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STABILITY OF VISCOELASTIC PLATES IN COMBINED FORCES

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1. Introduction

The stability of elastic plate in combined forces is investigated [1]. For the viscoelastic plate the problem becomes more complicated because of taking into account time effect. The stability of the plate falls with increasing time. When the viscoelastic plate is subjected to many forces simultaneously it is necessary to find a moment (time) the plate lost stability (critical time). In order to solve this problem the authors have used the theory of pseudo-bifurcation points and method "elastic analogy" [2] that allows to use the result of the problem of elastic stability in solving the problem of creep stability.

2. Construction of "elastic analogy"

Let us rewrite the equation of state of viscoelastic material in the form for "stimulus" $\Delta\sigma$, Δe

$$\Delta e(t) = \frac{\Delta\sigma(t)}{E} - \int\limits_0^t \Delta\sigma(t_1)K(t,t_1)dt_1, \tag{2.1}$$

where $\Delta\sigma = \sigma - \sigma^0$, $\Delta e = e - e^0$, σ^0, e^0 - stress and strain in the basis state, σ, e - stress and strain in the adjacent state.

Expanding the function $\Delta\sigma(t_1)$ into a series in the neighbourhood of $t_1 = t$ and using the definition of pseudo - bifurcation of N - degree (PBN: $\Delta \overset{(N)}{\sigma} \neq 0$; $\Delta \overset{(N)}{e} \neq 0$; $\Delta \overset{(K)}{\sigma} = 0$; $\Delta \overset{(K)}{e} = 0$; $K = 0, 1, \ldots, M$, $K \neq N$, $N \leq M$) after transformations, we obtain "elastic analogy"

$$\Delta \overset{(N)}{\sigma} = \widetilde{E}_N \Delta \overset{(N)}{e}. \tag{2.2}$$

where

$$\widetilde{E}_N = E\left[1 - \frac{E}{N!}\int_0^t (t_1 - t)^N \overset{(N)}{K}(t, t_1)dt_1\right]^{-1}. \tag{2.3}$$

We find that (2.2) is similar in form to Hooke's law, where fictive modulus of elasticity $\widetilde{E}_N$ depends on time.

3. Viscoelastic plate compressed in two directions

Let us consider the elastic stability of rectangular plate $(a \times b)$ compressed in two directions, when σ_1, σ_2 are small the equilibrium of plate is stable (Fig. 1). We have the domain OACB - the stable domain (Fig. 2). When σ_1, σ_2 are big enough the plate is not stable and we have a unstable domain. The curve ACB called boundary curve.

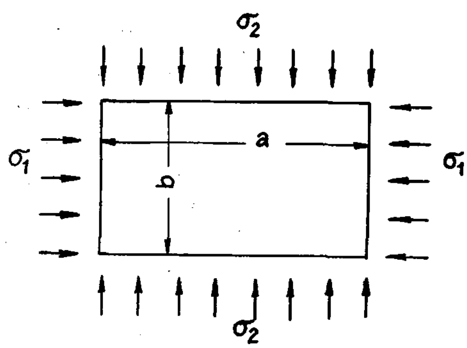

Fig. 1

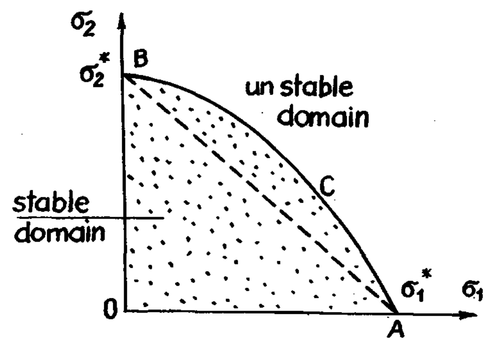

Fig. 2

Replacing the curve ACB by the straight line AB allows to determine the stable domain OAB - the lower bound of the real stable domain (as stable domain is a convex domain after the Papcovic's theorem).

The equation of the line AB can by written in the form

$$\frac{\sigma_1}{\sigma_1^*} + \frac{\sigma_2}{\sigma_2^*} = 1; \tag{3.1}$$

where σ_1^*, σ_2^* - called simple critical stresses

$$\sigma_1^* = k_1 \frac{\pi^2 D}{b^2 h}, \quad \sigma_2^* = k_2 \frac{\pi^2 D}{b^2 h}, \tag{3.2}$$

and the modulus

$$D = \frac{Eh^3}{12(1 - \nu^2)}; \tag{3.3}$$

k_1, k_2 determined by using the table [1].

Denoting the left - hand side of (3.1) by ω, we get

$$\omega = \frac{\sigma_1}{\sigma_1^*} + \frac{\sigma_2}{\sigma_2^*} \,. \tag{3.4}$$

If the coordinates of the point $M(\sigma_1, \sigma_2)$ satisfied (3.1), M is on the line AB and $\omega = 1$. If M is inside the stable domain OAB then $\omega < 1$ (as $\sigma_1 \leq \sigma_1^*$, $\sigma_2 \leq \sigma_2^*$)

Substituting (3.2) in (3.1) we have

$$\left(\frac{\sigma_1}{k_1} + \frac{\sigma_2}{k_2}\right)\frac{b^2 h}{\pi^2 D} = 1; \tag{3.5a}$$

or

$$\left(\frac{\sigma_1}{k_1} + \frac{\sigma_2}{k_2}\right)\frac{12(1-\nu^2)b^2}{\pi^2 h^2 E} = 1. \tag{3.5b}$$

In this case $(\sigma_1, \sigma_2) \in AB$.

Let us now consider viscoelastic plate. Using the criterion of creep stability [2] (PB2 is the limit of the creep stable domain) from (2.3) we have

$$\widetilde{E}_2 = E\Big[1 - \frac{E}{2}\int_0^t (t_1 - t)^2 \ddot{K}(t, t_1) dt_1\Big]^{-1}. \tag{3.6}$$

E - modulus of elasticity in the elastic stable condition (3.5). $\widetilde{E}_2$ - fictive modulus of elasticity in the creep stable condition (3.6), that depends on t. If $t = 0$, $\widetilde{E}_2 = E$, $(t \geq 0)$.

We replace E in (3.5b) by $\widetilde{E}_2$ from (3.6), it means we replace $(\sigma_1, \sigma_2) \in AB$ by (σ_1, σ_2) given in the stable domain. Thus, we can get

$$\omega = \Big[1 - \frac{E}{2}\int_0^t (t_1 - t)^2 \ddot{K}(t, t_1) dt_1\Big]^{-1}. \tag{3.7}$$

where $\omega \leq 1$.

Let us consider the case: $\sigma_1 = \alpha_1 \sigma_0$, $\sigma_2 = \alpha_2 \sigma_0$ where $\sigma_0 = \dfrac{\pi^2 D}{b^2 h}$, α_1, α_2 - are given, $\alpha_1 \leq k_1$, $\alpha_2 \leq k_2$, (σ_1, σ_2) in the stable domain.

From (3.7) and (3.4) we get

$$\omega = \left(\frac{\alpha_1}{k_1} + \frac{\alpha_2}{k_2}\right) = \left[1 - \frac{E}{2}\int_0^t (t_1 - t)^2 \ddot{K}(t, t_1) dt_1\right]^{-1} \tag{3.8}$$

In the case of square plate ($a = b$) $\sigma_1^* = \sigma_2^* = \dfrac{4\pi^2 D}{a^2 h}$, we obtain

$$\omega = \frac{\alpha_1 + \alpha_2}{4} = \left[1 - \frac{E}{2}\int_0^t (t_1 - t)^2 \ddot{K}(t, t_1) dt_1\right]^{-1}. \tag{3.9}$$

Using the creep kernel $K(t, t_1)$ we can obtain the relation $\omega \sim t_{cr}$. Thus, we can find t_{cr} when the applied stresses σ_1, σ_2 are given.

4. Viscoelastic plate compressed in one direction and in shear

Let us consider elastic rectangular plate in compression and in shear (Fig. 3).

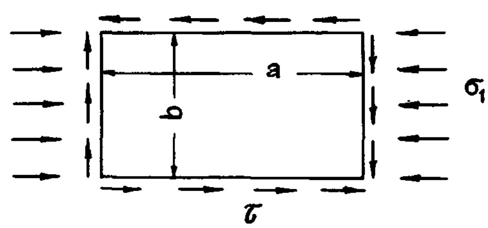

Fig. 3

It can be seen from [1] the values of simple critical stresses

$$\sigma_1^* = k_1 \frac{\pi^2 D}{b^2 h}, \qquad \tau^* = k_3 \frac{\pi^2 D}{b^2 h}; \tag{4.1}$$

we determine k_1, k_3, using the table.

The equation of limit straight line is written as followes

$$\frac{\sigma_1}{\sigma_1^*} + \frac{\tau}{\tau^*} = 1; \tag{4.2}$$

we denote

$$\omega = \frac{\sigma_1}{\sigma_1^*} + \frac{\tau}{\tau^*}; \tag{4.3}$$

where σ_1, τ - really acting stresses

Let us now consider viscoelastic plate acted on by the stresses

$$\sigma_1 = \alpha_1 \sigma_0, \qquad \tau = \alpha_3 \sigma_0, \qquad \sigma_0 = \frac{\pi^2 D}{b^2 h}$$

where α_1, α_3 are given ($\alpha_1 \leq k_1$, $\alpha_3 \leq k_3$).

In a similar manner, we get the equality

$$\omega = \left(\frac{\alpha_1}{k_1} + \frac{\alpha_3}{k_3}\right) = \left[1 - \frac{E}{2}\int_0^t (t_1 - t)^2 \ddot{K}(t, t_1) dt_1\right]^{-1}.$$

Choosing the creep kernel $K(t, t_1)$ we can find the relation $\omega \sim t_{cr}$. Thus, we have t_{cr} when the applied stresses σ_1, τ are given.

5. Plate in shear and in compression in two directions

Lut us consider elastic plate ($a \times b$) (Fig. 4). The values of simple critical stresses are given [1]

$$\sigma_1^* = k_1 \frac{\pi^2 D}{b^2 h}, \quad \sigma_2^* = k_2 \frac{\pi^2 D}{b^2 h}, \quad \tau^* = k_3 \frac{\pi^2 D}{b^2 h}, \tag{5.1}$$

where k_1, k_2, k_3 - determined with the help of the tables [1].

The equation of limit plane is written in the form (Fig. 5)

$$\frac{\sigma_1}{\sigma_1^*} + \frac{\sigma_2}{\sigma_2^*} + \frac{\tau}{\tau^*} = 1, \tag{5.2}$$

We denote

$$\omega = \frac{\sigma_1}{\sigma_1^*} + \frac{\sigma_2}{\sigma_2^*} + \frac{\tau}{\tau^*}, \tag{5.3}$$

where σ_1, σ_2, τ really acting stresses.

If the point $M(\sigma_1, \sigma_2, \tau)$ is on the limit surface $\omega = 1$; if M is inside the stable domain $\omega < 1$.

Let us consider the viscoelastic plate subjected to 3 stresses

$$\sigma_1 = \alpha_1 \sigma_0, \ \sigma_2 = \alpha_2 \sigma_0, \ \tau = \alpha_3 \sigma_0, \ \sigma_0 = \frac{\pi^2 D}{b^2 h}, \ (\alpha_1 \leq k_1, \ \alpha_2 \leq k_2, \ \alpha_3 \leq k_3)$$

In a similar manner, we get

$$\omega = \left(\frac{\alpha_1}{k_1} + \frac{\alpha_2}{k_2} + \frac{\alpha_3}{k_3}\right) = \left[1 - \frac{E}{2}\int_0^t (t_1 - t)^2 \ddot{K}(t, t_1) dt_1\right]^{-1}. \tag{5.4}$$

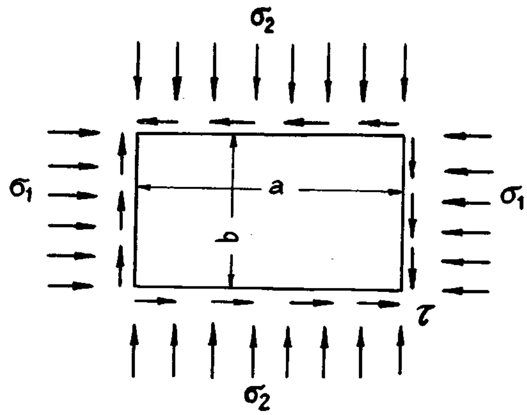

Fig. 4

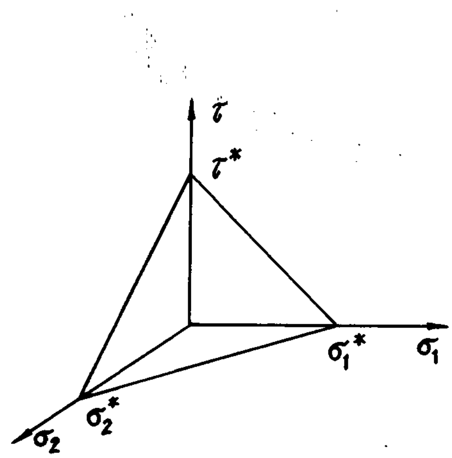

Fig. 5

Choosing the creep kernel $K(t,t_1)$ in a determined form we can find the relation $\omega \sim t_{cr}$. Thus, we obtain t_{cr} when the applied stresses σ_1, σ_2, τ are given.

- If we choose the creep kernel in the form (used for concrete)

$$K(t,t_1) = \frac{\partial}{\partial t_1}\Big[(C_0 + A_0 e^{-\beta t})(1 - e^{-\gamma(t-t_1)})\Big]; \tag{5.5}$$

substituting (5.5) in (5.4), we get

$$\omega = \Big(\frac{\alpha_1}{k_1} + \frac{\alpha_2}{k_2} + \frac{\alpha_3}{k_3}\Big) = \Big\{1 + EC_0\Big[1 - e^{-\gamma t}\big(1 + \gamma t + \frac{\gamma^2 t^2}{2}\big)\Big] + \\ + \frac{EA_0\gamma^2}{(\gamma-\beta)^2}\Big[e^{-\beta t} - e^{-\gamma t}\Big(1 + (\gamma-\beta)t + \frac{(\gamma-\beta)^2 t^2}{2}\Big)\Big]\Big\}^{-1}. \tag{5.6}$$

- If we choose the creep kernel in the form (for polymer)

$$K(t,t_1) = \frac{-A}{(t-t_1)^\alpha}, \tag{5.7}$$

where $0 < \alpha < 1$, $A > 0$; A, α - material constants.

substituting (5.7) in (5.4), we get

$$t_{cr} = \left[\frac{2(1-\alpha)(\frac{1}{\omega} - 1)}{AE\alpha(\alpha+1)}\right]^{\frac{1}{1-\alpha}}; \tag{5.8}$$

Consequently, if the stresses acting on the plate σ_1, σ_2, τ are given. We can determine the critical time t_{cr} with the help of (5.6), (5.8).

6. Conclusion

If the given stresses acting on the viscoelastic plate is smaller than the simple critical stresses we can use (3.9), (4.4), (5.4) to determine critical time t_{cr}. These formulas are obtained with the help of Cliusnhicov's theory of pseudo - bifurcation points.

In a similar manner we can solve the problems on stability of viscoelastic plate subjected to different combined forces, with different boundary conditions.

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ỔN ĐỊNH CỦA TẤM ĐÀN NHỚT CHỊU TẢI TRỌNG TỔ HỢP

Bằng việc sử dụng lý thuyết về các điểm phân nhánh giả và phương pháp xây dựng các "tương tự đàn hồi" các tác giả đã giải bài toán ổn định của tấm đàn nhớt chịu đồng thời các tải trọng khác nhau (chịu nén theo hai phương, chịu kéo và chịu cắt...). Kết quả là tìm được biểu thức giải tích của thời gian tới hạn ứng với các tải trọng xác định.