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ANALYSIS OF EXCEEDANCE PROBABILITY OF DISPLACEMENT RESPONSE OF RANDOMLY NONLINEAR STRUCTURES

LUU XUAN HUNG
Institute of Mechanics, NCST of Vietnam

SUMMARY. The paper presents the estimation of the exact exceedance probability (EEP) of stationary responses of some white noise-randomly excited nonlinear systems whose exact probability density function can be known. Consequently, the approximate exceedance probabilities (AEPs) are evaluated based on the analysis of equivalent linearized systems using the traditional Caughey method and the extension technique of LOMSEC. Comparisons of the AEPs versus the EEP are demonstrated. The obtained results indicate important characters of the exceedance probability (EP) as well as the influence of nonlinearity over EP. The evaluation of the applied possibility of the proposed linearization techniques for estimating AEPs are made.

1. Introduction

One of the most concerned problems in the design process of types of structures, is the estimation of the extreme demands on the structure during a specified period of time. This is the same meaning with the estimation of exceedance probability of the extreme responses during the period of time. In general, this is a very difficult problem and usually, only indicative answers can be obtained in practice. However, in the context of civil engineering, structures subjected to environmental loads such as wind and ocean waves, a remarkable developments over the last two decades in modelling both the structure, the loading process and the interaction between them has been made.

The framework usually adopted for the estimation of extreme responses of civil engineering structures for the purpose of design, is that of modelling the loading processes on the structure as stochastic processes. In cases where the dynamic behaviour of the structure can be modelled by linear equations of motion, the response statistics can be analysed in a rather satisfactory manner. However, this is usually an exception, especially for the estimation of extreme responses. Since stochastic response analysis of nonlinear structures is very difficult, methods of stochastic linearization have been developed.

When analysing nonlinear random systems using the equivalent linearization techniques, the analysis of the second-order moments were very much investigated; whereas the researches on the exceedance probability of the extreme responses were rarely made. Naess [2-4] has presented results from initial efforts to develop a stochastic linearization procedure specifically designed for making predictions of large responses.

This paper presents the estimation of the EEP of the stationary responses of some white noise-randomly excited nonlinear systems, whose exact probability density function can be found. Through the obtained EEP, some important characters of the EP, especially the influence of nonlinearity over EP are introduced. Consequently, the AEPs are evaluated based on the analysis of equivalent linearized systems using the traditional Caughey method [1] and the extension technique of LOMSEC [9-11]. Comparisons of the AEPs versus the EEP are given in order to evaluate the applied possibility of the proposed linearization techniques for estimating AEPs. The systems considered in this paper, and relative data for estimation of the exceedance probability are originated from the previous publications of the Author himself [10-12]. The numerical calculations are added by a special software of Mathematica 3.0 [13].

2. Estimation of exceedance probability of displacement response

The extreme value distribution of a stationary process $X(t)$ is assumed, for simplicity, to be given as [7]:

$$F(x) = \text{Prob}\{M(T) \le x\} = \exp\{-\nu(x)T\}, \tag{2.1}$$

where $M(T) = \max\{X(t); 0 \le t \le T\}$ is the largest value of $X(t)$ during a time interval of length T; $\nu(x)$ denotes the mean up-crossing rate of $X(t)$. At level x_p the exceedance probability p^+ during time T is defined by:

$$F(x_p) = 1 - p^+. \tag{2.2}$$

From (2.1), (2.2) we get:

$$\begin{aligned} p^+ &= 1 - F(x) = 1 - \exp\{-\nu(x)T\} \\ \Rightarrow 100(1 - F(x)) &= 100[1 - \exp\{-\nu(x)T\}]\%. \end{aligned} \tag{2.3}$$

A typical range of exceedance probabilities for design purposes is from 1% to 20%. The time interval chosen here is $T = 3h$.

The mean up-crossing rate $\nu(x)$ is defined by [8]:

$$\nu(x) = \int_0^\infty \dot{x} p(x, \dot{x}) d\dot{x}, \tag{2.4}$$

where $p(x, \dot{x})$ is joint probability density function (PDF) of responses $X(t)$ and $\dot{X}(t)$, $\dot{x} = dx/dt$. However, it is difficult to obtain the exact PDF of randomly excited nonlinear systems in practice. Even if the response can be modeled as a Markov process, the possibility of an exact PDF solution is still limited. Therefore, some approximate methods were developed and investigated for estimating the mean up-crossing rate [5, 6, 8, 12].

Exact exceedance probability (EEP)

The EEP of the stationary responses of the randomly nonlinear structures is defined by using formulas (2.3), (2.4); with the assumption that the exact joint probability density function $p_e(x, \dot{x})$ of the responses $x(t)$, $\dot{x}(t)$ can be known by solving Fokker-Planck equation [10-12]:

$$p_e^+ = 100[1 - \exp\{-\nu_e(x)T\}]\%, \tag{2.5}$$

where $\nu_e(x)$ is the exact mean up-crossing rate (EMCR):

$$\nu_e(x) = \int_0^\infty \dot{x} p_e(x, \dot{x}) d\dot{x}. \tag{2.6}$$

Approximate exceedance probability (AEP)

The AEPs are also defined by using formulas (2.3), (2.4) but for the equivalent linearized systems (obtained by Caughey or LOMSEC, respectively). In this case, the joint probability density functions are approximate and considered as the norm:

$$p_A(x, \dot{x}) = \frac{1}{2\pi\sigma_x\sigma_{\dot{x}}} \exp\Big\{ - \Big(\frac{x^2}{2\sigma_x^2} + \frac{\dot{x}^2}{2\sigma_{\dot{x}}^2} \Big) \Big\}, \tag{2.7}$$

where $\sigma_x^2 = \langle x^2 \rangle$, $\sigma_{\dot{x}} = \langle \dot{x}^2 \rangle$ are second moments of the equivalent linearized system. The approximate mean up-crossing rates (AMCRs) as follows:

$$\nu_A(x) = \int_0^\infty \dot{x} p_A(x, \dot{x}) d\dot{x}. \tag{2.8}$$

Using the linearization method of Caughey or LOMSEC, $\langle x^2 \rangle_G$, $\langle \dot{x}^2 \rangle_G$ and $\langle x^2 \rangle_{LG}$, $\langle \dot{x}^2 \rangle_{LG}$ can be calculated [10-11]. Consequently the AMCRs of $\nu_G(x)$, $\nu_{LG}(x)$ are evaluated by using formulas (2.7), (2.8) [12]. Then from (2.3), (2.4), (2.7), (2.8), the AEPs are defined as follows:

$$p_A^+ = 100[1 - \exp\{-\nu_A(x)T\}]\%. \tag{2.9}$$

3. Illustrative examples

Example 1. Consider the Duffing oscillator with Gaussian white noise excitation:

$$\ddot{x} + 2h\dot{x} + \omega_0^2 x + \varepsilon x^3 = \sigma w(t). \tag{3.1}$$

A series of calculation procedures to obtain the mean up-crossing rates (EMCR and AMCR) has been done in [12]. The results are:

The EMCR as follows:

$$\nu_e(x) = \frac{\sqrt{h}\Big(\int\limits_0^\infty \dot{x}\exp\Big\{-\frac{2h\dot{x}^2}{\sigma^2}\Big\}d\dot{x}\Big)\exp\Big\{-\frac{4h}{\sigma^2}\Big(\frac{\omega_0^2}{2}x^2+\frac{\varepsilon}{4}x^4\Big)\Big\}}{\sigma\sqrt{2\pi}\int\limits_0^\infty \exp\Big\{-\frac{4h}{\sigma^2}\Big(\frac{\omega_0^2}{2}x^2+\frac{\varepsilon}{4}x^4\Big)\Big\}dx}. \tag{3.2}$$

The AMCRs according to the linearization criterion of Caughey and LOMSEC are:

$$\nu_G(x) = \frac{\Big(\int\limits_0^\infty \dot{x}\exp\Big\{-\frac{\dot{x}^2}{2\langle x^2\rangle_G(\omega_0^2+\lambda_G)}\Big\}d\dot{x}\Big)\exp\Big\{-\frac{x^2}{2\langle x^2\rangle_G}\Big\}}{2\pi\langle x^2\rangle_G\sqrt{\omega_0^2+\lambda_G}}, \tag{3.3}$$

where $\lambda_G = 3\varepsilon\langle x^2\rangle_G$

$$\nu_{LG}(x) = \frac{\Big(\int\limits_0^\infty \dot{x}\exp\Big\{-\frac{\dot{x}^2}{2\langle x^2\rangle_{LG}(\omega_0^2+\lambda_{LG})}\Big\}d\dot{x}\Big)\exp\Big\{-\frac{x^2}{2\langle x^2\rangle_{LG}}\Big\}}{2\pi\langle x^2\rangle_{LG}\sqrt{\omega_0^2+\lambda_{LG}}}, \tag{3.4}$$

where $\lambda_{LG} = K_r\varepsilon\langle x^2\rangle_{LG}$ and $K_r = \dfrac{\int\limits_0^r t^4 n(t)dt}{\int\limits_0^r t^2 n(t)dt}$; $n(t) = \dfrac{1}{\sqrt{2\pi}}e^{-t^2/2}$.

Substitutions (3.2) into (2.5) and (3.3), (3.4) into (2.9) yields formulas for calculating the EEP and AEPs respectively. Table 1 shows the terminal expressions corresponding with specific value of the parameters.

Fig 1 - 4 show the EEP and AEPs. In order to distinguish graphics, we use different lines as follows [—— p_e^+ (the EEP), - - - p_G^+ (the AEP of Caughey), and ------ p_{LG}^+ (the AEP of LOMSEC)]. Numerical values are given in table 2.

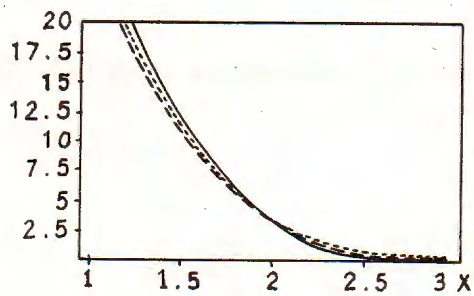

Fig. 1

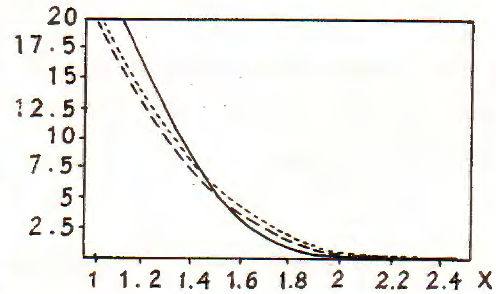

Fig. 2

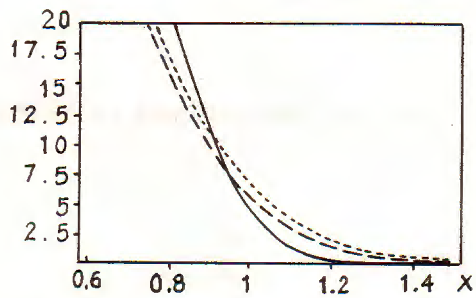

Fig. 3

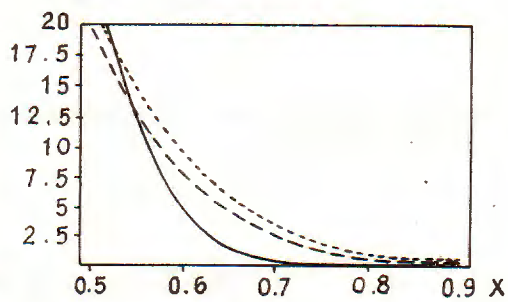

Fig. 4

Table 1. Expressions of the exceedance probabilities
($\omega_0^2 = 1$; $h = 0.25$, $\sigma = 1$; ε varies)

ε	p_e^+	p_G^+	p_{LG}^+
0.2	$100[1 - \exp\{-0.17528\times \exp\{-(0.5x^2 + 0.05x^4)\}T\}]$	$100[1 - \exp\{-0.18979 \times \exp\{-0.71097x^2\}T\}]$	$100[1 - \exp\{-0.18744 \times \exp\{-0.67996x^2\}T\}]$
1	$100[1 - \exp\{-0.20615\times \exp\{-(0.5x^2 + 0.25x^4)\}T\}]$	$100[1 - \exp\{-0.24151 \times \exp\{-1.15138x^2\}T\}]$	$100[1 - \exp\{-0.23252 \times \exp\{-1.07117x^2\}T\}]$
10	$100[1 - \exp\{-0.30597\times \exp\{-(0.5x^2 + 2.5x^4)\}T\}]$	$100[1 - \exp\{-0.38985 \times \exp\{-2.99994x^2\}T\}]$	$100[1 - \exp\{-0.36130 \times \exp\{-2.73613x^2\}T\}]$
100	$100[1 - \exp\{-0.50867\times \exp\{-(0.5x^2 + 25x^4)\}T\}]$	$100[1 - \exp\{-0.67199 \times \exp\{-8.91424x^2\}T\}]$	$100[1 - \exp\{-0.61060 \times \exp\{-8.07624x^2\}T\}]$

Table 2. Some numerical values ($\omega_0^2 = 1$; $h = 0.25$; $\sigma = 1$; ε varies)

ε	$p_e^+(\%)$	x	$p_G^+(\%)$	$\text{Error}_G(\%)$	$p_{LG}^+(\%)$	$\text{Error}_{LG}(\%)$
0.2	20	1.22140	17.89220	-10.539	18.44680	-7.766
	15	1.40109	13.15230	-12.318	13.75770	-8.282
	10	1.59997	8.81236	-11.876	9.39274	-6.073
	5	1.85974	4.75265	-4.947	5.21268	4.254
	1	2.28151	1.39671	39.671	1.61923	61.923
1	20	1.11953	15.72910	-21.355	16.65550	-16.723
	15	1.23250	11.84100	-21.060	12.80820	-14.612
	10	1.35736	8.31875	-16.813	9.23862	-7.614
	5	1.51999	4.94108	-1.178	5.70291	14.058
	1	1.78346	1.84307	84.307	2.28502	128.502
10	20	0.81164	14.96330	-25.184	16.36670	-18.167
	15	0.85917	11.99080	-20.061	13.39600	-10.693
	10	0.91427	9.08760	-9.124	10.42380	4.238
	5	0.98935	6.01695	20.339	7.17509	43.502
	1	1.11692	2.73322	173.322	3.50632	250.632
100	20	0.51720	16.95030	-15.249	19.03630	-4.819
	15	0.53803	14.16100	-5.593	16.20830	8.055
	10	0.56315	11.24750	12.475	13.18780	31.878
	5	0.59877	7.91912	58.382	9.62826	92.565
	1	0.66208	3.96942	296.942	5.17520	417.520

Comments. From the figures and from table 2, it is shown that: at a specific level of p_e^+, the extreme response $X(t)$ reduces when the nonlinearity increases. For larger values of p_e^+ (equal to small values of $X(t)$), one gets p_{LG}^+ more improved than p_G^+, especially with a strong nonlinearity. For specified small values of p_e^+, both p_G^+ and p_{LG}^+ have high relative errors though the absolute errors are not so high; however, one gets p_G^+ better than p_{LG}^+.

Example 2. Consider the case $\omega_0^2 = -1$ of the Duffing oscillator (3.1).

The formulas for calculating EMCR and AMCRs, then EEP and AEPs are quite the same as (3.2), (3.3), (3.4). Table 3 shows the terminal expressions corresponding with specific value of the parameters. Fig 5 - 8 show the EEP and AEPs. The graphic symbols are similar to the above-mentioned. Some numerical values given in table 4.

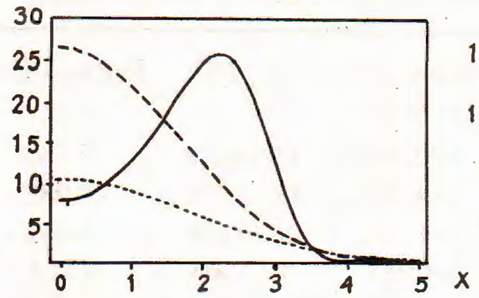

Fig. 5

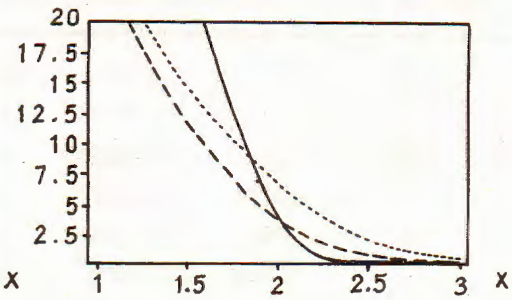

Fig. 6

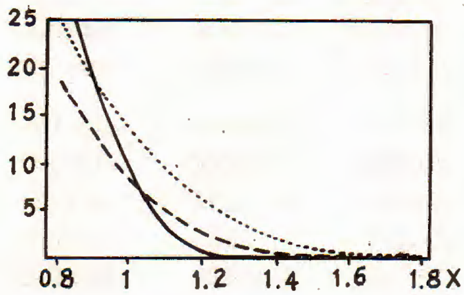

Fig. 7

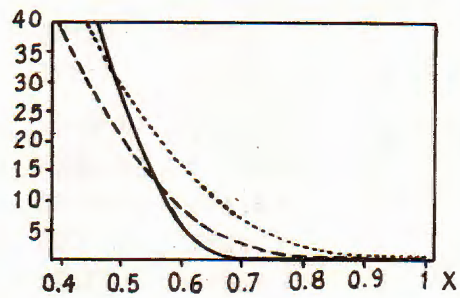

Fig. 8

Table 3. Expressions of the exceedance probabilities
($\omega_0^2 = -1$; $h = 0.25$; $\sigma = 1$; ε varies)

ε	p_e^+	p_G^+	p_{LG}^+
0.2	$100[1-\exp\{-0.02856\times \exp\{0.5x^2-0.05x^4\}T\}]$	$100[1-\exp\{-0.10338 \times\exp\{-0.21099x^2\}T\}]$	$100[1-\exp\{-0.03713 \times\exp\{-0.14387x^2\}T\}]$
1	$100[1-\exp\{-0.10216\times \exp\{0.5x^2-0.25x^4)\}T\}]$	$100[1-\exp\{-0.18166 \times\exp\{-0.65139x^2\}T\}]$	$100[1-\exp\{-0.15756 \times\exp\{-0.47504x^2\}T\}]$
10	$100[1-\exp\{-0.24688\times \exp\{0.5x^2-2.5x^4)\}T\}]$	$100[1-\exp\{-0.35588 \times\exp\{-2.5x^2\}T\}]$	$100[1-\exp\{-0.33650 \times\exp\{-1.91666x^2\}T\}]$
100	$100[1-exp\{-0.47541\times \exp\{0.5x^2-25x^4)\}T\}]$	$100[1-\exp\{-0.65290 \times\exp\{-8.41326x^2\}T\}]$	$100[1-\exp\{-0.62589 \times\exp\{-6.55996x^2\}T\}]$

Table 4. Some numerical values ($\omega_0^2 = -1$; $h = 0.25$; $\sigma = 1$; ε varies)

ε	$p_e^+(\%)$	x	$p_G^+(\%)$	$\text{Error}_G(\%)$	$p_{LG}^+(\%)$	$\text{Error}_{LG}(\%)$
0.2	20	1.60626	16.46840	-17.658	7.39704	-63.015
	15	2.91416	5.03748	-66.417	3.22938	-78.471
	10	3.09318	4.03581	-59.642	2.77294	-72.271
	5	3.30728	3.03807	-39.239	2.28249	-54.350
	1	3.63837	1.88125	88.125	1.64487	64.487
1	20	1.58317	10.10190	-49.491	13.38560	-33.072
	15	1.69729	8.00607	-46.626	11.33380	-24.441
	10	1.81546	6.16933	-38.307	9.40445	-5.956
	5	1.96339	4.32792	-13.442	7.29347	45.869
	1	2.19776	2.31670	131.670	4.65331	365.331
10	20	0.89438	13.45640	-32.718	19.57970	-2.102
	15	0.94090	11.01840	-26.544	16.89010	12.601
	10	0.99441	8.61734	-13.827	14.07560	40.756
	5	1.06689	6.01417	20.283	10.76760	115.352
	1	1.18966	3.05551	205.551	6.47990	547.990
100	20	0.53158	16.61880	-16.906	25.48320	27.416
	15	0.55220	13.98130	-6.791	22.43460	49.564
	10	0.57704	11.21360	12.136	19.04980	90.498
	5	0.61224	8.02291	60.458	14.83580	196.716
	1	0.67477	4.16017	316.017	9.03707	803.707

Comments. Numerical results show that: at a specific level of p_e^+, the influence of nonlinearity over the extreme response $X(t)$ is similar to the example 1. At $\varepsilon = 0.2$ (weakly nonlinearity) a maximum value of p_e^+ exists: $\text{Max}(p_e^+) \approx 25\%$ at $x \approx 2.4$. Error_G and Error_{LG} both are higher than that of the case $\omega_0^2 = 1$ respectively. For weakly nonlinearity $\varepsilon = 0.2$ and strong nonlinearity $\varepsilon = 100$, p_G^+ is generally better than p_{LG}^+; for medium nonlinearity $\varepsilon =$ 1-10 one gets p_{LG}^+ more improved than p_G^+ at larger values of p_e^+ and the contrary.

Example 3. Consider the oscillator of nonlinear stiffness and damping with Gaussian white noise excitation:

$$\ddot{x} + 4h\Big(\frac{\dot{x}^2}{2} + \omega_0^2\frac{x^2}{2} + \varepsilon\frac{x^4}{4}\Big)\dot{x} + \omega_0^2 x + \varepsilon x^3 = \sigma w(t). \qquad (3.5)$$

Similarly, the calculation procedures to obtain the mean up-crossing rates (EMCR and AMCR) to be done in [12]. The results are:

$$\nu_e(x) = \frac{\int_0^\infty \dot{x} \exp\left\{-\frac{4h}{\sigma^2}\left(\frac{1}{2}\dot{x}^2 + \frac{\omega_0^2}{2}x^2 + \frac{\varepsilon}{4}x^4\right)^2\right\} d\dot{x}}{4\int_0^\infty \int_0^\infty \exp\left\{-\frac{4h}{\sigma^2}\left(\frac{1}{2}\dot{x}^2 + \frac{\omega_0^2}{2}x^2 + \frac{\varepsilon}{4}x^4\right)^2\right\} dx d\dot{x}}. \tag{3.6}$$

Using (2.5), one receives the formula for estimation of the exact exceedance probability p_e^+. The formulas for calculation of the AMCRs of Caughey and LOMSEC are quite the same as the above Duffing example (3.3), (3.4). (2.9) is used for estimating the approximate exceedance probability p_G^+ and p_{LG}^+.

The terminal expressions corresponding with the specific value of the parameters are given in table 5. Fig 9 - 10 show the EEP and AEPs. Numerical values are given in table 6.

Table 5. Expressions of the exceedance probabilities ($\omega_0^2 = 1$; $\sigma = 1$; h, ε varies)

h, ε	p_e^+	p_G^+	p_{LG}^+
$h = 0.1$ $\varepsilon = 0.01$	$100[1 - \exp\{-0.11378 \times \int_0^\infty \dot{x}\exp\{-0.4 \times (0.5\dot{x}^2 + 0.5x^2 + 0.0025x^4)^2\}d\dot{x}T\}]$	$100[1 - \exp\{-0.16022 \times \exp\{-1.12135x^2\}T\}]$	$100[1 - \exp\{-0.16072 \times \exp\{-0.56990x^2\}T\}]$
$h = 1$ $\varepsilon = 0.2$	$100[1 - \exp\{-0.37257 \int_0^\infty \dot{x}\exp\{-4 \times (0.5\dot{x}^2 + 0.5x^2 + 0.05x^4)^2\}d\dot{x}T\}]$	$100[1 - \exp\{-0.16822 \times \exp\{-2.56187x^2\}T\}]$	$100[1 - \exp\{-0.16551 \times \exp\{-2.09899x^2\}T\}]$

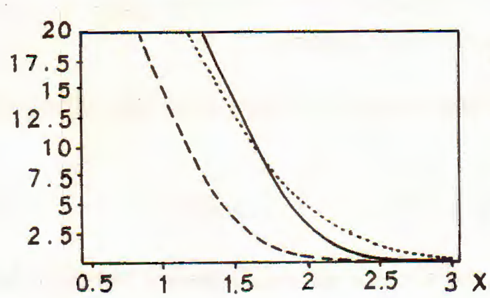

Fig. 9

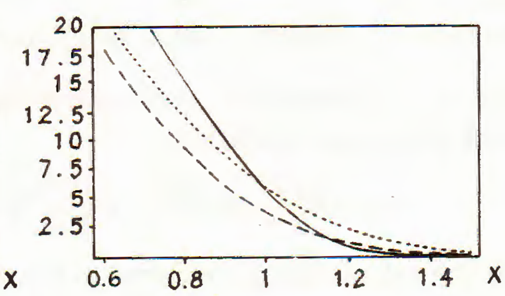

Fig. 10

Table 6. Some numerical values ($\omega_0^2 = 1$; $\sigma = 1$; h, ε varies)

h, ε	$p_e^+(\%)$	x	$p_G^+(\%)$	$\text{Error}_G(\%)$	$p_{LG}^+(\%)$	$\text{Error}_{LG}(\%)$
$h = 0.1$	21.23780	1.22474	8.55215	-59.732	18.54220	-12.693
$\varepsilon = 0.01$	16.07240	1.41421	4.97521	-69.045	14.29290	-11.072
	11.59350	1.58114	2.87103	-75.236	10.95180	-5.535
	5.18161	1.87083	0.94470	-81.768	6.34977	22.544
	1.04236	2.23607	0.17639	-83.078	2.75184	164.001
$h = 1$	20.28710	0.70711	13.08010	-35.525	15.95710	-21.344
$\varepsilon = 0.2$	14.97660	0.80623	9.10420	-39.211	11.91670	-20.431
	10.40120	0.89443	6.29326	-39.495	8.84557	-14.956
	5.76053	1.00000	3.81913	-33.702	5.90495	2.507
	1.18175	1.18322	1.38778	17.434	2.59441	119.540

Comments. We also get the rule that at a specific level of p_e^+, the extreme response $X(t)$ reduces,when the nonlinearity increases. In any case of the nonlinearity we get p_{LG}^+ more improved than p_G^+ at larger and medium values of p_e^+ and the contrary at smaller values of p_e^+.

Example 4. Consider the oscillator with nonlinear damping following x, $\dot{x}$ under Gaussian white noise excitation, which obtained from (3.5) with $\varepsilon = 0$:

$$\ddot{x} + 4h\Big(\frac{\dot{x}^2}{2} + \omega_0^2\frac{x^2}{2}\Big)\dot{x} + \omega_0^2 x = \sigma w(t). \tag{3.7}$$

Similarly, we have:

$$\nu_e(x) = \frac{\displaystyle\int_0^\infty \dot{x}\exp\Big\{-\frac{4h}{\sigma^2}\Big(\frac{1}{2}\dot{x}^2 + \frac{\omega_0^2}{2}x^2\Big)^2\Big\}d\dot{x}}{\displaystyle 4\int_0^\infty\int_0^\infty \exp\Big\{-\frac{4h}{\sigma^2}\Big(\frac{1}{2}\dot{x}^2 + \frac{\omega_0^2}{2}x^2\Big)^2\Big\}dx\,d\dot{x}}. \tag{3.8}$$

The calculation of the AMCRs of Caughey and LOMSEC, plus the exact exceedance probability p_e^+ and the approximate exceedance probability p_G^+, p_{LG}^+ are the same as the discussions in example 3.

The terminal expressions corresponding with specific value of the parameters are given in table 7. Fig 11 - 12 show the EEP and AEPs. Numerical values are given in table 8.

Table 7. Expressions of the exceedance probabilities ($\omega_0^2 = 1$; $\sigma = 1$; h varies)

h	p_e^+	p_G^+	p_{LG}^+
$h = 0.25$	$100[1 - \exp\{-0.17959 \times \int_0^\infty \dot{x} \exp\{-(0.5\dot{x}^2 + 0.5x^2)^2\} d\dot{x} T\}]$	$100[1 - \exp\{-0.15915 \times \exp\{-x^2\} T\}]$	$100[1 - \exp\{-0.15915 \times \exp\{-0.87058x^2\} T\}]$
$h = 1$	$100[1 - \exp\{-0.35917 \times \int_0^\infty \dot{x} \exp\{-4(0.5\dot{x}^2 + 0.5x^2)^2\} d\dot{x} T\}]$	$100[1 - \exp\{-0.15915 \times \exp\{-2x^2\} T\}]$	$100[1 - \exp\{-0.15915 \times \exp\{-1.82862x^2\} T\}]$

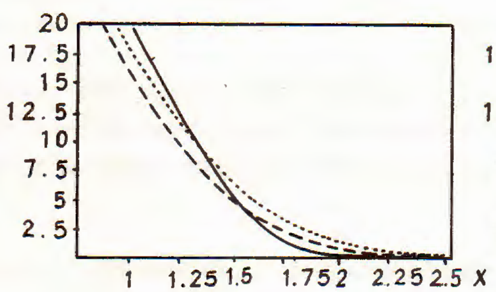

Fig. 11

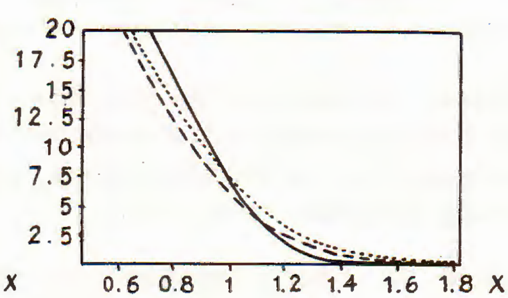

Fig. 12

Table 8. Some numerical values ($\omega_0^2 = 1$; $\sigma = 1$; h varies)

h	$p_e^+(\%)$	x	$p_G^+(\%)$	$\text{Error}_G(\%)$	$p_{LG}^+(\%)$	$\text{Error}_{LG}(\%)$
$h = 0.25$	20.46300	1.00000	16.10830	-21.281	18.11980	-11.451
	14.97600	1.16190	11.64210	-22.262	13.705507	-8.484
	10.37180	1.30384	8.35266	-19.468	10.29890	-0.703
	5.18964	1.50000	4.90776	-5.432	6.51164	25.474
	1.02397	1.80278	1.83424	79.130	2.77993	171.485
$h = 1$	20.46250	0.70711	16.10830	-21.279	17.41640	-14.886
	15.71080	0.80623	12.20100	-22.340	13.53700	-13.836
	10.37150	0.921957	8.35266	-19.465	9.59768	-7.461
	4.83868	1.07238	4.67409	-3.402	5.66300	17.036
	1.12292	1.26491	1.92738	71.640	2.52770	125.101

Comments. Comments for this case are the same as for the above in example 3.

4. Conclusions

Through the analysis of the exact exceedance probability of the nonlinear systems considered, some important characters of the exceedance probability, especially the influence of nonlinearity over the exceedance probability are investigated:

- It is obvious that the exceedance probability is generally a contra-variant function versus the extreme response (p^+ goes down when $X(t)$ increases). However it is a type of system whose exceedance probability contains a maximum-peak at a value of the extreme response $X(t)$; which is not as small as the case of Duffing with $\omega_0^2 = -1$.
- At a specified level of p^+, the extreme response $X(t)$ reduces when the nonlinearity increases; in other words, the nonlinearity effect causes a reduction of the exceedance probability.

The obtained result shows the applied possibility of the proposed linearization techniques for estimating the approximate exceedance probability:

- In a specified large domain of p^+, it is usually to get p^+_{LG} more improved than p^+_G. When p^+ goes down at a specified smaller value, both p^+_G and p^+_{LG} have rather high relative errors though the absolute errors are not so high, however one gets p^+_G better than p^+_{LG}.
- The influence of the nonlinearity effect over the degree of accuracy of the approximate exceedance probability p^+_G and p^+_{LG} is rather complicated and in general it is only possible to obtain individual answers for each type of system.

In short, according to design purposes (p^+ required is large or small) as well as basing upon each specific system, we apply either the Caughey or the LOMSEC for estimating the approximate exceedance probability.

The results and comments in this paper are supplementary to the previous researches on the approximate exceedance probability through the linearization using the Caughey. Further more, this is the first research conducted on the analysis of the approximate exceedance probability using the LOMSEC linearization. The above tendency of research can be enlarged for other nonlinear systems to aim at discovering more natures of the exceedance probability. The expansion also can be used for multi-degree of freedom randomly nonlinear systems.

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PHÂN TÍCH XÁC SUẤT VƯỢT CỦA ĐÁP ỨNG CHUYỂN VỊ CỦA CÁC CẤU TRÚC PHI TUYẾN NGẪU NHIÊN

Bài báo trình bày việc tính toán xác suất vượt chính xác của các đáp ứng một số hệ phi tuyến ngẫu nhiên chịu kích động ồn trắng mà có hàm mật độ xác suất chính xác có thể tìm được. Tiếp theo, các xác suất vượt gần đúng được xác định trên cơ sở phân tích các hệ tuyến tính hóa tương đương dùng phương pháp Caughey truyền thống và "tiêu chuẩn sai số bình phương trung bình khu vực" (LOMSEC). Các so sánh xác suất vượt gần đúng đối với xác suất vượt chính xác được đưa ra. Kết quả nhận được cho ra một số tính chất quan trọng của xác suất vượt, ảnh hưởng của tính phi tuyến đối với xác suất vượt. Việc đánh giá khả năng áp dụng các tiêu chuẩn tuyến tính hóa Caughey và LOMSEC để tính xác suất vượt gần đúng được thực hiện.