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MIXED REGIME IN A QUASI-LINEAR SYSTEM

NGUYEN VAN DINH - TRAN KIM CHI
Institute of Mechanics, NCST of Vietnam

ABSTRACT. A quasi-linear system with cubic nonlinearity under two external excitations in subharmonic responces of order 1/2 and 1/3 was examined. In the system under consideration there appears mixed oscillation due to the interaction between derived excitations. Various forms of the resonance curve were identified. The stability study was based on an abbreviated form of the second stability condition [2].

1. Introduction

The present article deals with the effect of two external excitations in subharmonic resonances on an oscillator which has weak (order ε) cubic non-linearity. The attention is focused on mixed regimes due to supplementary excitations of ε-order, which are introduced by the "original" excitations through the cubic non-linearity.

The asymptotic method [1] and some remarks given in [2] are used. The so-called associated equations are established; the analytical identification of the resonance curves is done; the classification of the resonance curves is based on the location of the critical part; stable branches of the resonance curve are determined by an abbreviate form of the second stability condition.

2. System under consideration - Original and Associated equations

Consider a quasi-linear system governed by the differential equation:

$$\ddot{x} + x = \varepsilon(-h\dot{x} - \gamma x^3) - 3b\omega^2 \cos 2\omega t - 8c\omega^2 \cos(3\omega t + \sigma), \tag{2.1}$$

where x is an oscillatory variable, over dots denote derivatives relative to time t, $\varepsilon > 0$ is a positive small parameter, $h \geq 0$, $b > 0$, $c > 0$, $\omega \approx 1$, $0 \leq \sigma < 2\pi$ and γ, for simplicity, is assumed to be positive.

In the first approximation [1], the oscillations are of the form:

$$x = a\cos(\omega t + \theta) + b\cos 2\omega t + c\cos(3\omega t + \sigma), \tag{2.2}$$

$$\dot{a} = -\frac{\varepsilon}{2\omega} f_0, \quad f_0 \equiv h\omega a - e\sin(\theta + \sigma) - pa^2 \sin(3\theta - \sigma),$$
$$a\dot{\theta} = -\frac{\varepsilon}{2\omega} g_0, \quad g_0 \equiv \delta a - e\cos(\theta + \sigma) - pa^2 \cos(3\theta - \sigma), \tag{2.3}$$

where a, θ are slowly varying variables (amplitude and dephase angle):

$$\delta = \omega^2 - 1 - \frac{3\gamma}{2}(b^2 + c^2) - \frac{3\gamma}{4}a^2, \quad p = \frac{3\gamma}{4}c, \quad e = \frac{3\gamma}{4}b^2 c.$$

Constant amplitude and dephase of stationary oscillations are determined by the original equations:

$$f_0 = 0, \quad g_0 = 0. \tag{2.4}$$

Higher harmonics ($\sin 3\theta, \cos 3\theta$) in (2.4) can be eliminated by the combinations:

$$f = \tau_{11} f_0 + \tau_{12} g_0, \quad g = \tau_{21} f_0 + \tau_{22} g_0, \tag{2.5}$$

where

$$\begin{aligned}
\tau_{11} &= u\sin\theta - v\cos\theta + pa\cos 2\theta, & \tau_{12} &= v\sin\theta + u\cos\theta - pa\sin 2\theta, \\
\tau_{21} &= v\sin\theta + u\cos\theta + pa\sin 2\theta, & \tau_{22} &= -u\sin\theta + v\cos\theta + pa\cos 2\theta, \\
u &= h\omega\cos\sigma - \delta\sin\sigma, & v &= \delta\cos\sigma + h\omega\sin\sigma,
\end{aligned}$$

and the associated equations are obtained:

$$f \equiv A\sin\theta + B\cos\theta - E = 0, \quad g \equiv G\sin\theta + H\cos\theta - K = 0, \tag{2.6}$$

where

$$\begin{aligned}
&A = a(pe - T)\cos\sigma, \quad B = -a(pe - T)\sin\sigma, \quad E = e(h\omega\cos 2\sigma - \delta\sin 2\sigma), \\
&G = -a(pe + T)\sin\sigma, \quad H = -a(pe + T)\cos\sigma, \quad K = e(h\omega\sin 2\sigma + \delta\cos 2\sigma), \\
&T = \tau_{11}\tau_{22} - \tau_{12}\tau_{21} = p^2a^2 - h^2\omega^2 - \delta^2.
\end{aligned}$$

If $T = 0$, the two systems (2.4) and (2.6) are not equivalent. Consequently, the original resonance curve C_0 (determined from (2.4)) is obtained from the associated resonance curve C (determined from (2.6)) by rejecting strange representative points; the latter located in the non-equivalence curve $T = 0$, corresponds to strange solutions.

3. A frequency - amplitude relationship and two resonance curves

From (2.6), by eleminating the phase θ the following frequence - amplitude relationship can easily be established

$$W(\omega^2, a^2) \equiv D_1^2 + D_2^2 - D^2 = 0, \tag{3.1}$$

or

$$W(\omega^2, a^2) \equiv TW_0(\omega^2, a^2) = 0, \tag{3.2}$$

where $D \equiv \begin{vmatrix} A & B \\ G & H \end{vmatrix} = a^2(T^2 - p^2e^2), \quad D_1 \equiv \begin{vmatrix} E & B \\ K & H \end{vmatrix}, \quad D_2 \equiv \begin{vmatrix} A & E \\ G & K \end{vmatrix}.$

From (3.1) the associated resonance curve C can be identified and it consists of two parts:

- the regular part C' located in the regular region:

$$D \neq 0, \tag{3.3}$$

- the irregular critical part C" located in the irregular region:

$$D = 0, \tag{3.4}$$

satisfies

$$D_1 = 0, \quad D_2 = 0, \quad \text{(compatibility conditions)} \tag{3.5}$$

$$A^2 + B^2 \geq E^2, \quad G^2 + H^2 \geq K^2, \quad \text{(trigonometric conditions)}. \tag{3.6}$$

From (3.2), with regard to $D\big|_{T=0} = -a^2p^2e^2 < 0$, we conclude that the non equivalence curve $T = 0$ is a regular branch. It has been demonstrated in [2], the original resonance curve C_0 (heavy curve) is obtained from C by rejecting $T = 0$ and can be identified from the simplified frequency - amplitude relationship

$$W_0(\omega^2, a^2) = \frac{W(\omega^2, a^2)}{T} = 0. \tag{3.7}$$

Note that, on the plane $(\Delta = \omega^2 - 1, a^2)$, the irregular region $D = 0$ is formed by two parabolas:

$$P_1 : pe - T = 0, \quad \text{and} \quad P_2 : pe + T = 0, \tag{3.8}$$

and they are respectively situated above and below the non equivalence curve $T = 0$ (broken curve).

4. Diverse forms of the resonance curves

The resonance curves are quite complicated and depend on the location of the critical part C". Below, for fixed values $\gamma = 0.04$, $b = 0.4$, $c = 0.2$ typical forms of the resonance curves will be plotted with different values h, σ.

For the system without damping ($h = 0$), the condition (3.4), (3.5) admit on the curve $P_1 : \sigma = k\frac{\pi}{2}$, $(k = 0, 1, 2, 3)$ and $\delta = 0$, and on the curve $P_2 : \sigma = \frac{\pi}{4} + k\frac{\pi}{2}$, with the restriction $a^2 \geq \frac{1}{3}a_*^2 = \frac{1}{3}b^2$.

Thus: - If $\sigma = k\frac{\pi}{2}$, $(k = 0, 1, 2, 3)$, the critical part C" coincides with the irregular parabola P_1 (Figure 1 for $\sigma = 0$).

- If $\sigma = \frac{\pi}{4} + k\frac{\pi}{2}$, $(k = 0, 1, 2, 3)$, the critical part C" consists of two upper portions (those bound below by $a^2 \geq \frac{1}{3}a_*^2$) of the irregular parabola P_2 and a single critical point I (Figure 4 corresponds to $\sigma = \pi/4$).

- For other values of σ, only the point I is critical (Figures 2, 3 respectively correspond to $\sigma = \pi/12$ and $\sigma = \pi/6$).

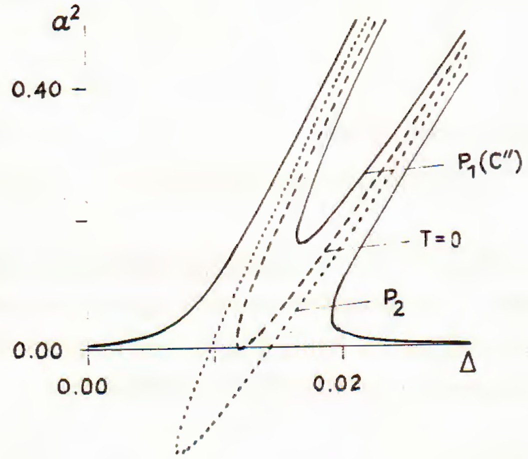

Fig. 1. $h = 0$, $\sigma = 0$,
$\gamma = 0.04$, $b = 0.4$, $C = 0.2$

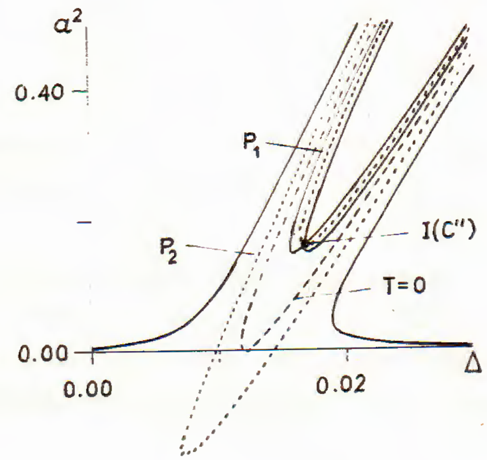

Fig. 2. $h = 0$, $\sigma = \frac{\pi}{12}$,
$\gamma = 0.04$, $b = 0.4$, $C = 0.2$

For the system with damping ($h > 0$), the critical part C" (if it exists) is reduced to a single point. Let us examine in detail the case $\sigma = 0$. The resonance curves shown in Figures 5, 6, 7, 8, 9 correspond to $h = 0.001$, 0.002753, 0.0033, 0.00357, 0.00362. In the case $h = 0.001$ the resonance curve C_0 only consists of the regular part C' (Figure 5). Successively a single returning critical point I on

P_2 (Figure 6), and a loop appear (Figure 7). At $h \approx 0.00357$, the loop connects with the left and right regular branches (Figure 8). The last form of the resonance curve is given in Figure 9.

The resonance curve in Figure 10 is plotted with $\sigma = \pi/4$, $h = 0.0033$; the only critical point I moves up. Figures 11, 12 show the resonance curves in the case $\sigma = \pi/6$. They are plotted with $h = 0.001, 0.002$ respectively: the critical point I moves along the irregular parabola P_1.

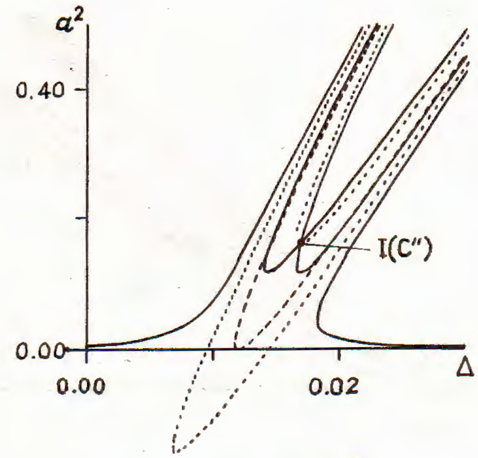

Fig. 3. $h = 0, \sigma = \dfrac{\pi}{6},$
$\gamma = 0.04, b = 0.4, C = 0.2$

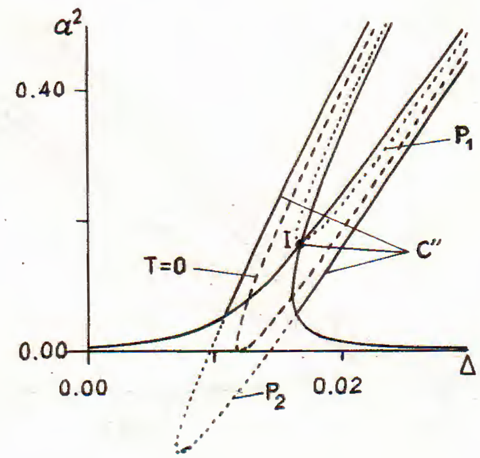

Fig. 4. $h = 0, \sigma = \dfrac{\pi}{4},$
$\gamma = 0.04, b = 0.4, C = 0.2$

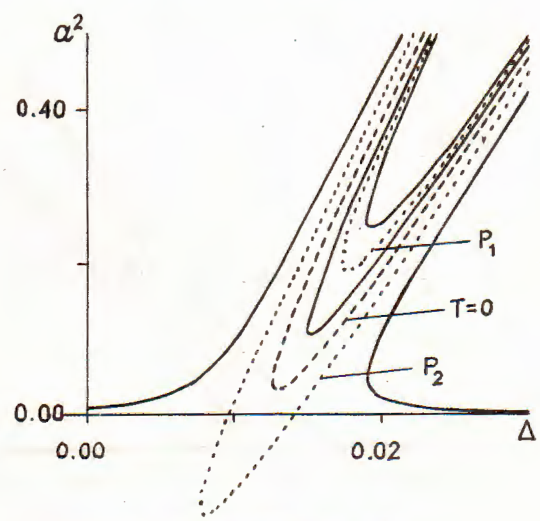

Fig. 5. $h = 0.001, \sigma = 0,$
$\gamma = 0.04, b = 0.4, C = 0.2$

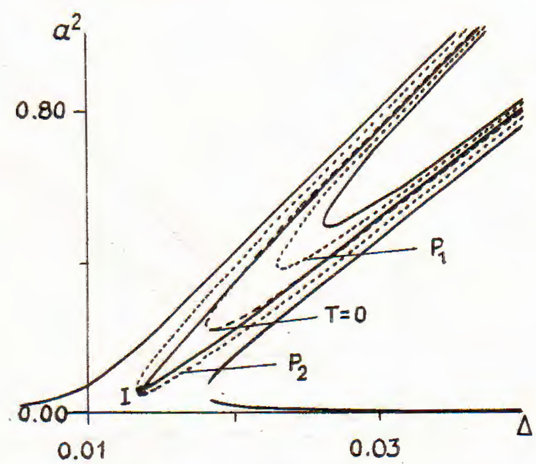

Fig. 6. $h = 0.002753, \sigma = 0,$
$\gamma = 0.04, b = 0.4, C = 0.2$

For ordinary stationary oscillations ($D \neq 0, h > 0$) with representative points in the equivalence domain ($T \neq 0$), the stability condition is:

$$\frac{1}{TD}\,\frac{\partial W}{\partial a^2} > 0.$$

For example, in Figure 7 the branches containing heavy points correspond to stable oscillations.

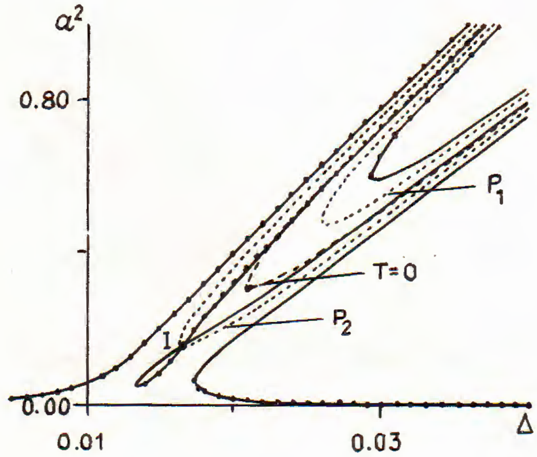

Fig. 7. $h = 0.0033$, $\sigma = 0$, $\gamma = 0.04$, $b = 0.4$, $C = 0.2$

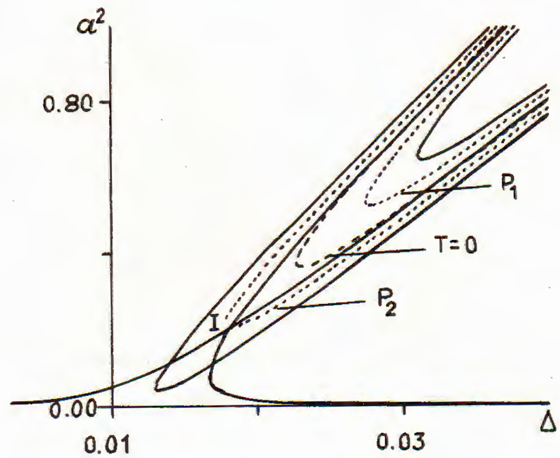

Fig. 8. $h = 0.00357$, $\sigma = 0$, $\gamma = 0.04$, $b = 0.4$, $C = 0.2$

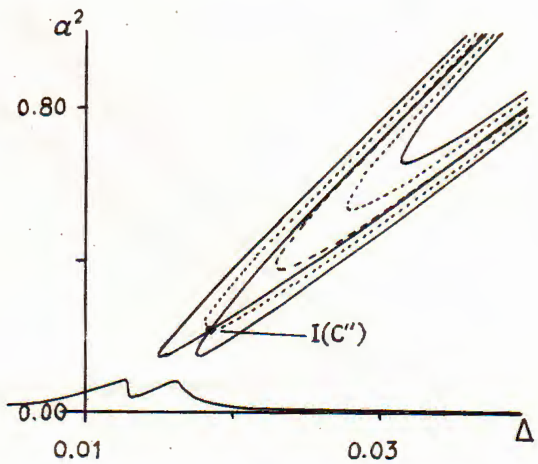

Fig. 9. $h = 0.00362$, $\sigma = 0$, $\gamma = 0.04$, $b = 0.4$, $C = 0.2$

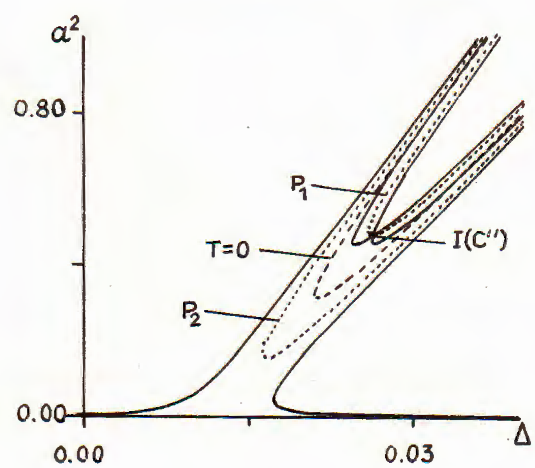

Fig. 10. $h = 0.0033$, $\sigma = \frac{\pi}{4}$, $\gamma = 0.04$, $b = 0.4$, $C = 0.2$

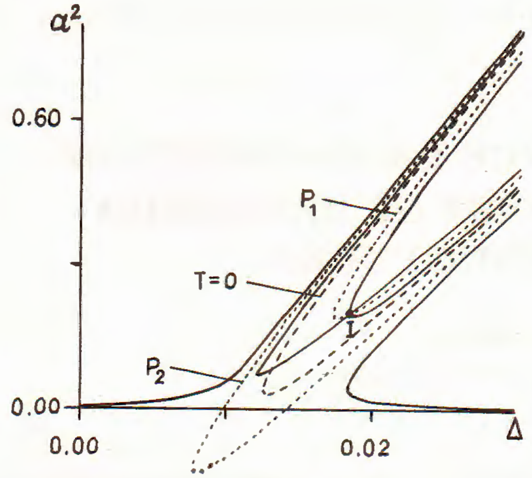

Fig. 11. $h = 0.001$, $\sigma = \dfrac{\pi}{6}$,
$\gamma = 0.04$, $b = 0.4$, $C = 0.2$

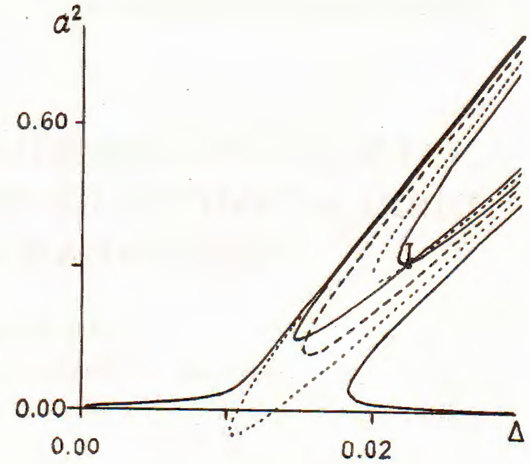

Fig. 12. $h = 0.002$, $\sigma = \dfrac{\pi}{6}$,
$\gamma = 0.04$, $b = 0.4$, $C = 0.2$

5. Conclusion

Mixed oscillations in a quasi-linear system were examined. Higher harmonics of the dephase in the equations of stationary oscillations were eliminated. The resonance curve was identified analytically. An extended abbreviated form of the second condition of stability facilitated the stability study. Diverse complicated resonance curve were obtained.

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DAO ĐỘNG HỖN HỢP Ở MỘT HỆ Á TUYẾN

Xét chấn tử phi tuyến yếu bậc ba chịu hai ngoại kích động cộng hưởng thứ điều hòa cấp 1/2 và 1/3. Trong hệ xuất hiện thành phần dao động hỗn hợp thông số - cưỡng bức do tương tác giữa các kích động tương ứng thứ cấp gây ra. Các đường cộng hưởng đã được xác định và tính ổn định đã được nhận biết dễ dàng nhờ dạng gọn mở rộng của điều kiện ổn định thứ hai.