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AN ALGORITHM FOR SIMULTANEOUS DETERMINATION OF PRESSURE AND TEMPERATURE DISTRIBUTION IN OIL-PRODUCING TUBE FLOW AND THE COUNTER FLOW IN GAS-LIFT ANNULUS

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ABSTRACT. An algorithm for simultaneous determination of pressure and temperature distribution in oil-producing tube flow and the counter flow in gas-lift annulus has been proposed. The algorithm is based on a double loop iterative procedure and shooting method. The two-phase flow calculation for pressure used in the algorithm is based on improved methods of Beggs and Brills, Orkiszewski or Hagedorn and Brown. The numerical implementation for an example is presented to assure the applicability of proposed algorithm.

1. Introduction

The problem of accurately predicting pressure drops in oil-producing well has been developed in the last 50 years. In general, heat transfer calculations were not been considered, and the temperature distribution was usually assumed to be linear between the surface temperature and bottom hole temperature. In many cases this assumption will not introduce significant errors. Nevertheless, the heat transfer calculations can be very important when the liquid rate of a well is changed, in gas-lift-injection wells, flow of high paraffin oil, etc.

Various algorithms for coupling pressure and heat losses calculation in oil-producing well were presented in [1, 2]. These algorithms have been performed without considering the case of counter flow of gas in gas-lift annulus.

This paper presents an algorithm for simultaneous calculation of pressure and heat losses in oil-producing tube as well as in gas-lift annulus. Assuming the liquid production rate and gas injection rate are known, the pressure and temperature distributions in the producing tube and gas-lift annulus can be determined if a given set of boundary conditions is sufficient, i.e. the set of conditions on the surface or in the reservoir. For example, the well-head pressure, well-bottom temperature, injection depth and the surface temperature of injection gas make a sufficient set of boundary conditions. In this case, the most important quantities need to be determined are the well-bottom pressure, well-head temperature, surface injection pressure, and the temperature and pressure at the injection point, i.e. at the injection valve.

At first, the basic equations that describe the flow and heat exchange in producing tube and in the gas-lift annulus are presented, and the boundary conditions will be discussed. The effects of the phase transition such as paraffin crystallisation and vaporisation are taken into account in the pressure equations as well as in the temperature equations.

Next, the algorithm for one case of the boundary conditions will be presented. Some known methods such as Beggs and Brills method[1, 3], Orkiszewski method[1, 4], and Hagerdorn and Brown[1] method are improved and used in the algorithm for solution of two-phase flow equations in the producing tube.

At the end, some results of calculation for one producing well are presented and conclusions are given.

2. Basic equations and boundary conditions

Figure 1 presents a schematic diagram of producing well with gas-lift injection.

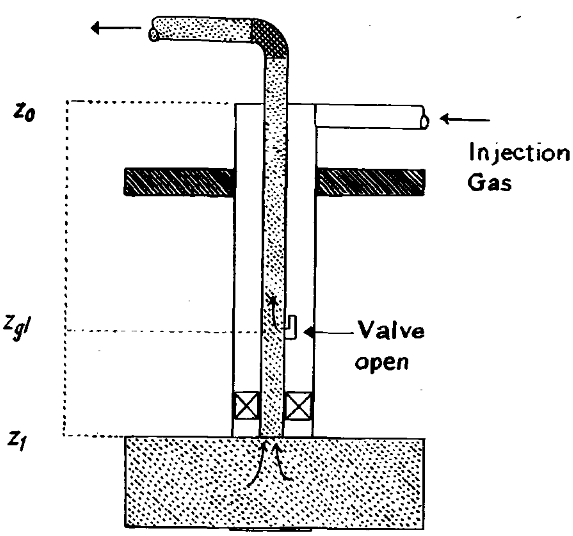
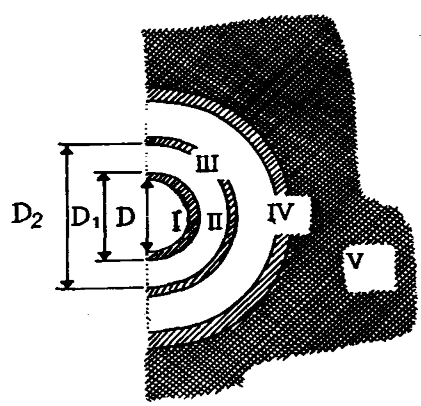


Fig. 1. Schematic diagram of a oil-producing well



I. Producing tube
II. Gas-lift annulus
III. First casing
IV. Second casing
V. Ground

Fig. 2. Well cross-section

Gas is continuously injected into tube through a gas-lift valve at a fixed depth $z_{g\ell}$. The injected gas is conducted to the valve through an annulus, which is modelled by equivalent circular pipe with hydraulic and thermal equivalent diameters. The flow in the producing tube is a multi-phase flow and the gas-liquid ratio is increased from the valve to the surface.

Under the steady conditions, the equations for pressure and temperature calculation in producing tube and in gas-lift annulus can be written as presented below.

2.1. Pressure and temperature equations in producing tube

a. *Pressure equation*

The total pressure gradient of fluid flowing in a pipe inclined at a given angle θ from horizontal, can be considered to be composed of three distinct components, that is [1]:

$$\frac{dp_O}{dz} = \left(\frac{dp_O}{dz}\right)_{el} + \left(\frac{dp_O}{dz}\right)_f + \left(\frac{dp_O}{dz}\right)_{acc} \tag{2.1}$$

where $\left(\frac{dp_O}{dz}\right)_{el} = g\rho_{tp}\sin\theta$ is the component due to potential energy or elevation change (hydrostatic component); $\left(\frac{dp_O}{dz}\right)_f = \frac{f_{tp}\rho_f v_{mf}^2}{2D} f_{corr}$ is the component due to friction losses. $\left(\frac{dp_O}{dz}\right)_{acc} = \frac{\rho v dv}{dz}$ is the component due to kinetic energy change or convective acceleration (This component is negligible for most practical cases). And under several simplifying assumptions it can be expressed as following[3], [4]:

$$\left(\frac{dp_O}{dz}\right)_{acc} = -\frac{\rho_{tp} v_m v_{sg}}{p_O} \frac{dp_O}{dz}$$

The quantities ρ_{tp}, ρ_f, v_{mf} and f_{corr} are determined dependently by flow pattern and method of calculations (see Appendix A).

b. *Temperature equation*

The total temperature gradient in producing tube can be considered to be composed of five distinct components, that is [2]:

$$\frac{dt_O}{dz} = \left(\frac{dt_O}{dz}\right)_p + \left(\frac{dt_O}{dz}\right)_{ex} + \left(\frac{dt_O}{dz}\right)_{Lg} + \left(\frac{dt_O}{dz}\right)_{Lf} + \left(\frac{dt_O}{dz}\right)_f \tag{2.2}$$

where

$$\left(\frac{dt_O}{dz}\right)_p = \frac{\left[\alpha_p \lambda_L T_O + (1-\lambda_L)(1+\eta_p T_O)\right]}{c_{pw}\rho_n} \frac{dp_O}{dz}$$

is the component due to expansion of the gas and the liquid.

$$\left(\frac{dt_0}{dz}\right)_{ex} = \frac{\pi D k_\tau}{w_t c_{pw}}(t_0 - t_{ex})$$

is the heat loss to surroundings;

$$t_{ex} = \begin{cases} t_g & \text{for } z < z_{gl} \\ t_{gr} & \text{for } z \geq z_{gl} \end{cases}$$

$$\left(\frac{dt_0}{dz}\right)_{Lg} = -L_g \frac{d}{dz}\left(\frac{(1-\lambda_L)\rho_g}{c_{pw}\rho_n}\right)$$

is the component due to vaporisation.

$$\left(\frac{dt_0}{dz}\right)_{Lf} = L_f \frac{d}{dz}\left(\frac{\lambda_L \varepsilon_p}{c_{pw}\rho_n}\right)$$

is the component due to paraffin crystallisation.

$$\left(\frac{dt_0}{dz}\right)_f = \frac{f_{tp} w_t v_m}{2DC c_{pw}\rho_n}$$

is the component concerned friction losses.

k_τ is the overall heat transfer coefficient, which can be determined as presented in Appendix B.

2.2. Pressure and temperature equations in gas-lift annulus

The flow of gas in the gas-lift annulus can be modelled by flow of gas in a circular pipe with a hydrodynamic equivalent diameter used for calculation of the friction component and with a thermal equivalent diameter used for calculation of overall heat transfer coefficient. According to these assumptions, the equations for pressure and temperature in the gas-lift annulus can be written as following [2]:

a. *Pressure equation*

$$w^2 \frac{1+\eta_p T_g}{T_g} \frac{dt_g}{dz} + \left(\frac{1}{\rho_g} - w^2 \frac{1-\xi_T}{P_g}\right)\frac{dp_g}{dz} = f_g \frac{w^2}{2(D_2 - D_1)} + g\sin\theta \qquad (2.3)$$

b. *Temperature equation*

$$c_{pg}\rho_g \frac{dt_g}{dz} - (1+\eta_T T_g)\frac{dp_g}{dz} = -\frac{\pi D_2 \rho_g k_{\tau g}}{G_g}(t_g - t_{gr}) - \frac{\pi D \rho_g k_\tau}{G_g}(t_g - t_0) + f_g \frac{\rho_g w^2}{2(D_2 - D_1)} \qquad (2.4)$$

where

$$\rho_g = \frac{Mp_g}{848 z_g T_g}, \quad w = \frac{4G_g}{\pi(D_2^2 - D_1^2)\rho_g}, \quad \eta_p = \frac{1}{z_g}\left(\frac{\partial z_g}{\partial T}\right), \quad \xi_T = \frac{1}{z_g}\left(\frac{\partial z_g}{\partial p}\right)_T. \tag{2.5}$$

The equivalent diameters for determination of f_g and k_{T_g} are:

$$D_{heq} = D_2 - D_1 \tag{2.6}$$

and

$$D_{teq} = \frac{D_2^2 - D_1^2}{2D_2} \tag{2.7}$$

respectively.

The determination of overall heat transfer coefficient k_{T_g} is presented in Appendix B.

2.3. Conditions at gas-lift valve

In steady condition a valve can be considered as a pipe with three equivalent parameters: the equivalent length L_{eq}; the equivalent diameter D_{eq} and the equivalent cross-section area A_{eq}.

The pressure difference between the annulus and producing tube at injection point can be calculated according to following expression:

$$\Delta p = -f_g \frac{\rho_g w^2}{2D_{eq}} L_{eq} \tag{2.8}$$

where $w = \dfrac{G_g}{A_{eq}\rho_g}$.

The temperature of flowing liquid in the tube at the injection point may be considered as discontinuous and can be determined by the relation:

$$t_{0+} = \frac{t_{0-} c_{pw} w_t + t_g c_{pg} G_g}{c_{pw} w_t + c_{pg} G_g} \tag{2.9}$$

where t_{0+} and t_{0-} are the temperatures of the fluid above and under the valve respectively.

2.4. Boundary conditions

Assuming the producing liquid rate and gas injection rate are known, the sufficient boundary conditions for determination of pressure and temperature distribution in producing tube and in gas-lift annulus can be formulated as following:

• for the producing tube: pressure and temperature either at the inlet or at the outlet of the tube must be required. It is not necessary that pressure and temperature must be known at the same end of the tube. There are, therefore, four possibilities of boundary conditions for this case:

1. Pressure and temperature are known at the top (outlet) of producing tube
2. Pressure and temperature are known at the bottom (inlet) of producing tube
3. Pressure at the top (outlet) and temperature at the bottom (inlet) of producing tube are known
4. Pressure at the bottom (inlet) and temperature at the top (outlet) of producing tube are known

• for gas-lift annulus: the temperature of injected gas at the surface must be required. Since the gas injection rate and the characteristic parameters of valve are given, the pressure of injected gas at the surface is, therefore, dependent on the position of the valve. If the pressure at surface is known then the position of the valve can be determined, and in reverse, if the position of valve is given then the pressure at surface can be determined. For the gas-lift annulus are, therefore, two possibilities:

1. The temperature and pressure of injected gas at surface are given
2. The temperature of injected gas at surface and position of the valve are given

The combination of the conditions for the producing tube and for gas-lift annulus makes eight sets of sufficient boundary conditions, which is summarised in the Table 1.

Table 1. Eight sets of boundary conditions

Set	Producing tube				Gas-lift annulus		
number	at Top		at Bottom		at Surface		Valve
	temp.	press.	temp.	press.	temp.	press.	position
1		known	known		known		known
2		known	known		known	known	
3	known	known			known		known
4	known	known			known	known	
5	known			known	known		known
6	known			known	known	known	
7			known	known	known		known
8			known	known	known	known	

In practice, the condition of imposing the temperature at the top of producing tube is very sensible condition. That means a little change in the temperature in the top causes a magnifical change of temperature at the bottom. Otherwise, the temperature at the top is strongly dependent on the liquid rate, so the condition of imposing this temperature is not commonly advised. Hence, the boundary conditions set 3-6 can be rejected from consideration.

The remained problems with the boundary conditions set 1, 2, 7 or 8 have practical meanings and are worth for consideration.

Now the procedure for solution of the problem with the boundary conditions 1 will be presented below. The problems with the sets 2, 7 or 8 may be treated similarly.

3. Procedure of solution

3.1. Mathematical formulation

The considered problem can be written in the form of a system of non-linear ordinary differential equations as following:

$$\left.\begin{aligned} \frac{dp_0}{dz} &= F_{p_0}(t_0, p_0) \\ \frac{dt_0}{dz} &= F_{t_0}\left(t_0, p_0, t_g, \frac{dp_0}{dz}\right) \end{aligned}\right\} \quad \text{for} \quad z_0 < z < z_\ell \tag{3.1}$$

$$\left.\begin{aligned} \frac{dp_g}{dz} &= F_{p_g}\left(t_g, p_g, \frac{dt_g}{dz}\right) \\ \frac{dt_g}{dz} &= F_{t_g}\left(t_g, p_g, \frac{dp_g}{dz}, t_0\right) \end{aligned}\right\} \quad \text{for} \quad z_0 < z < z_{g\ell} \tag{3.2}$$

with the boundary conditions:

$$\begin{aligned} p_0(z_0) &= p_{0z0}, \\ t_0(z_\ell) &= t_{0z\ell}, \\ t_g(z_0) &= t_{0z0} \end{aligned} \tag{3.3}$$

and the condition at injection point $z = z_{g\ell}$

$$\begin{aligned} p_g - p_0 &= F_{val}(t_g, p_g, p_0) \\ t_{0+} &= \frac{t_0 - c_{pw} w_t + t_g c_{pg} G_g}{c_{pw} w_t + c_{pg} G_g} \end{aligned} \tag{3.4}$$

where F_{p_0}, F_{t_0}, F_{p_g}, F_{t_g} and F_{val} are no-linear functions determined by (2.1), (2.2), (2.3), (2.4) and (2.8) respectively.

3.2. Finite difference formulation

Dicretize the z variable by $z_0, z_1, \ldots, z_n$ such that $z_0 = z_0$, $z_n = z_\ell$ and $\Delta z = z_{i+1} - z_i$ for $i = 0, 1, \ldots, n-1$.

An approximation to the function θ at the point z_i will be denoted by:

$$\theta_i = \theta(z_i) \quad \text{and} \quad \theta_{i+1/2} = \frac{1}{2}(\theta_{i+1} + \theta_i). \tag{3.5}$$

The equations (3.1)-(3.2) approximated implicitly by backward Euler approximations will get the form:

$$\frac{p_{0i+1} - p_{0i}}{\Delta z} = F_{p_0}(t_{0i+1/2}, p_{0i+1/2}) \tag{3.6}$$

$$\frac{t_{0i+1} - t_{0i}}{\Delta z} = F_{t_0}\left(t_{0i+1/2}, p_{0i+1/2}, t_{gi+1/2}, \frac{p_{0i+1} - p_{0i}}{\Delta z}\right) \tag{3.7}$$

$$\frac{p_{gi+1} - p_{gi}}{\Delta z} = F_{p_g}\left(t_{gi+1/2}, p_{gi+1/2}, \frac{t_{gi+1} - t_{gi}}{\Delta z}\right) \tag{3.8}$$

$$\frac{t_{gi+1} - t_{gi}}{\Delta z} = F_{t_g}\left(t_{gi+1/2}, p_{gi+1/2}, \frac{P_{gi+1} - p_{gi}}{\Delta z}, t_{0i+1/2}\right) \tag{3.9}$$

The finite difference equations (3.6)-(3.9) are strongly non-linear. And an iterative procedure will be used for solving this system of non-linear equations.

3.3. The iterative procedure

The iterative procedure with double loop is used for solving the system of non-linear finite difference equations (3.6)-(3.9). The inner loop is used for pressure calculations (with the iterative index ℓ), and the outer loop is used for temperature calculations (with the iterative index k).

Assuming the values of temperature and pressure are known in k-th iterative step at point z_{i+1}, that means the values $p_{0i+1}^{(k)}$, $t_{0i+1}^{(k)}$, $p_{gi+1}^{(k)}$, $t_{gi+1}^{(k)}$ are known. The inner iterative loop with index ℓ allows to determine the pressure values in the following way:

Set $p_{0i+1}^{k,0} = p_{0i+1}^{(k)}$, $p_{gi+1}^{k,0} = p_{gi+1}^{(k)}$. The values of pressure in $\ell+1$-th iterative step are obtained from the equations:

$$p_{0i+1}^{k,\ell+1} = p_{0i} + \Delta z F_{p_0}\left(t_{0i+1/2}^{(k)}, p_{oi+1/2}^{k,\ell}\right), \tag{3.10}$$

$$p_{gi+1}^{k,j+1} = p_{gi} + \Delta z F_{p_g}\left(t_{gi+1/2}^{(k)}, p_{gi+1/2}^{k,\ell}, \frac{t_{gi+1}^{(k)} - t_{gi}^{(k)}}{\Delta z}\right). \tag{3.11}$$

The convergence of this loop gives the values: $\bar{p}_{0i+1}^{(k)}$ and $\bar{p}_{gi+1}^{(k)}$, which will be used for the outer loop.

Similarly, for the outer loop, the values of temperature in the $k+1$-th step can be calculated from the equations:

$$t_{0i+1}^{(k+1)} = t_{0i} + \Delta z F_{t_0}\left(t_{0i+1/2}, \overline{p}_{0i+1/2}^{(k)}, t_{gi+1/2}^{(k)}, \frac{\overline{p}_{0i+1}^{(k)} - p_{0i}}{\Delta z}\right) \tag{3.12}$$

$$t_{gi+1}^{(k+1)} = t_{gi} + \Delta z F_{t_g}\left(t_{gi+1/2}^{(k)}, \overline{p}_{gi+1/2}^{(k)}, \frac{\overline{p}_{gi+1}^{(k)} - p_{gi}}{\Delta z}, t_{0i+1/2}^{(k)}\right). \tag{3.13}$$

The convergence of this loop gives the solution at point z_{i+1}:

$$p_{0i+1} = \overline{p}_{0i+1}^{(k)}, \quad p_{gi+1} = \overline{p}_{gi+1}^{(k)}$$
$$t_{0i+1} = t_{0i+1}^{(k+1)}, \quad t_{gi+1} = t_{gi+1}^{(k+1)}$$

3.4. Using the shooting method

The iterative algorithm described above uses the values of point z_i to calculate the values of point z_{i+1}. While the boundary conditions sets 1, 2 or 7, 8 do not give the conditions at the same point (at the top or bottom of the tube). Some boundary conditions are, therefore, to be assumed to start the calculation loops, and the shooting method has to be used for getting the given boundary conditions. For the boundary conditions set 2, the temperature on the top of producing tube and the injection pressure at the surface are to be assumed, then the temperature at bottom of producing tube and the position of the gas-lift valve can be determined. And an iterative procedure like the shooting method [6] will be used for matching the given temperature at bottom and given position of the valve.

4. Example of calculation

The algorithm described above is realized in a computer program and applied for one producing well with gas-lift injection. The main characteristic parameters of the well are given below:

Oil rate :	72.02(m^3/day)
Gas rate:	12243(m^3/day)
Gas injection rate:	6446(m^3/day)
Depth of gas-lift valve:	3600(m)
Pressure at top of producing tube:	4(atm)
Temperature of liquid at bottom:	137.8(°C)
Temperature of injected gas at surface:	28(°C)
Length of producing tube:	3850(m)

The results of calculation are presented in figure 3. The calculated values at boundaries are good in comparison with the measured data:

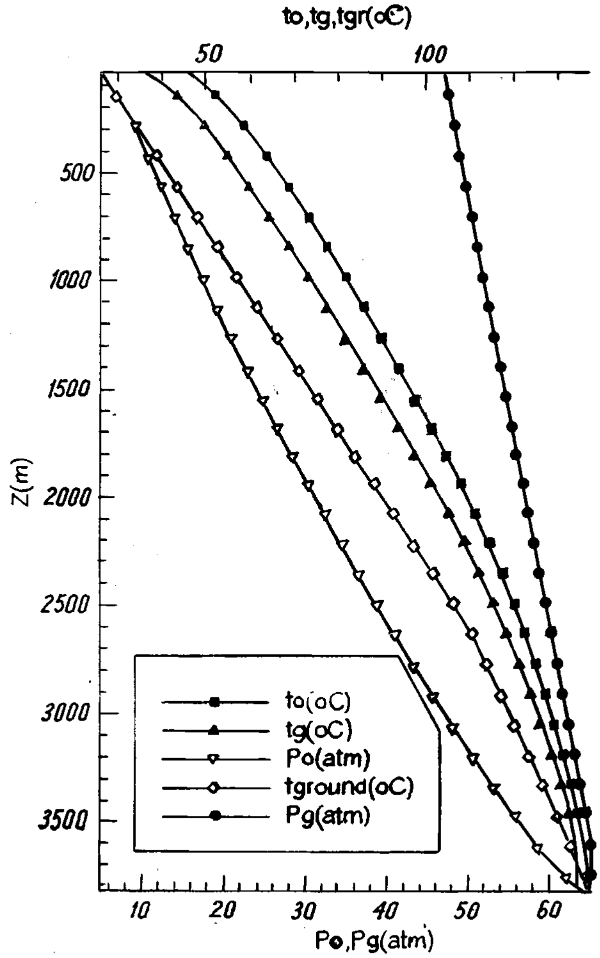


Fig. 3. Pressure and temperature distributions of the tube and annulus

	Measured	Calculated	Error(%)
Pressure at bottom of prod. tube (atm)	66.64	63.2	5.16
Temp. at top of prod. tube (°C)	39	42.57	-9.15

The pressure and temperature at which the gas-lift valve is operating can be determined:

Temp. at operating gas-lift valve 134.4 (°C)
Press. at operating gas-lift valve 65.03(atm)
This information is important for gas-lift design of the well.

The computer program allows to analyse the contributions of each component in pressure equation (2.1), and the temperature equation (2.2) at each point of the producing tube. The contributions of components change significally along the tube. And it can be seen in the figure 4 and 5

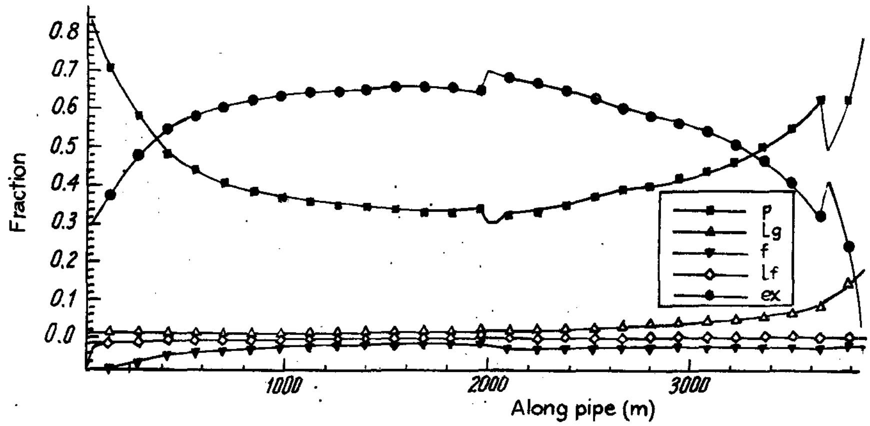


Fig. 4. Component fraction of total temperature change

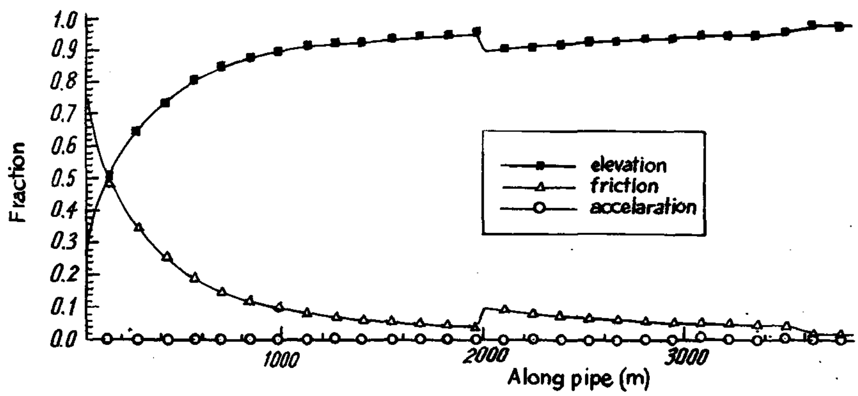


Fig. 5. Component fraction of total pressure change

5. Conclusion

The algorithm for solving simultaneously the system of equations governing the hydrodynamic and thermal processes in producing tube and gas-lift annulus was presented. The algorithm was based on double loop iterative procedure to solve the system of non-linear finite difference equations. An iterative procedure based on shooting method was been also used for the case of complicated boundary conditions.

The algorithm was implemented by a computer program on PC, and the results of calculations of an example show that it can be used for multi-purposes analysis of oil producing well.

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Nomenclature

A	area	m^2
c_p	specific heat	J/kg
c_{wp}	effective specific heat of flowing fluid in tube	J/kg
c_{pg}	specific heat of injected gas	J/kg
D	inner diameter of proc. tube	m
D_1	outer diameter of proc. tube	m
D_2	inner diameter of first casing	m
f_n	friction factor	
g	acceleration of gravity	m/sec^2
G_g	mass rate of injected gas	kg/m^3
H_L	liquid holdup	
k_r	overall heat transfer coeff. for producing tube	$w/m°C$
k_{r_g}	overall heat transfer coeff. for gas-lift annulus	$w/m°C$
L_g	latent heat of vaporisation	J/kg
L_f	latent heat of paraffin crystallisation	J/kg
M	molecule weight	kg
p	pressure	Pas
p_0	pressure of iquid in proc. tube	Pas
p_g	pressure of injected gas	Pas
q_g	gas volumetric rate	m^3/sec
q_0	oil volumetric rate	m^3/sec
q_t	total volumetric rate	m^3/sec
r_i	inner radius m ro outer radius	m
t_0	temperature of liquid in proc. tube	°C
t_g	temperature of gas in gas-lift annulus	°C
v	velocity	m/sec
v_{sg}	superficial gas velocity	m/sec
v_{s0}	superficial oil velocity	m/sec
v_{sL}	superficial liquid velocity	m/sec
v_m	total superficial velocity	m/sec
w	gas velocity in gas-lift annulus	m/sec
w_t	total mass velocity	kg/sec
z	elevation referred to datum	m
z_g	Z-factor of gas	

ε_p	paraffin content in fluid	kg/m^3
λ_g	gas void fraction (non-slip)	
λ_L	liquid holdup (non-slip)	
μ	viscosity	Pas.sec
μ_g	gas viscosity	Pas.sec
μ_O	oil viscosity	Pas.sec
μ_L	effective liquid viscosity	Pas.sec
ρ	density	kg/m^3
ρ_g	gas density	kg/m^3
ρ_O	oil density	kg/m^3
ρ_L	effective liquid density	kg/m^3
ρ_n	non-slip two-phase density	kg/m^3
ρ_m	two-phase density	kg/m^3

Appendix A. Determination of two-phase flow parameters

The values of quantities in equation (2.1), i.e. the value of function $F_{p_O}(t_O, p_O)$ can be determined by known methods based on empirical correlations for liquid holdup H_L and friction factor f_{tp} such as Beggs and Brills method, Orkiszewski method and Hagerdorn and Brown method [1], [3], [4].

1. Determination of friction component

$$\left(\frac{dp_O}{dz}\right)_f = \frac{f_{tp}\rho_f v_{mf}^2}{2D} f_{corr}$$

- Beggs and Brill method

1. $\rho_f = \rho_n = \rho_L\lambda_L + \rho_g\lambda_g$
2. $f_{tp} = e^s f_n;\ \ f_n = f_n(N_{Re});\ \ N_{Re} = \dfrac{\rho_n v_m D}{\mu_n}$
3. $v_{mf} = v_{sL} + v_{sg} = v_m$
4. $f_{corr} = 1.0$

where s is a correlation factor (see [3]).

- Orkiszewski method
  * for bubble flow

1. $\rho_f = \rho_L$
2. $f_{tp} = f_n;\ \ f_n = f_n(N_{Re});\ \ N_{Re} = \dfrac{\rho_L v_L D}{\mu_L}$
3. $v_{mf} = \dfrac{q_L}{A(1-F_g)}$
4. $f_{corr} = 1.0$

where F_g is the void fraction of gas.

* for slug flow

1. $\rho_f = \rho_L$
2. $f_{tp} = f_n;\ f_n = f_n(N_{Re});\ N_{Re} = \dfrac{\rho_L v_m D}{\mu_L}$
3. $v_{mf} = v_m$
4. $f_{corr} = \dfrac{q_L + v_b A}{q_t + v_b A} + \Gamma$

where Γ is the liquid distribution coefficient and v_b is the rise-bubble velocity, which can be determined by correlations (see [4]).

* for mist flow

1. $\rho_f = \rho_g$
2. $f_{tp} = f_n;\ f_n = f_n(N_{Re});\ N_{Re} = \dfrac{\rho_g v_{sg} D}{\mu_g}$
3. $v_{mf} = v_{sg}$
4. $f_{corr} = 1.0$

* transition flow

$$\left(\frac{dp}{dz}\right)_f = S\left[\left(\frac{dp}{dz}\right)_f\right]_{slug} + M\left[\left(\frac{dp}{dz}\right)_f\right]_{mist}$$

where the weighting factors S, M are obtained by correlations (see [4])

- Hagerdorn and Brown method

1. $\rho_f = \dfrac{\rho_n^2}{\rho_{sm}};\ \rho_s = \rho_L H_L + \rho_g(1 - H_L)$
2. $f_{tp} = f_n;\ f_n = f_n(N_{Re});\ N_{Re} = \dfrac{\rho_n v_m D}{\mu_s};\ \mu_s = (\mu_L)^{H_L}(\mu_g)^{1-H_L}$
3. $v_{mf} = v_m$
4. $f_{corr} = 1.0$

2. Determination of elevation component

$$\left(\frac{dp_0}{dz}\right)_{el} = g\rho_{tp}\sin\theta$$

- Beggs and Brill method

$$\rho_{tp} = \rho_s$$

- Orkiszewski method

* for bubble flow

$$\rho_{tp} = (1 - F_g)\rho_L + F_g\rho_g$$

* for slug flow

$$\rho_{tp} = \rho_L\left(\frac{q_L + v_b A}{q_t + v_b A} + \Gamma\right) + \rho_g\left(\frac{q_g}{q_t + v_b A}\right)$$

* for mist flow

$$\rho_{tp} = (1 - F_g)\rho_L + F_g\rho_g$$

* transition flow

$$\left(\frac{dp}{dz}\right)_{el} = S\left[\left(\frac{dp}{dz}\right)_{el}\right]_{slug} + M\left[\left(\frac{dp}{dz}\right)_{el}\right]_{mist}$$

where the weighting factors S, M are obtained by **correlations (see [4])**

- Hagerdorn and Brown method

$$\rho_{tp} = \rho_s$$

3. Determination of acceleration component

$$\left(\frac{dp_0}{dz}\right)_{acc} = -\frac{\rho_{tp} v_m v_{sg}}{p_0}\frac{dp_0}{dz}$$

- Beggs and Brill method

$$\rho_{tp} = \rho_s$$

- Orkiszewski method

ρ_{tp} is determined in the same way as for the elevation component.

- Hagerdorn and Brown method

$$\left(\frac{dp_0}{dz}\right)_{acc} = 0$$

Appendix B. Determination of overall heat transfer coefficient

The overall heat transfer coefficient of a fluid in a tube can be a combination of three components. They are convective heat losses between the flowing fluids and the tube wall, conductive losses through the wall and through any insulation or coating material, and conductive losses to the environment. For a buried tube, the overall heat transfer coefficient U can be expressed in the form:

$$U = \frac{1}{R_g + R_p + R_f}$$

where R_g = resistance to conductive heat transfer from the tube to the ground, R_p = resistance to conductive heat transfer through the tube wall and coatings, R_f = resistance to convective heat transfer between the flowing fluid and the tube wall.

1. For the gas-lift annulus

$$k_{\tau g} = \frac{1}{R_g + R_p + R_f}$$

a. $R_g = \dfrac{f(t)}{2k_g D_2}$ where $f(t)$ is a time-dependent, dimensionless function. This function for intervals longer than a week can be expressed as:

$$f(t) = -\ln\left(\frac{r_c}{2\sqrt{a_\infty t}}\right) - 0.29$$

r_c = outer radius of casing; a_α = thermal diffusivity of the earth; t = time since well began flowing.

b. $R_f = \dfrac{D_2}{k_{fg} Nu}$ where $Nu = 0.027 Re^{0.8}\ Pr^{1/3}$ = Nusselt number; and Reynold number Re is calculated based on D_{teq} from (2.7)

c. $R_p = \sum R_{p(i)}$ where $R_{p(i)} = \dfrac{\ln(r_{O(i)}/r_{i(i)})}{2k_{p(i)}/D_2}$

2. for the section of producing tube below the injection point (where $z > z_{gl}$)

The calculation of k_τ is similar to the case of gas-lift annulus. But the outer diameter D_2 and the equivalent thermal diameter D_{teq} of gas-lift annulus must be replaced by the inner diameter D of producing tube.

3. for the section of producing tube above the injection point (where $z < z_{gl}$)

$$k_\tau = \frac{1}{R_g + R_p + R_f}$$

a. $R_g = 0$

b. $R_p = \dfrac{\ln(r_O/r_i)}{2k_p/D}$

c. $R_f = R_{ftug} + R_{fgsl}$

where R_{ftub} = resistance to convective heat transfer between the flowing fluid and the producing tube wall with Re based on inner diameter of the tube, R_{fgsl} = resistance to convective heat transfer between the flowing injected gas and the producing tube wall with Re based on thermal equivalent diameter.

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MỘT THUẬT TOÁN XÁC ĐỊNH ĐỒNG THỜI PHÂN BỐ ÁP SUẤT VÀ NHIỆT ĐỘ TRONG ỐNG KHAI THÁC DẦU VÀ TRONG VÀNH KHUYÊN GASLIFT

Bài báo đưa ra một thuật toán xác định đồng thời phân bố áp suất và nhiệt độ dòng chảy trong ống khai thác dầu và dòng chảy ngược của khí trong vành khuyên gaslift. Thuật toán dựa trên quy trình lặp kép và phương pháp "bắn". Việc tính toán áp suất của dòng chảy hai pha trong thuật toán dựa trên các phương pháp Beggs và Brills, Orkiszewski hoặc phương pháp Hagedorn và Brown đã được cải tiến. Một ví dụ tính toán số được trình bày để chứng tỏ khả năng áp dụng của thuật toán vào trong thực tế.

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