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TRANSITION LENGTH OF TWO STATIONARY RANDOM FUNCTIONS IN INVESTIGATION OF RAILWAY STABILITY

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ABSTRACT. In this paper the scientific justification for determination of transition length of two stationary random functions is presented on the basis of the weight function and properties of stationary random function in wide sense. The application of the results for practice gives the transition lengths of straight and curved railway, which it is necessary to throw away in measurement of initial horizontal displacement of railway in its stability estimation under action of longitudinal compression load.

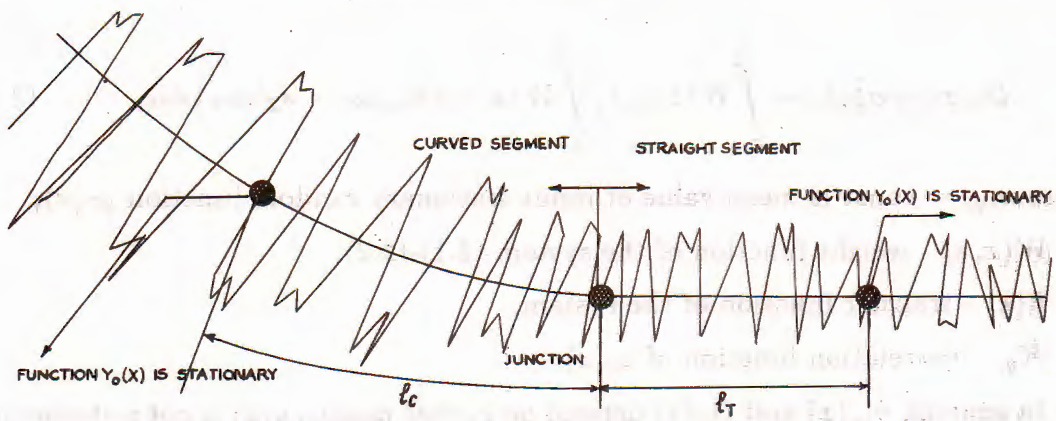

Fig. 1

1. Introduction

Measurement data of the initial horizontal displacement of the neutral axis of straight and curved railway are different stationary gaussian random functions. The connection of the two kinds of the railway makes them to become unstationary, and it is difficult for processing measurement data for research of railway stability. To overcome the difficulty it is necessary to find the reasonable set of measurement data near to connection place, which must be no used in its processing.

The problem will be solved below by theory and application in practice.

2. Scientific justification

2.1. Let us consider the mechanical system governed by random differential equation

$$F_x y(x) = H_x y_0(x), \tag{2.1}$$

where

$$F_x = \sum_{j=0}^{n} a_j \frac{d^{(j)}}{dx^j}, \quad H_x = \sum_{k=0}^{m} b_k \frac{d^{(k)}}{dx^k} \tag{2.2}$$

and a_j $(j = 0, \ldots, n)$, b_k $(k = 0, \ldots, m)$ are real constant coefficients, $y_0(x)$ is input random function of independent variable x, $y(x)$ is output function.

The mean value $m_y(x)$ and variance $D_y(x)$ of $y(x)$ can be found by the following formulae [2]

$$m_y(x) = m_{y_0} \int_0^x W(x,s)ds = m_{y_0} h(x), \tag{2.3}$$

$$D_y(x) = \sigma_y^2(x) = \int_0^x W(x,s_1)\Big[\int_0^x W(x,s_2)K_{y_0}(s_1 - s_2)ds_2\Big]ds_1, \tag{2.4}$$

where $m_{y_0} = \text{const}$ is mean value of input stationary random function $y_0(x)$,

$W(x,s)$ - weight function of the system (2.1)-(2.2),

$h(x)$ - transfer function of the system,

K_{y_0} - correlation function of $y_0(x)$.

In general, $m_y(x)$ and $D_y(x)$ depend on x, that means, $y(x)$ is not a stationary random process. The conditions for stationariness in wide sense of the process $y(x)$ are following

$$m_y(x) = \text{const} \quad \text{or} \quad \frac{dm_y(x)}{dx} = 0, \tag{2.5}$$

$$D_y(x) = \text{const} \quad \text{or} \quad \frac{dD_y(x)}{dx} = 0. \tag{2.6}$$

2.2. Determination of weight function

First of all consider the homogeneouss linear different equation

$$F_x y(x) = 0. \tag{2.7}$$

The weight function $g(x,s)$ of the equation could be found as follows [3]

$$F_x g(x,s) = 0 \tag{2.8}$$

with the initial conditions:

$$\begin{aligned} g(s,s) &= \frac{d}{dx}\big[g(x,s)\big]_{x=s} = \cdots = \frac{d^{(n-2)}}{dx^{n-2}}\big[g(x,s)\big]_{x=s} = 0 \\ \frac{d^{(n-1)}}{dx^{n-1}}\big[g(x,s)\big]_{x=s} &= \frac{1}{a_n(s)} \end{aligned} \tag{2.9}$$

The weight function $W(x,s)$ for nonhomogeneous equation (2.1) could be determined from formula

$$W(x,s) = H_s^* g(x,s) \tag{2.10}$$

here, H_s^* is the conjugate operator of H_x and

$$H_s^* g(x,s) = \sum_{k=0}^{m} (-1)^k \frac{d^{(k)}}{ds^k}[b_k g(x,s)] \tag{2.11}$$

2.3. Determination of mean value m_{y_0} and correlation function K_{y_0}.

In the formulae (2.3) and (2.4) the values m_{y_0} and K_{y_0} are approximately calculated by the following expressions

$$m_{y_0} = \frac{\sum_{i=0}^{N} y_{0i}}{N}, \tag{2.12}$$

$$K_{y_0}(s) = \frac{1}{N-s} \sum_{i=0}^{n-s-1} (y_{0i} - m_{y_0})(y_{0i+s} - m_{y_0}). \tag{2.13}$$

2.4. After determining mean value m_{y_0}, correlation function K_{y_0} and weight function $W(x,s)$ can use the equations (2.5) and (2.6) for obtaining the transition lengths.

$$\frac{dm_y(x)}{dx} = m_{y_0} \frac{d}{dx} \int_0^x W(x,s)ds = 0, \tag{2.14}$$

$$\frac{dD_y(x)}{dx} = \frac{d}{dx}\Big\{ \int_0^x W(x,s_1)\Big[\int_0^x W(x,s_2)K_{y_0}(s_1 - s_2)ds_2\Big]ds_1\Big\} = 0. \tag{2.15}$$

Note, that

$$|W(x,s)| < \infty, \tag{2.16}$$

$$\left| W(x,s_1)\Big[\int_0^x W(x,s_2) K_{y_0}(s_1 - s_2) ds_2 \Big] \right| < \infty, \tag{2.17}$$

then from (2.14) and (2.15) we have

$$W(x,s) = 0. \tag{2.18}$$

The condition (2.18) could be replaced with acceptable error in practice by inequality

$$W(x,s) \leq 0.05 \tag{2.19}$$

for determining the transition lengths.

3. Application

In this section we try to apply previous results for straight railway and ℓ_c on the curved one.

First of all on the basis of the works [4, 5] it is able to get the equation for straight railway

$$EJ_t \frac{d^{(4)}y}{dx^4} + (P_t - m_{mo}) \frac{d^{(2)}y}{dx^2} + m_u y = -P_t \frac{d^{(2)}y_0}{dx^2} \tag{3.1}$$

and the equation for curved railway

$$EJ_c \frac{d^{(5)}y}{dx^5} + \Big(\frac{EJ_c}{R^2} + P_c - m_{mo} \Big) \frac{d^{(3)}y}{dx^3} + \Big(\frac{P_c}{R^2} + m_u \Big) \frac{dy}{dx} = -\Big(\frac{EJ_c}{R^2} + P_c \Big) \frac{d^{(3)}y_0}{dx^3} \tag{3.2}$$

where, denote

$$EJ_t = a_t; \quad (P_t - m_{mo}) = b_t; \quad m_u = C_t$$

$$EJ_c = a_c; \quad \Big(\frac{EJ_c}{R^2} + P_c - m_{mo} \Big) = b_c; \quad \Big(\frac{P_c}{R^2} + m_u \Big) = C_c$$

and

E - material elasticity modulus of railway,

J_t, J_c - inertia moment of straight and curved railway beam respectively,

P_t, P_c - longitudinal compression load on straight and curved railway respectively,

m_{mo} - distributive moment coefficient on the beam,

m_{mu}: elasticity modulus of the ballast,

R - curved radius of railway.

Using results in section 2, the weight function of (3.1) and (3.2) can be written as follows

$$W_{x_t}(x,s) = \frac{-P_t}{\beta_t(3\alpha_t^2 - \beta_t^2)a_t} e^{-\alpha_t(x-s)}\Big[(\alpha_t^2 - \beta_t^2)\sin\beta_t(x-s) - 2\alpha_t\beta_t\cos\beta_t(x-s)\Big] \tag{3.3}$$

$$W_{x_c}(x,s) = \frac{-\left(\dfrac{EJ_c}{R^2} + P_c\right)}{4\alpha_c\beta_c(\alpha_c^2 - \beta_c^2)a_c} \times \tag{3.4}$$

$$\times e^{-\alpha_c(x-s)}\Big[\alpha_c(3\beta_c^2 - \alpha_c^2)\sin\beta_c(x-s) - \beta_c(\beta_c^2 - 3\alpha_c^2)\cos\beta_c(x-s)\Big],$$

where α_t, α_c and β_t, β_c are determined by the formulae

$$\alpha = \sqrt{\left(\frac{c}{4a}\right)^{1/2} - \frac{b}{4a}}\,; \quad \beta = \sqrt{\left(\frac{c}{4a}\right)^{1/2} + \frac{b}{4a}}\,. \tag{3.5}$$

For the real positive parameters $a > 0$, $b > 0$, $c > 0$, we have from (2.16) and (3.3), (3.4)

$$C_t \neq \frac{b_t^2}{a_t}, \quad C_c > \frac{b_c^2}{4a_c}. \tag{3.6}$$

In order to satisfy the condition (2.19) and from (3.3) and (3.4) we have

$$|-2\alpha_t\beta_t| > |\alpha_t^2 - \beta_t^2| \quad \text{and} \quad |\alpha_c(3\beta_c - \alpha_c^2)| > |-\beta_c(\beta_c^2 - 3\alpha_c^2)|.$$

Finally, we obtain the following equations for determining ℓ_t and ℓ_c respectively:

$$\left|\frac{-P_t}{(3\alpha_t^2 - \beta_t^2)a_t} 2\alpha_t \cdot e^{-\alpha_t(x-s)}\right| = 0.05, \tag{3.7}$$

$$\left|\frac{-\left(\dfrac{EJ_c}{R^2} + P_c\right)}{4\beta_c(\alpha_c^2 - \beta_c^2)a_c}(3\beta_c^2 - \alpha_c^2)e^{-\alpha_c(x-s)}\right| = 0.05, \tag{3.8}$$

in which $s = \dfrac{1}{2}x$.

The numerical resultats are given on the tables 1 and 2

Table 1. Given data: $EJ_c = EJ_t = a_c = a_t = 158.34\,(\mathrm{Tm}^2)$, $s = \frac{1}{2}x$, $m_u = 2000\,(\mathrm{T/m}^2)$, $m_{mo} = 0,728\,(\mathrm{T})$

P (T)	ℓ_t (m)	R (m)				
		300	500	700	1000	1200
80	3.33145	5.27542	5.27542	5.27542	5.27542	5.27543
100	3.76558	5.35896	5.35896	5.35896	5.35896	5.35897
150	4.65766	5.57562	5.57562	5.57562	5.57562	5.57562
200	5.4262	5.80255	5.80255	5.80255	5.80255	5.80256

Table 2. Given data: $EJ_c = EJ_t = a_c = a_t = 2 \times 158.34$ (Tm^2), $s = \frac{1}{2}x$, $m_u = 2 \times 2000$ $(\mathrm{T/m}^2)$, $m_{mo} = 2 \times 0.728$ (T)

P (T)	ℓ_t (m)	R (m)				
		300	500	700	1000	1200
80	2.11821	5.11900	5.11900	5.11900	5.11901	5.11901
100	2.49289	5.15577	5.15577	5.15577	5.15578	5.15578
150	3.21106	5.25490	5.25490	5.25490	5.25490	5.25490
200	3.76558	5.35896	5.35896	5.35896	5.35896	5.35896

4. Conclusion

The presented method can be applied to determining the transition lengths of straight and curved railway and has given numerical values, which could be used in practice.

For the curved railway the transition length ℓ_c directly proportional to compression load P_c and inversly proportional to curved radius R. The lengths ℓ_t and ℓ_c are inversely proportional to parameters E, J, m_{mo}. The value ℓ_c is always greater than ℓ_t at the same longitudinal load and structure parameters.

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ĐỘ DÀI CHUYỂN TIẾP GIỮA HAI HÀM NGẪU NHIÊN DỪNG
TRONG NGHIÊN CỨU ỔN ĐỊNH CỦA ĐƯỜNG RAY

Bài báo này trình bày cơ sở khoa học của việc xác định độ dài chuyển tiếp của 2 hàm ngẫu nhiên dừng khác nhau dựa trên cơ sở hàm trọng và tính chất của hàm ngẫu nhiên dừng theo nghĩa rộng. Áp dụng các kết quả này vào thực tế ta nhận được độ dài chuyển tiếp của đường ray thẳng và cong. Đó là các đoạn phải bỏ qua khi đo dịch chuyển ngang ban đầu của đường ray trong bài toán đánh giá ổn định của chúng dưới tác dụng của tải trọng dọc.