STATIC DIAGNOSIS OF MULTIPLE CRACKED BEAM

Tran Van Lien
University of Civil Engineering

Nguyen Tien Khiem
Institute of Mechanics, NCST of Vietnam

1. Model of multiple cracked beam

Model of crack was studied by many authors. Adams R. D. and Cawley P. in 1978 have proposed an axial spring model [1] to investigate the problem of location of crack in a bar using natural frequencies and mode shape. However, in [1] the stiffness of the equivalent spring has not been calculated from the crack depth. The formula relating the spring stiffness and crack depth has been established due to study of Ju and others [2] in 1982. Subsequent studies by Haisty B. S. and Springer W. T. [3] and Dimaroganas A. D. and Chondros T. G. [4] have made a great progress in improvement of the formula. In this paper the rotational spring model of transverse crack in beam developed in studies of Dimaroganas A. D. and his coworkers is adopted and used for solving the crack detection problem. The beam with single transverse crack has been studied in a lot of publications [5, 6]. Less amount of works [7, 8] devoted to the case of multiple cracked beam, especially to the problem of multi crack detection. Following our study in [8] concerned the multiple cracked beam, the problem of multi crack detection using static displacements measured in a beam is considered in the present paper. The theoretical investigation will be illustrated by a numerical example for a cantilever beam with two and three cracks.

Thus, a crack of the depth a at the position x^* (Figure 1.a), following to the [4] may be modeled as a rotational spring of stiffness

$$K = \frac{1}{\alpha}; \quad \alpha = \frac{6\pi(1-\nu^2)h}{EI_z} I_c\left(\frac{a}{h}\right), \tag{1.1}$$

where

$$I_c(z) = 0.6272z^2 - 1.04533z^3 + 4.5948z^4 - 9.973z^5 + 20.2948z^6 - 33.0351z^7$$
$$+ 47.1063z^8 - 40.7556z^9 + 19.6z^{10}, \tag{1.2}$$

which is determined experimentally. Therefore, a beam with n cracks of depth a_j at x_j, $j = 1, \ldots, n$ can be modeled as shown in Figure 1.b.
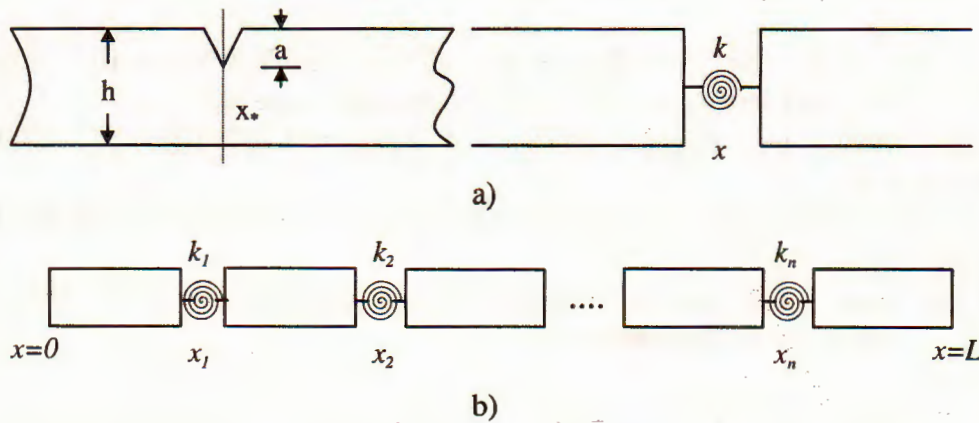
Fig. 1

2. Static response of multiple cracked beam

Let us consider the beam in bending only, described by the equation:

$$EI_z \frac{d^4\Phi(x)}{dx^4} = q(x), \tag{2.1}$$

where $\Phi(x)$ is the flexural displacement of the beam at section x and $q(x)$ is a distributed load Introduce the notations

$$U_1 = \Phi(0); \quad U_2 = \Phi'(0); \quad U_3 = \Phi(L); \quad U_4 = \Phi'(L). \tag{2.2}$$

which is the generalized displacement of the beam and represents vector of degree of freedom $U = \{U_1, U_2, U_3, U_4\}^T$ and

$$P_1 = EI_z\Phi'''(0); \quad P_2 = -EI_z\Phi''(0); \quad P_3 = -EI_z\Phi'''(L); \quad P_4 = EI_z\Phi''(L) \tag{2.3}$$

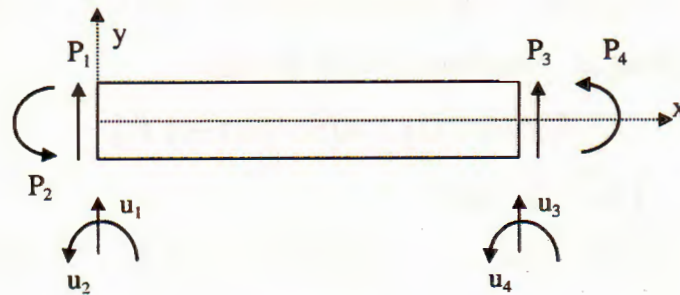being a vector of the end forces. These displacements and forces are shown in Figure 2.

Fig. 2

For displacements and forces at the section x_j, $j = 1, \ldots, n$ one uses the notations

$$\{Z^-(j)\} = \{Z_1^-(j), Z_2^-(j), Z_3^-(j), Z_4^-(j)\}$$
$$\{Z^+(j)\} = \{Z_1^+(j), Z_2^+(j), Z_3^+(j), Z_4^+(j)\}; \quad j = 1, 2, \ldots, n$$

225

$Z_1^\pm(j) = \Phi(x_j \pm 0); \quad Z_2^\pm(j) = \Phi'(x_j \pm 0); \quad Z_3^\pm(j) = \pm EI_z\Phi'''(x_j \pm 0); \quad z_4^\pm(j) = \mp EI_z\Phi''(x_j \pm 0)$, and letting $x_0 = 0$, $x_{n+1} = L$ one will have also

$Z_1^+(0) = \Phi(0) = U_1; \quad Z_2^+(0) = \Phi'(0) = U_2; \quad Z_3^+(0) = EI_2\Phi'''(0) = P_1; \quad Z_4^+(0) = -EI_2\Phi''(0) = P_2;$

$Z_1^-(n+1) = \Phi(L) = U_3; \quad Z_2^-(n+1) = \Phi'(L) = U_4; \quad Z_3^-(n+1) = -EI_z\Phi'''(L) = P_3; \quad Z_4^-(n+1) = EI_z\phi''(L) = P_4.$

On the other hand, general solution of equation (2.1) for $x \in (x_{j-1}, x_j)$, $j = 1, 2, \ldots, n+1$ can be represented as

$$\Phi(x) = \Phi_0(\overline{x}) + \frac{1}{6EI_z}\int_0^{\overline{x}}(\overline{x}-\tau)^3 q(x_{j-1}+\tau)d\tau; \quad x \in [x_{j-1}, x_j]; \quad \overline{x} = x - x_{j-1}$$

$$(2.4)$$

where

$$\Phi_0(x) = Z_1^+(j-1) + \overline{x}Z_2^+(j-1) + \frac{\overline{x}^3}{6EI_z}Z_3^+(j-1) - \frac{\overline{x}^2}{2EI_z}Z_4^+(j-1), \quad \overline{x} = x - x_{j-1}.$$

Furthermore, introducing the functions

$$\overline{q}_1(\ell_j) = \frac{1}{6EI_z}\int_0^{\ell_j}(\ell_j - \tau)^3 q(x_{j-1}+\tau)d\tau, \quad \overline{q}_3(\ell_j) = -\int_0^{\ell_j} q(x_{j-1}+\tau)d\tau,$$

$$\overline{q}_2(\ell_j) = \frac{1}{2EI_z}\int_0^{\ell_j}(\ell_j - \tau)^2 q(x_{j-1}+\tau)d\tau, \quad \overline{q}_4(\ell_j) = \int_0^{\ell_j}(\ell_j - \tau)q(x_{j-1}+\tau)d\tau$$

one will have a vector of generalized load

$$\{\overline{q}(j)\} = \{\overline{q}_1(\ell_j), \overline{q}_2(\ell_j), \overline{q}_3(\ell_j), \overline{q}_4(\ell_j)\}; \quad j = 1, 2, \ldots, n+1.$$

Following the method of transfer matrix [8] one gets

$$\{Z^-(n+1)\} = \mathbf{Q}\{Z^+(0)\} + \{\tilde{F}\} \tag{2.5}$$

where the matrix $\mathbf{Q}$ is of the form

$$\mathbf{Q} = T_{n+1}Q(n)Q(n-1)\ldots Q(1) = T_{n+1}J_nT_nJ_{n-1}\ldots J_2T_2J_1T_1 \tag{2.6}$$

with $Q(j) = J_jT_j$ and

$$T_j(\ell_j) = \begin{pmatrix} 1 & \ell_j & \ell_j^3/6EI_z & -\ell_j^2/2EI_z \\ 0 & 1 & \ell_j^2/2EI_z & -\ell_j/EI_z \\ 0 & 0 & -1 & 0 \\ 0 & 0 & \ell_j & -1 \end{pmatrix}; \quad \ell_j = x_j - x_{j-1}; \ j = 1, \ldots, n+1, \tag{2.7}$$

$$J_j = J(\alpha_j) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_j \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \quad j = 1, \ldots, n; \quad \alpha_j = \frac{6\pi(1-\nu^2)h}{EI_z} I_c\left(\frac{a_j}{h}\right).$$

(2.8)

The vector $\{\tilde{F}\}$ is calculated by the formula:

$$\{\tilde{F}\} = \bar{q}(n+1) + T_{n+1}J_n\bar{q}(n) + T_{n+1}J_nT_nJ_{n-1}\bar{q}(n-1) +$$
$$+ T_{n+1}J_nT_nJ_{n-1}T_{n-1}J_{n-2}\bar{q}(n-2) + \cdots + T_{n+1}J_nT_n \ldots J_2T_2J_1\bar{q}(1). \quad (2.9)$$

Writing the matrix $[Q]$ into the form $\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \\ \mathbf{Q}_3 & \mathbf{Q}_4 \end{pmatrix}$, where

$$\mathbf{Q}_1 = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}; \quad \mathbf{Q}_2 = \begin{pmatrix} Q_{13} & Q_{14} \\ Q_{23} & Q_{24} \end{pmatrix}; \quad \mathbf{Q}_3 = \begin{pmatrix} Q_{31} & Q_{32} \\ Q_{41} & Q_{42} \end{pmatrix}; \quad \mathbf{Q}_4 = \begin{pmatrix} Q_{33} & Q_{34} \\ Q_{43} & Q_{44} \end{pmatrix}$$

and using the notations introduced above, we have from (2.5)

$$\begin{pmatrix} U_3 \\ U_4 \end{pmatrix} = \mathbf{Q}_1 \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} + \mathbf{Q}_2 \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} + \begin{pmatrix} \tilde{F}_1 \\ \tilde{F}_2 \end{pmatrix}; \quad \begin{pmatrix} P_3 \\ P_4 \end{pmatrix} = \mathbf{Q}_3 \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} + \mathbf{Q}_4 \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} + \begin{pmatrix} \tilde{F}_3 \\ \tilde{F}_4 \end{pmatrix}$$

that can be written in the matrix form,

$$\mathbf{K}\{U\} = \{P\} + \mathbf{G}\{\tilde{F}\}, \tag{2.10}$$

where

$$\mathbf{K} = \begin{pmatrix} -\mathbf{Q}_2^{-1}\mathbf{Q}_1 & \mathbf{Q}_2^{-1} \\ \mathbf{Q}_3 - \mathbf{Q}_4\mathbf{Q}_2^{-1}\mathbf{Q}_1 & \mathbf{Q}_4\mathbf{Q}_2^{-1} \end{pmatrix}; \quad \mathbf{G} = \begin{pmatrix} \mathbf{Q}_2^{-1} & 0 \\ \mathbf{Q}_4\mathbf{Q}_2^{-1} & -I \end{pmatrix}. \tag{2.11}$$

$\mathbf{K}$ is the stiffness matrix and $\{F\} = \mathbf{G}\{\tilde{F}\}$ is vector of generalized load. Suppose furthermore that the boundary conditions are linear of the form

$$\mathbf{B}_u\{U\} + \mathbf{B}_p\{P\} = 0 \tag{2.12}$$

where

$$\mathbf{B}_u = \begin{pmatrix} B_{11}^0 & B_{12}^0 & 0 & 0 \\ B_{21}^0 & B_{22}^0 & 0 & 0 \\ 0 & 0 & B_{11}^L & B_{12}^L \\ 0 & 0 & B_{21}^L & B_{22}^L \end{pmatrix}; \quad \mathbf{B}_p = \begin{pmatrix} B_{13}^0 & B_{14}^0 & 0 & 0 \\ B_{23}^0 & B_{24}^0 & 0 & 0 \\ 0 & 0 & B_{13}^L & B_{14}^L \\ 0 & 0 & B_{23}^L & B_{24}^L \end{pmatrix}, \tag{2.13}$$

then (2.10) can be rewritten as

$$\mathbf{K}_u\{U\} + \mathbf{K}_p\{P\} = -\{\tilde{F}\} \tag{2.14}$$

227

with

$$K_u = \begin{pmatrix} Q_1 & -I \\ Q_3 & 0 \end{pmatrix}; \quad K_p = \begin{pmatrix} Q_2 & 0 \\ Q_4 & -I \end{pmatrix}. \tag{2.15}$$

Finally, equations (2.12) and (2.14) must be solved together, so they form a system

$$\overline{K}\{\overline{U}\} = -\{\overline{P}\}, \tag{2.16}$$

where

$$\overline{K} = \begin{pmatrix} K_u & K_p \\ B_u & B_p \end{pmatrix}; \quad \{\overline{U}\} = \begin{pmatrix} \{U\} \\ \{P\} \end{pmatrix}; \quad \{\overline{P}\} = -\begin{pmatrix} \{\tilde{F}\} \\ \{0\} \end{pmatrix}. \tag{2.17}$$

Solving equation (2.16) gives as vector of generalized displacements, as well as the vector of friction end forces or in fact the state vectors $\{Z^+(0)\}$, $\{Z^-(n+1)\}$. Using the result obtained one can determine fully the field of displacement, slope, bending moment and shear force:

$$\{Z^+(j)\} = Q(J)\{Z^+(j-1)\} + J_j\{\overline{q}(\ell_j)\}, \quad j = 1, \ldots, n;$$

$$\{Z(x)\} = [T_j(\lambda, \overline{x})]\{Z^+(j-1)\} + \{\overline{q}(\overline{x})\}. \quad \overline{x} - x_{j-1}. \tag{2.18}$$

Here

$$\{Z(x)\} = \{Z_1(x), Z_2(x), Z_3(x), Z_4(x)\}^T = \{\Phi(x), \Phi'(x), -EI_z\Phi'''(x), EI_z\Phi''(x)\}^T. \tag{2.19}$$

Let's consider a cantilever beam loaded by a force F applied to the free end. In the case of uncracked beam one has

$$\Phi_0^F(x) = -\frac{Fx^3}{6EI_z} + \frac{FLx^2}{2EI_z}; \quad U_3 = \Phi_0^F(L) = -\frac{FL^3}{3EI_z}; \quad \Phi_0'^F(L) = -\frac{FL^2}{2EI_z}. \tag{2.20}$$

For cantilever beam with cracks at positions $\{x_j; j = 1, 2, \ldots, n\}$ and with depth $\{a_j; j = 1, 2, \ldots, n\}$, the given load F leads to the field of displacement

$$\Phi(x) = \begin{cases} \Phi_0(x) = \Phi_0^F(x) = -\dfrac{Fx^3}{6EI_z} + \dfrac{FLx^2}{2EI_z} & \text{for } 0 \le x \le x_1 \\ \Phi_1(c) = \Phi_0^F(x) - F\alpha_1(x - x_1)(L - x_1) & \text{for } x_1 \le x \le x_2 \\ \ldots \quad \ldots \quad \ldots & \ldots \quad \ldots \\ \Phi_n(x) = \Phi_0^F(x) - \sum\limits_{j=1}^{n} F\alpha_j(x - x_j)(L - x_j) & \text{for } x_n \le x \le L. \end{cases} \tag{2.21}$$

In particular, at free end $x = L$ the displacement and slope are

$$U_3 = \Phi_0^F(L) = -\frac{FL^3}{3EI_z}\left[1 + \frac{3EI_z}{L^3}\sum_{j=1}^{n}\alpha_j(L - x_j)^2\right]$$

$$(2.22)$$

$$U_4 = \Phi_0'^F(L) = -\frac{FL^2}{2EI_z}\left[1 + \frac{2EI_z}{L^3}\sum_{j=1}^{n}\alpha_j(L - x_j)\right].$$

Comparing (2.22) and (2.20), one can obtain the factors representing amplification due to cracks of displacement and slope as follow

$$A_3^F = \left[1 + \frac{3EI_z}{L^3}\sum_{j=1}^{n}\alpha_j(L - x_j)^2\right]; \quad A_4^F = \left[1 + \frac{2EI_z}{L^3}\sum_{j=1}^{n}\alpha_j(L - x_j)\right] \quad (2.23)$$

Analogously, in the case of uniformly distributed load q, the displacement field can be found in the form

$$\Phi(x) = \begin{cases} \Phi_0(x) &= \Phi_0^q(x) = -\dfrac{qx^4}{24EI_z} + \dfrac{qLx^3}{6EI_z} - \dfrac{qL^2x^2}{4EI_z} \quad \text{for } 0 \le x \le x_1 \\[2mm] \Phi_1(x) &= \Phi_0^q(x) - \dfrac{q\alpha_1}{2}(x - x_1)(L - x_1)^2 \quad \text{for } x_1 \le x \le x_2 \\[2mm] \Phi_2(x) &= \Phi_0^q(x) - \dfrac{q\alpha_1}{2}(x - x_1)(L - x_1)^2 \\[1mm] & \quad - \dfrac{q\alpha_2}{2}(x - x_2)(L - x_2)^2 \quad \text{for } x_2 \le x \le x_3 \\[2mm] \cdots & \quad \cdots \quad \cdots \\[1mm] \Phi_n(x) &= \Phi_0^q(x) - \displaystyle\sum_{j=1}^{n}\dfrac{q\alpha_j}{2}(x - x_j)(L - x_j)^2 \quad \text{for } x_n \le x \le L. \end{cases}$$

$$(2.24)$$

In this case the amplification factors for displacement and slope at the free end have the analogous to the (2.23) form

$$A_3^q = \left[1 + \frac{4EI_z}{L^4}\sum_{j=1}^{n}\alpha_j(L - x_j)^3\right]; \quad A_4^q = \left[1 + \frac{3EI_z}{L^3}\sum_{j=1}^{n}\alpha_j(L - x_j)^2\right]. \quad (2.25)$$

3. Crack detection by using static displacement

In this section the problem of crack detection for a cantilever beam by displacement measured at various positions under static load applied to the free end is studied. To identify the crack parameters $\{\lambda_j\} = \{a_j, x_j\}$, $j = 1, \ldots, n$ it is necessary to measure displacements at $2n$ points. Moreover, the function-displacement as shown in (2.21) can be calculated at a point if amount of the cracks on the left of the point is known. This fact constrains one to assume that measurement points

must be distributed as shown in Figure 3. It means that the measurement points are coupled (u_j, u_{j+1}), $j = 1, \ldots, n$, no crack exists between the points in the pair and in the interval between the pairs there is only single crack. In practice the assumption is not much strict, because one can choose the measurement points mesh freely.
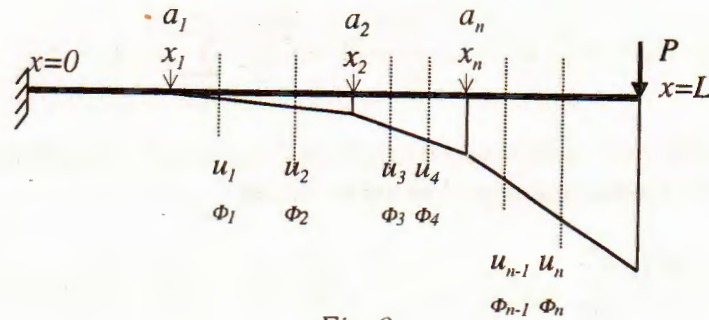

Fig. 3

Thus, it is supposed that the static displacements measured at the chosen mesh $(u_1, u_2, \ldots, u_{2n-1}, u_{2n})$ are $(\Phi_1, \Phi_2, \ldots, \Phi_{2n-1}, \Phi_{2n})$. We Introduce the function of crack parameters $\lambda = \{a_1, x_1, a_2, x_2, \ldots, a_n, x_n\}$:

$$
\begin{aligned}
f(\lambda) = f(a_1, x_1, a_2, x_2, \ldots, a_n, x_n) = \\
= \left|\overline{\Phi}_1 + F\alpha_1(u_1 - x_1)(L - x_1)\right|^2 + \left|\overline{\Phi}_2 + F\alpha_1(u_2 - x_1)(L - x_1)\right|^2 + \cdots + \\
+ \left|\overline{\Phi}_{2n-1} + \sum_{j=1}^{n} F\alpha_j(u_{2n-1} - x_j)(L - x_j)\right|^2 \\
+ \left|\overline{\Phi}_{2n} + \sum_{j=1}^{n} F\alpha_j(u_{2n} - x_j)(L - x_j)\right|^2.
\end{aligned}
\tag{3.1}
$$

$$
\overline{\Phi}_k = \Phi_k - \Phi_0^F(u_k), \quad k = 1, 2, \ldots, 2n.
$$

The arguments of the function (3.1) are constrained by

$$
0 \leq x_1 \leq u_1 < u_2 \leq x_2 \leq u_3 \leq u \cdots \leq x_n \leq u_{2n-1} < u_{2n} \leq L
$$
$$
0 \leq a_j < h; \quad j = 1, 2, \ldots, n
\tag{3.2}
$$

where

$$
\alpha_j = \frac{6\pi(1 - \nu^2)h}{EI_z} I_c\left(\frac{a_j}{h}\right), \quad j = 1, 2, \ldots, n;
$$
$$
I_c(z) = 0.6272z^2 - 1.04533z^3 + 4.5948z^4 - 9.973z^5 + 20.2948z^6 - 33.0351z^7
$$
$$
+ 47.1063z^8 - 40.7556z^9 + 19.6z^{10}.
$$

The diagnosis problem now can be formulated as a nonlinear programming problem

230

$$\begin{cases} f(\lambda) \to \min \\ G_j(\lambda) = 0; \quad j = 1, 2, \ldots, m_e \\ G_j(\lambda) \le 0; \quad j = m_e + 1, \ldots, m \\ \lambda_{\ell b} \le \lambda \le \lambda_{ub} \end{cases} \tag{3.3}$$

For solving the problem (3.3), algorithms given in MATLAB [10] can be used and in result one obtains crack position x_j and crack depth a_j. Input for the algorithm is static displacements measured at various points along the beam.

4. Numerical example and discussion

To illustrate the procedure proposed for crack detection, a cantilever beam with following properties is studied.

The length $L = 3\,\mathrm{m}$, cross section area $A = 0.2m \times 0.2m$, Young's Modulus $E = 2.1 \cdot 10^{11}\,N/m^2$, the force applied to free end $P = 3kN$.

Let's consider four scenarios of cracked beam as follow, the first scenario (S1) is the case of two cracks (at positions $1\,\mathrm{m}$ and $2\,\mathrm{m}$ from the fixed end of beam with depth of $0.06\,\mathrm{m}$ and $0.04\,\mathrm{m}$ respectively) and in the diagnosis problem only two cracks are sought. In this case the influence of the measurement noise on the diagnosis result is studied. The subsequence three cases under consideration are shown in Table 1.

Table 1

Scenario	S_2 (two cracks)			S_3 (three cracks)			S_4 (three cracks)		
	1^{st}cr.	2^{nd}cr.	3^{rd}cr.	1^{st}cr.	2^{nd}cr.	3^{rd}cr.	1^{st}cr.	2^{nd}cr.	3^{rd}cr.
Crack Position	$1\,\mathrm{m}$	$1.5\,\mathrm{m}$	$2.0\,\mathrm{m}$	$1\,\mathrm{m}$	$1.5\,\mathrm{m}$	$2.0\,\mathrm{m}$	$1.0\,\mathrm{m}$	$1.5\,\mathrm{m}$	$2.0\,\mathrm{m}$
Crack depth	$0.0\,\mathrm{m}$	$0.04\,\mathrm{m}$	$0.06\,\mathrm{m}$	$0.02\,\mathrm{m}$	$0.04\,\mathrm{m}$	$0.06\,\mathrm{m}$	$0.02\,\mathrm{m}$	$0.04\,\mathrm{m}$	$0.06\,\mathrm{m}$

In the case of scenario S_1 the measurement points mesh is $u_1 = 1.5\,\mathrm{m}$, $u_2 = 1.8\,\mathrm{m}$, $u_3 = 2.1\,\mathrm{m}$, $u_4 = 2.4\,\mathrm{m}$, $u_5 = 2.6\,\mathrm{m}$, $u_6 = 2.8\,\mathrm{m}$. For the scenarios S_2, S_3 the measurements are made at the positions $u_1 = 1.2\,\mathrm{m}$, $u_2 = 1.4\,\mathrm{m}$, $u_3 = 1.6\,\mathrm{m}$, $u_4 = 1.8\,\mathrm{m}$, $u_5 = 2.2\,\mathrm{m}$, $u_6 = 2.6\,\mathrm{m}$ and for the last case, $u_1 = 1.2\,\mathrm{m}$, $u_2 = 1.4\,\mathrm{m}$, $u_3 = 1.5\,\mathrm{m}$, $u_4 = 1.8\,\mathrm{m}$, $u_5 = 2.2\,\mathrm{m}$, $u_6 = 2.6\,\mathrm{m}$. It is necessary to note here that, in the second case (S_2 in table 1) only two cracks are presented, but in the diagnosis problem three cracks are sought. Of course, among the cracks one must be of zero depth. This case is considered to test the detection of zero depth crack. The last

scenario (S_4) is carried out for testing the case when measurement point is identical to the crack position.

The result of crack detection for the case of first scenario S_1 for different levels of measurement error, which has been introduced by addition of the term $\varepsilon.rand$, to the input of the crack detection program is given in Table 2. Here, $\varepsilon = 0.0$; $0.01; \ldots; 0.1$ corresponding to the 0%, 1%, 2%,..., 10% of measurement error and *rand* is a random number uniformly distributed in the interval $(0, 1)$. The cracks parameters detected in the three last scenarios are shown in Table 3.

Table 2

ε	0%	1%	2%	3%	4%	5%	6%	10%
x_1	0.9998	0.9747	0.9540	0.9363	0.9216	0.9089	0.8979	0.8652
a_1	0.0600	0.0625	0.0649	0.0671	0.0692	0.0713	0.0732	0.0800
x_2	1.9995	1.9979	1.9969	1.9941	1.9937	1.9921	1.9907	1.9875
a_2	0.0400	0.0420	0.0439	0.0457	0.0474	0.0490	0.0505	0.0562

actual crack parameters: $x_1 = 1.0$, $a_1 = 0.06$, $x_2 = 2.0$, $a_2 = 0.04$

Table 3

Scenario	S_2 (two cracks)			S_3 (three cracks)			S_4 (three cracks)		
	1^{st}cr.	2^{nd}cr.	3^{rd}cr.	1^{st}cr.	2^{nd}cr.	3^{rd}cr.	1^{st}cr.	2^{nd}cr.	3^{rd}cr.
Position	0.800	1.5002	1.9999	1.0094	1.5013	2.001	1.002	1.499	2.0
depth	0.0005	0.0400	0.0600	0.0203	0.0400	0.0599	0.02	0.0399	0.06

5. Conclusion

In this paper the following results are presented:

1. General solution for static displacement of multiple cracked beam subjected to concentrated as well as distributed load is obtained in an analytical form.

2. The problem for crack detection of multiple cracked cantilever by measurements of static displacement has been formulated in a form of the constrained non-linear programming, that may be solved by using MATLAB code.

3. An example has been investigated numerically to validate the developed herein theory. Different scenarios of cracks have been arranged and in each case the crack detection was tried. Results of detection given in Table 2, 3 show acceptability of the procedure proposed.

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CHẨN ĐOÁN TĨNH DẦM CÓ NHIỀU VẾT NỨT

Trong bài báo này các tác giả trình bày một thuật toán chẩn đoán nhiều vết nứt trong dầm bằng phương pháp đo độ võng tĩnh. Việc chẩn đoán này dựa trên cơ sở kết quả nghiên cứu phản ứng của dầm có nhiều vết nứt đối với tải trọng ngoài, mà trong trường hợp tải trọng tĩnh cho kết quả giải tích. Thuật toán này được minh họa bằng một ví dụ số, trong đó xem xét thử nghiệm nhiều phương án vết nứt khác nhau. Đã nghiên cứu ảnh hưởng của sai số đo đạc đến kết quả chẩn đoán. Xét trường hợp chẩn đoán vết nứt ở độ sâu bằng không (nghĩa là không có nứt) và thử nghiệm trường hợp điểm đo trùng với vị trí vết nứt. Các kết quả cho thấy thuật toán này có độ chính xác và tính ổn định chấp nhận được.