibration amplitude for the case without trial mass and A_1, A_2 are amplitudes for the cases with trial mass.

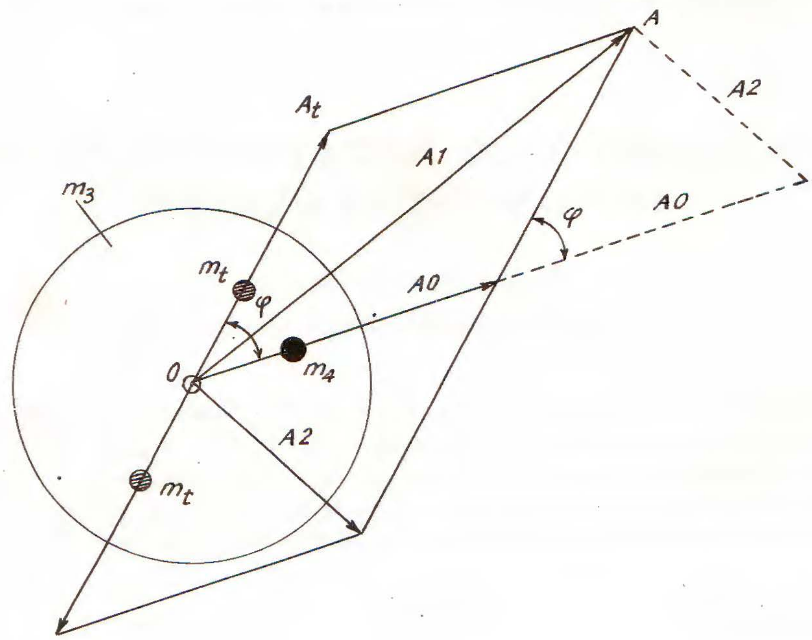

Fig. 1

3. Balancing rotational part on non-linear system

3.1. Equation of motion

Let us investigate the system in Fig. 2

If the system has imbalance mass m_4 with distance r from shaft line and the stiffness of the system is described by function $f(x)$ then the differential equation of motion in the direction of axis X is governed as follows

$$m_3\ddot{x} + h_1\dot{x} + f(x) = m_4 r\omega_1^2 \cos\omega_1 t \tag{3.1}$$

Denoting $\dfrac{h_1}{m_3} = 2h\omega_0$,

$$\frac{f(x)}{m_3} = \omega_0^2 x + g(x), \quad \frac{m_4}{m_3} r = P,$$

we have

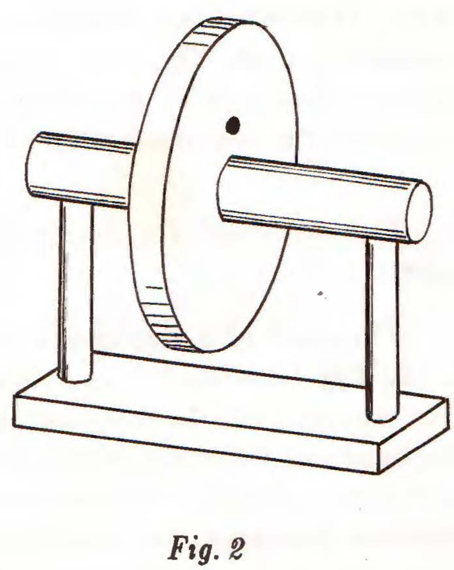

Fig. 2

$$\ddot{x} + 2h\omega_0\dot{x} + \omega_0^2 x + g(x) = P\omega_1^2 \cos\omega_1 t. \tag{3.2}$$

In practice the function $f(x)$ and therefore the function $g(x)$ is symmetric for two sides of vibration with respect to axis X, therefore in mathematical expression the function $f(x)$ is odd, that means

$$f(-x) = -f(x)$$

Then the expand of $g(x)$ is of the form

$$g(x) = b_3x^3 + b_5x^5 + \dots \tag{3.3}$$

Since the vibration of the system is supposed to be small, then we restrict ourself by first term and the equation (2) becomes

$$\ddot{x} + 2h\omega_0\dot{x} + \omega_0^2x + b_3x^3 = P\omega_1^2\cos\omega_1 t, \tag{3.4}$$

where h, ω_0, ω_1 are given constants and b_3 is to be determined below.

3.2. Identification of the system by measurement and calculation of vibration

Suppose that b_3 is small and can be written

$$b_3 = \varepsilon b, \tag{3.5}$$

where ε is a small positive parameter, we can write the equation (3.4) in the form

$$\ddot{x} + 2h\omega_0\dot{x} + \omega_0^2x = P\omega_1^2\cos\omega_1 t - \varepsilon bx^3. \tag{3.6}$$

On the basis of small parameter method [1] the solution of the equation (3.6) can be sought in the form

$$x(t) = x_0(t) + \varepsilon x_1(t) + \varepsilon^2 x_2(t) + \dots \tag{3.7}$$

Substituting (3.7) into (3.6) then balancing the terms with the same order of ε we obtain the following equations system:

$$\ddot{x}_0 + 2h\omega_0\dot{x}_0 + \omega_0^2x_0 = P\omega_1^2\cos\omega_1 t. \tag{3.8}$$
$$\ddot{x}_1 + 2h\omega_0\dot{x}_1 + \omega_0^2x_1 = -bx_0^3 \tag{3.9}$$
$$\ddot{x}_2 + 2h\omega_0\dot{x}_2 + \omega_0^2x_2 = -3bx_0^2x_1 \tag{3.10}$$
$$\dots \quad \dots \quad \dots \quad \dots$$

We are interested in the stationary solutions of equations system (3.8)-(3.10). The stationary solution of Eq. (3.8) is described as follows

$$x_0(t) = \frac{P\omega_1^2}{D_1^{1/2}} \cos\phi_0, \tag{3.11}$$

where

$$D_1 = (\omega_0^2 - \omega_1^2)^2 + 4h^2\omega_0^2\omega_1^2, \quad \phi_0 = \omega_1 t + \varphi_0, \quad \tan\varphi_0 = \frac{2h\omega_0\omega_1}{\omega_1^2 - \omega_0^2} \cdot \tag{3.11}$$

Equation (3.9) becomes

$$\ddot{x}_1 + 2h\omega_0\dot{x}_1 + \omega_0^2 x_1 = -\frac{bP^3\omega_1^6}{D_1^{3/2}} \cos^3\phi_0, \tag{3.13}$$

$$\cos^3\phi_0 = \frac{3}{4}\cos\phi_0 + \frac{1}{4}\cos 3\phi_0, \tag{3.14}$$

and stationary solution of the Eq. (3.13) is

$$x_1(t) = -\frac{3bP^3\omega_1^6}{4D_1^2}\cos(\phi_0 + \varphi_{11}) - \frac{bP^3\omega_1^6}{4D_1^{3/2}D_2^{1/2}}\cos(3\phi_0 + \varphi_{13}), \tag{3.15}$$

where

$$D_2 = (\omega_0^2 - 9\omega_1^2)^2 + 36h^2\omega_0^2\omega_1^2, \quad \tan\varphi_{11} = \tan\varphi_0 \text{ and } \tan\varphi_{13} = \frac{6h\omega_0\omega_1}{9\omega_1^2 - \omega_0^2} \cdot$$

It is similar for $x_2(t)$, which will be consisted of harmonics of first, third and fifth order of frequency ω_1. We restrict ourselves by the first approximation. We have

$$\begin{aligned} x(t) &= x_0(t) + \varepsilon x_1(t) \\ &= \frac{P\omega_1^2}{D_1^{1/2}}\cos\phi_0 - \frac{3P^3\omega_1^6 b_3}{4D_1^2}\cos(\phi_0 + \varphi_{11}) - \frac{b_3P^3\omega_1^6}{4D_1^{3/2}D_2^{1/2}}\cos(3\phi_0 + \varphi_{13}). \end{aligned} \tag{3.16}$$

After some simple transformations the amplitudes of the first and the third order harmonics can be expressed by the following formulae.

$$A_1^2 = \frac{P^2\omega_1^4}{D_1} + \frac{9b_3^2P^6\omega_1^{12}}{16D_1^4} - \frac{3P^4\omega_1^8 b_3(\omega_1^2 - \omega_0^2)}{2D_1^3}, \tag{3.17}$$

$$A_3 = \frac{b_3P^3\omega_1^6}{4D_1^{3/2}D_2^{1/2}} \cdot \tag{3.18}$$

If it is possible to measure the vibration of the system along the axis X and describe it in frequency domain, then it is able to obtain A_1 for frequency ω_1 and A_3 for frequency $3\omega_1$ from measurement data.

Using two equations (3.17) and (3.18) we have the following equation for $q = P\omega_1^2$:

$$q^2 - 6A_3(\omega_1^2 - \omega_0^2)\frac{D_2^{1/2}}{D_1^{1/2}}q + 9A_3^2 D_2 - A_1^2 D_1 = 0. \tag{3.19}$$

From here we obtain

$$q = P\omega_1^2 = 3A_3(\omega_1^2 - \omega_0^2)\frac{D_2^{1/2}}{D_1^{1/2}} \pm \sqrt{\Delta}, \tag{3.20}$$

where $\Delta = 9A_3^2(\omega_1^2 - \omega_2^2)^2 \dfrac{D_2}{D_1} - 9A_3^2 D_2 + A_1^2 D_1$ and from (3.18) we have the value b_3.

3.3. Determination of balancing parameters

Adding a trial mass m_t alternatively at two radial symmetric places of rotational part, measuring vibrations for every case and using similar formula as (3.20) we can calculate the values P_1 and P_2 which are proportional to centrifugal forces.

Draw the triangle, which is similar to triangle OAB on Fig. 1 and allows us to determine the angle φ, that means, the location of imbalance mass m_4. The magnitude of m_4 is calculated by formula

$$m_4 = \frac{m_3 P}{r}. \tag{3.21}$$

We can check it by computing P_t from the expression

$$P_t^2 = \frac{P_1^2 + P_2^2 - 2P^2}{2}, \tag{3.22}$$

$$m_4 = \frac{m_t P}{P_t}, \tag{3.23}$$

and the angle φ can be calculated by the formula

$$\varphi = \arccos\left[\frac{P_1^2 - P_2^2}{4PP_t}\right]. \tag{3.24}$$

3.4. Procedure for balancing rotational part in non-linear system

- Measure three times vibrations of bearing on imbalance system, determine amplitudes at frequency ω_1 and $3\omega_1$ on graphics of vibrations in frequency domain for initial imbalance mass and two times of attachment of the trial mass.

- Determine P, P_1, P_2 for three times of measurement by formula similar to (3.20)

- Calculate P_t and φ from formulas (3.22) and (3.24). Two balancing parameters m_4 and φ are determined.

- Attach the balance mass m_4 (calculated by (3.21)) in the place of radial symmetry with respect to position found by angle φ, the rotational part will be balanced.

4. Results of simulation and calculation

- The equation (3.4) is solved directly by computer programme with given parameters including b_3 and P, and the solution (vibration) and its spectrum can be obtained. We can determine two values A_1 and A_3 (Figs 3-5).

- Using input parameters: ω_1, ω_0, h, A_1, A_3.

That means, it is not necessary to know nonlinear coefficient b_3 and intense of force P, but we can get these values from equations (3,18) and (3.20).

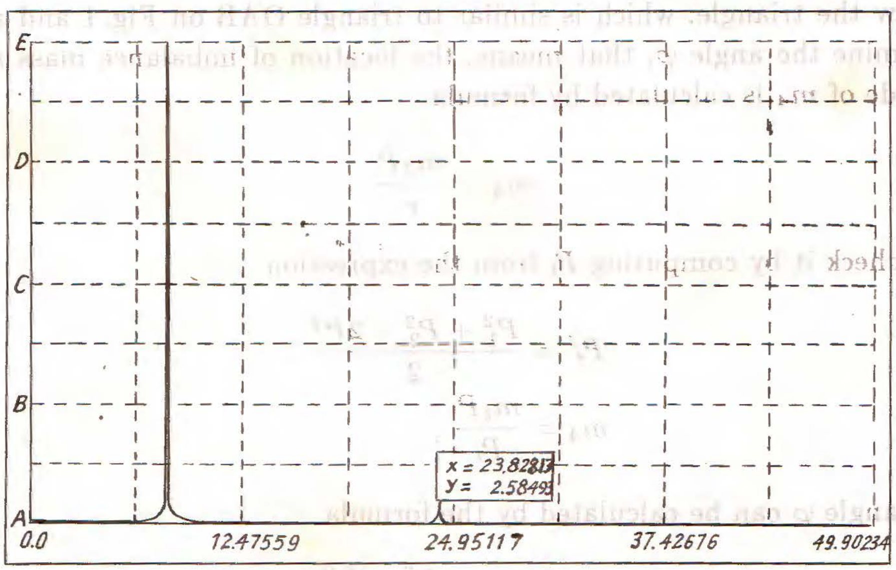

Fig. 3. Vibration and spectrum

$h_1 = 0.029811,\ m_4 = 1.,\ f = 4.74185,\ f_1 = 8.,\ \beta = 0.5,\ r = 0.6$

Ordinate level: E = 56.4559, D = 42.34192, C = 28.22795, B = 14.11397, A = 0.0

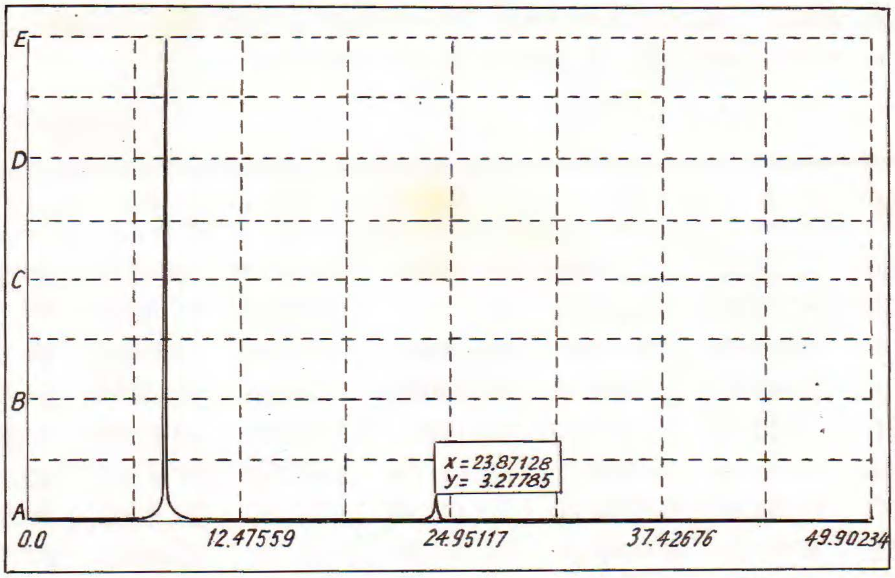

Fig. 4. Vibration and spectrum

$h_1 = 0.02981, \quad m_4 = 1.5, \quad f = 4.74173, \quad f_1 = 8., \quad \beta = 0.5, \quad r = 0.6$

Ordinate level : E = 60.19252, D = 45.14439, C = 30.09626, B = 15.04813, A = 0.0

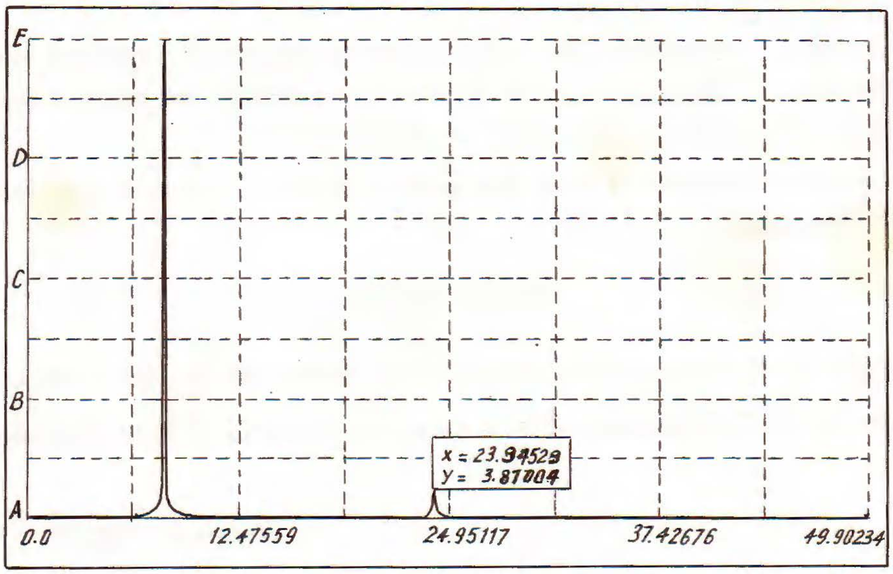

Fig. 5. Vibration and spectrum

$h_1 = 0.029809, \quad m_4 = 2., \quad f = 4.74162, \quad f_1 = 8., \quad \beta = 0.5, \quad r = 0.6$

Ordinate level : E = 63.34972, D = 47.51229, C = 31.67486, B = 15.83743, A = 0.0

- Comparison of the values P_g (calculated by formula $P_g = \dfrac{m_4}{m_3} r$) with computational results P_c (by (3.20)) is given in the Table 1 with amplify coefficient 10^4, in which $\omega_1 = 2\pi f_1$, $\omega_0 = 2\pi f$ and $f_1 = 8$, f and h are calculated from simulation model, $m_3 = 20,000\,\text{kg}$. The values in two final columns are closed each other, that shows the exactitude of the method.

Table 1

m_4	f	h	A_1	A_3	P_g	P_c
2.50	4.74150	0.029809	66.09504	4.55542	75.5727	74.8783
2.25	4.74156	0.029809	64.76664	4.22026	67.6920	67.3913
2.00	4.74162	0.029809	63.34972	3.87782	59.9041	59.6823
1.75	4.74167	0.029810	61.83076	3.54005	52.4168	51.8845
1.50	4.74173	0.029810	60.19252	3.20013	44.9292	44.0758
1.25	4.74179	0.029811	58.41179	2.85299	37.4415	36.1874
1.00	4.74185	0.029811	56.45590	2.51983	29.9536	28.7850
0.75	4.74191	0.029811	54.27776	2.19330	22.4855	21.7034
0.50	4.74196	0.029812	51.80022	1.84879	14.9772	14.3523

5. Conclusions

The method presented in this paper is rather simple for practical application, after computing P_c (final column of Table 1) the results for linear system could be applied and the balancing parameters are found out.

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CÂN BẰNG MÁY QUAY TRONG TRƯỜNG HỢP PHI TUYẾN

Bài này đề xuất một phương pháp cân bằng động máy quay trong trường hợp phi tuyến. Sau khi phân tích và nhận dạng hệ phi tuyến, một quy trình cân bằng động bao gồm đo dao động, xử lý tín hiệu và dùng các tham số để tính toán độ lớn và vị trí khối lượng mất cân bằng đã được trình bày. Một ví dụ về mô phỏng và tính toán đã được thực hiện để minh họa cho phương pháp.