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ELASTOPLASTIC STABILITY OF THIN RECTANGULAR PLATES UNDER COMPLEX AND IMPURE LOADING

VU CONG HAM
Le Quy don Technical University

ABSTRACT. This paper deals with investigation of the elastoplastic stability of thin rectangular plates. The plate considered herein is subjected to the biaxial compressive forces which are assumed to be linearly distributed along every its edge.

The governing equations of the problem are formulated with applying the elastoplastic process theory whereas Bubnov - Galerkin method is used to calculate the critical forces.

In the paper the author proposes a new method to determine the elements of the matrix concerned with the instability moment of the structure and applies the Gaussian quadric method for integral calculation. Some results of numerical calculations are also presented in the paper.

1. Introduction

Let's consider a thin rectangular plate which has the biaxial dimensions a, b and the thickness h. A coordinate orthogonal system $Oxyz$ (or $Ox_1x_2x_3$ in tensor notations) is attached to the plate so that the plane Oxy coincides with the middle surface and the four edges can be mathematically described as $x = 0$, $y = 0$, $x = a$, $y = b$, respectively.

In [2, 3, 4, 5]. the so called pure loaded state is considered. According to this loaded state, the plate is subjected to one or any combination of biaxial compressive forces p, q and shear force τ (figure 1). These external forces are assumed to act in the middle surface and to be evenly distributed along every edge of the plate. Because of this, the prebuckling stress-strain state is pure at any point in the plate

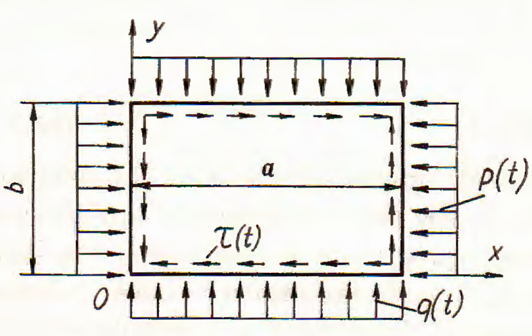

Fig. 1

This paper is concerned with the impure loading. The plate in the considered case is subjected to biaxial compressive forces p, q which are also assumed to act in the middle surface, but to be unevenly distributed along each edge, respectively. Because of mathematical difficulties, the expressions of p and q are not general, but accepted to be the product of two functions as follows

$$p = p_1(t) \cdot p_2(y), \quad q = q_1(t) \cdot q_2(x) \tag{1.1}$$

where $p_1(t)$, $q_1(t)$ can be called as the *process functions* which depend only on the process parameter t and increase with respect to the increment of t; $p_2(y)$, $q_2(x)$ are called as the *distribution functions* which characterize the dependence of p and q on the coordinates. According to the form of the distribution functions $p_2(y)$, $q_2(x)$, the problem can be considered in three cases:

a) $p_2(y) = q_2(x) \equiv 1$:

$$p = p_1(t), \quad q = q_1(t) \tag{1.2}$$

the loading (1.1) returns to the pure loading without shear force τ.

b) $p_2(y)$, $q_2(x)$ are linear

$$p = p_1(t) \cdot (a_1 y + b_1), \quad q = q_1(t) \cdot (a_2 x + b_2) \tag{1.3}$$

(where a_1, b_1, a_2, b_2 are constant) the loading can be called as the *linear loading* (figure 2)

c) $p_2(y)$, $q_2(x)$ are arbitrarily continuous, the loading can be called as the *curve loading* (figure 3)

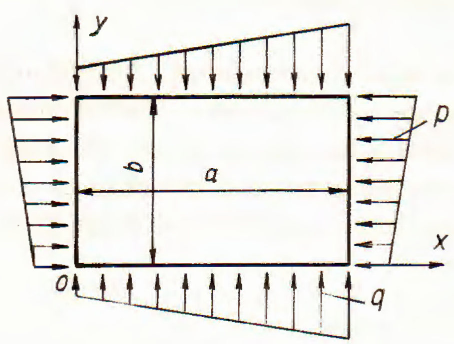

Fig. 2

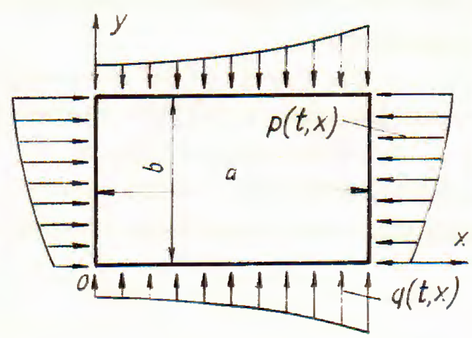

Fig. 3

Because the loading is assumed to be known, the functions $p_1(t)$, $q_1(t)$, $p_2(y)$, $q_2(x)$ are known as well. In the fact of solving the problem on PC, these functions are always chosen before the programme is performed. The loading is really complex if the process functions $p_1(t)$, $q_1(t)$ are chosen in linear independence.

The governing equations of the problem are formulated with applying the theory of the elsatoplastic process [1]. Some results received from solving numerically the problem are also given out in this paper.

2. Stability of rectangular plates subjected to linear loading

2.1. Governing equations of the problem

2.1.1. Prebuckling stage

Let's consider an arbitrary point $M(x, y, z)$ $(0 \le x \le a,\ 0 \le y \le b,\ -h/2 \le z \le h/2)$ in the plate. At any moment t in the prebuckling stage, there exists a plane stress state

$$\begin{aligned} \sigma_{11} &= -p_1(t) \cdot (a_1 y + b_1) \equiv -p, \\ \sigma_{22} &= -q_1(t) \cdot (a_2 x + b_2) \equiv -q, \\ \sigma_{12} &= \sigma_{23} = \sigma_{13} = \sigma_{33} = 0, \end{aligned} \tag{2.1}$$

so that

$$\sigma = \frac{\sigma_{11} + \sigma_{22}}{3}, \quad \sigma_u = \sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2}\,.$$

The stress components (2.1) satisfy the equilibrium equations, the boundary conditions of the plate and the Beltrami's equation of continuity.

The corresponding components of deformation velocity are determined according to the theory of elastoplastic process [1]

$$\dot{\varepsilon}_{11} = \frac{1}{A}\Big(\dot{\sigma}_{11} - \frac{1}{2}\dot{\sigma}_{22}\Big) + \Big(\frac{1}{P} - \frac{1}{A}\Big)\frac{\sigma_{11}\dot{\sigma}_{11} + \sigma_{22}\dot{\sigma}_{22} - \frac{1}{2}\sigma_{11}\dot{\sigma}_{22} - \frac{1}{2}\sigma_{22}\dot{\sigma}_{11}}{\sigma_u^2}\Big(\sigma_{11} - \frac{1}{2}\sigma_{22}\Big)$$

$$\dot{\varepsilon}_{22} = \frac{1}{A}\Big(\dot{\sigma}_{22} - \frac{1}{2}\dot{\sigma}_{11}\Big) + \Big(\frac{1}{P} - \frac{1}{A}\Big)\frac{\sigma_{11}\dot{\sigma}_{11} + \sigma_{22}\dot{\sigma}_{22} - \frac{1}{2}\sigma_{11}\dot{\sigma}_{22} - \frac{1}{2}\sigma_{22}\dot{\sigma}_{11}}{\sigma_u^2}\Big(\sigma_{22} - \frac{1}{2}\sigma_{11}\Big)$$

$$\varepsilon_{12} = \varepsilon_{23} = \varepsilon_{31} = 0, \quad \varepsilon_{33} = -(\varepsilon_{11} + \varepsilon_{22}) \tag{2.2}$$

where, for the processes of average curvature

$$A = \frac{\sigma_u}{s}, \quad P = \Phi'(s), \quad \frac{ds}{dt} = \frac{2}{\sqrt{3}}\big(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{11}\dot{\varepsilon}_{22} + \dot{\varepsilon}_{12}^2\big)^{1/2} \tag{2.3}$$

$\Phi'(s)$ - a known function concerned with the material used, s - the arc-length of the strain trajectory. The mark (.) is derivative of the corresponding quantity with respect to the process parameter t.

Solving the system of differential equation (2.2), (2.3) in the combination with the initial conditions of the problem one can determine the stress-strain state at any point M in the plate at any moment of the prebuckling stage.

2.1.2. Postbuckling stage

Let t increase until it reaches the value $t = t^*$ at which a bifurcation of equilibrium states appears. It means: with an infinitesimal small increment of the external forces there are possible increments of deformation (including the bending deformation) in the plate.

Suppose deformation increments of the middle surface are equal to zero at the moment the instability appears. According to the assumption of straight normal, we can get the increments of deformation at the considered point M

$$\delta\varepsilon_{ij} = -z\delta\chi_{ij} = -z\frac{\partial^2\delta w}{\partial x_i \partial x_j} \tag{2.4}$$

where

$\delta\chi_{ij} = \dfrac{\partial^2\delta w}{\partial x_i \partial x_j}$ - increments of curvature and torsion associated with instability,

δw - deflection increment of the middle surface.

The corresponding stress increments can be determined according to the theory of the elastoplastic process

$$\begin{aligned}\delta\sigma_{ij} &= \frac{2}{3}A(\delta\varepsilon_{ij} + \delta_{ij}\delta\varepsilon_{kk}) + (P-A)\frac{\sigma_{k\ell}\delta\varepsilon_{k\ell}}{\sigma_u^2}\sigma_{ij},\\ \delta s &= \frac{2}{\sqrt{3}}\left(\delta\varepsilon_{11}^2 + \delta\varepsilon_{11}\delta\varepsilon_{22} + \delta\varepsilon_{22}^2\right)^{1/2}\\ &\quad (i,j,k,\ell = 1,2)\end{aligned} \tag{2.5}$$

The increments of membrane forces and bending moments are determined as

$$\delta N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}dz, \quad \delta M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}zdz. \tag{2.6}$$

Using (2.4), (2.5) and note that A and P do not depend on z in the case that the increments of the middle surface are absent [2], we reach

$$\begin{aligned}\delta N_{ij} &= -\left[\frac{2}{3}A\Big(\frac{\partial^2\delta w}{\partial x_i\partial x_j} + \delta_{ij}\frac{\partial^2\delta w}{\partial x_k\partial x_k}\Big) + (P-A)\frac{\sigma_{ij}\sigma_{kl}}{\sigma_u^2}\cdot\frac{\partial^2\delta w}{\partial x_k\partial x_\ell}\right]\int_{-h/2}^{h/2} = 0,\\ \delta M_{ij} &= -\left[\frac{2}{3}A\Big(\frac{\partial^2\delta w}{\partial x_i\partial x_j} + \delta_{ij}\frac{\partial^2\delta w}{\partial x_k\partial x_k}\Big) + (P-A)\frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_u^2}\cdot\frac{\partial^2\delta w}{\partial x_k\partial x_\ell}\right]\int_{-h/2}^{h/2} z^2dx\\ &= -\left[\frac{2}{3}A\Big(\frac{\partial^2\delta w}{\partial x_i\partial x_j} + \delta_{ij}\frac{\partial^2\delta w}{\partial x_k\partial x_k}\Big) + (P-A)\frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_u^2}\cdot\frac{\partial^2\delta w}{\partial x_k\partial x_\ell}\right]\frac{h^3}{12}\end{aligned} \tag{2.7}$$

or in the development form

$$\begin{aligned}
\delta M_{11} &= -\frac{Gh^3}{4}\Big(R_1\frac{\partial^2\delta w}{\partial y^2} + R_2\frac{\partial^2\delta w}{\partial y^2}\Big)\\
\delta M_{12} = \delta M_{21} &= -\frac{Gh^3}{4}\cdot R_3\frac{\partial^2\delta w}{\partial x\partial y}\\
\delta M_{22} &= -\frac{Gh^3}{4}\Big(R_4\frac{\partial^2\delta w}{\partial x^2} + R_5\frac{\partial^2\delta w}{\partial y^2}\Big)
\end{aligned} \tag{2.8}$$

where

$$\begin{aligned}
R_1 &= \frac{4}{3}\varphi_A + (\varphi_P - \varphi_A)\hbar,\\
R_2 &= \frac{2}{3}\varphi_A + (\varphi_P - \varphi_A)\rho,\\
R_3 &= \frac{2}{3}\varphi_A,\\
R_4 &= \frac{2}{3}\varphi_A + (\varphi_P - \varphi_A)\rho,\\
R_5 &= \frac{4}{3}\varphi_A + (\varphi_P - \varphi_A)\lambda,\\
\hbar &= \frac{\sigma_{11}^2}{\sigma_u^2},\quad \rho = \frac{\sigma_{11}\sigma_{22}}{\sigma_u^2},\quad \lambda = \frac{\sigma_{22}^2}{\sigma_u^2},\quad \varphi_A = \frac{A}{3G},\quad \varphi_P = \frac{P}{3G}
\end{aligned} \tag{2.9}$$

Now the stability equation of the plate

$$\frac{\partial^2\delta M_{ij}}{\partial x_i\partial x_j} + N_{ij}\delta\chi_{ij} = 0$$

becomes

$$\begin{aligned}
&R_1\frac{\partial^4\delta w}{\partial x^4} + (R_2 + 2R_3 + R_4)\frac{\partial^4\delta w}{\partial x^2\partial y^2} + R_5\frac{\partial^4\delta w}{\partial y^4} + 2\frac{\partial R_1}{\partial x}\cdot\frac{\partial^3\delta w}{\partial x^3} +\\
&+ 2\frac{\partial(R_3 + R_4)}{\partial y}\cdot\frac{\partial^3\delta w}{\partial x^2\partial y} + 2\frac{\partial(R_2 + R_3)}{\partial x}\cdot\frac{\partial^3\delta w}{\partial x\partial y^2} + 2\frac{\partial R_5}{\partial y}\frac{\partial^3\delta w}{\partial y^3} +\\
&\Big(\frac{\partial^2 R_1}{\partial x^2} + \frac{\partial^2 R_4}{\partial y^2} - \frac{4\sigma_{11}}{Gh^2}\Big)\frac{\partial^2\delta w}{\partial x^2} + 2\frac{\partial^2 R_3}{\partial x\partial y}\cdot\frac{\partial^2\delta w}{\partial x\partial y} + \Big(\frac{\partial^2 R_2}{\partial x^2} + \frac{\partial^2 R_5}{\partial y^2} - \frac{4\sigma_{22}}{Gh^2}\Big)\frac{\partial^2\delta w}{\partial y^2} = 0
\end{aligned}$$

or

$$\begin{aligned}
&\alpha_1\frac{\partial^4\delta w}{\partial x^4} + \alpha_2\frac{\partial^4\delta w}{\partial x^2\partial y^2} + \alpha_3\frac{\partial^4\delta w}{\partial y^4} + \alpha_4\frac{\partial^3\delta w}{\partial x^3} + \alpha_5\frac{\partial^3\delta w}{\partial x^2\partial y} + \alpha_6\frac{\partial^3\delta w}{\partial x\partial y^2} +\\
&+ \alpha_7\frac{\partial^3\delta w}{\partial y^3} + \alpha_8\frac{\partial^2\delta w}{\partial x^2} + \alpha_9\frac{\partial^2\delta w}{\partial x\partial y} + \alpha_{10}\frac{\partial^2\delta w}{\partial y^2} = 0
\end{aligned} \tag{2.10}$$

where

$$\alpha_1 = R_1 = \frac{4}{3}\varphi_A + (\varphi_P - \varphi_A)\hbar,$$
$$\alpha_2 = (R_2 + 2R_3 + R_4) = \frac{8}{3}\varphi_A + 2(\varphi_P - \varphi_A)\rho,$$
$$\alpha_3 = R_5 = \frac{4}{3}\varphi_A + (\varphi_P - \varphi_A)\lambda,$$

$$\begin{aligned}
\alpha_4 &= 2\frac{\partial R_1}{\partial x} = \frac{8}{3}\frac{\partial \varphi_A}{\partial x} + 2\frac{\partial(\varphi_P - \varphi_A)}{\partial x}\hbar + 2(\varphi_P - \varphi_A)\frac{\partial \hbar}{\partial x},\\
\alpha_5 &= 2\frac{\partial(R_3 + R_4)}{\partial y} = \frac{8}{3}\frac{\partial \varphi_A}{\partial y} + 2\frac{\partial(\varphi_P - \varphi_A)}{\partial y}\rho + 2(\varphi_p - \varphi_A)\frac{\partial \rho}{\partial y},\\
\alpha_6 &= 2\frac{\partial(R_3 + R_2)}{\partial x} = \frac{8}{3}\frac{\partial \varphi_A}{\partial x} + 2\frac{\partial(\varphi_P - \varphi_A)}{\partial x}\rho + 2(\varphi_P - \varphi_A)\frac{\partial \rho}{\partial x},\\
\alpha_7 &= 2\frac{\partial R_5}{\partial y} = \frac{8}{3}\frac{\partial \varphi_A}{\partial y} + 2\frac{\partial(\varphi_P - \varphi_A)}{\partial y}\lambda + 2(\varphi_P - \varphi_A)\frac{\partial \lambda}{\partial y},\\
\alpha_8 &= \frac{\partial^2 R_1}{\partial x^2} + \frac{\partial^2 R_4}{\partial y^2} - \frac{4\sigma_{11}}{Gh^2}\\
&= \frac{\partial^2}{\partial x^2}\Big[\frac{4}{3}\varphi_A + (\varphi_P - \varphi_A)\hbar\Big] + \frac{\partial^2}{\partial y^2}\Big[\frac{2}{3}\varphi_A + (\varphi_P - \varphi_A)\rho\Big] - \frac{4\sigma_{11}}{Gh^2},\\
\alpha_9 &= 2\frac{\partial^2 R_3}{\partial x \partial y} = \frac{4}{3}\frac{\partial^2 \varphi_A}{\partial x \partial y},\\
\alpha_{10} &= \frac{\partial^2 R_2}{\partial x^2} + \frac{\partial^2 R_5}{\partial y^2} - \frac{4\sigma_{22}}{Gh^2}\\
&= \frac{\partial^2}{\partial x^2}\Big[\frac{2}{3}\varphi_A + (\varphi_P - \varphi_A)\rho\Big] + \frac{\partial^2}{\partial y^2}\Big[\frac{4}{3}\varphi_A + (\varphi_P - \varphi_A)\lambda\Big] - \frac{4\sigma_{22}}{Gh^2},
\end{aligned} \tag{2.11}$$

2.2. Determining the critical forces by using Bubnov-Galerkin method

2.2.1. The Bubnov-Galerkin method

The values of external forces at the moment $t = t^*$ when the instability occurs are called the critical forces. To determine the critical forces, we have to determine the stress-strain state of any point in the plate at any moment in the prebuckling stage and solve the stability equation (2.10).

The stability equation (2.10) is a partial differential equation of the fourth order with the coefficients α_k ($k = 1 \div 10$) depending on R_j ($j = 1 \div 5$) and their up-to-the-second-order derivatives. According to (2.9), R_j are depend on φ_A, φ_P, $\hbar$, ρ, λ, i.e. R_j depend on A, P, σ_{11}, σ_{22} and σ_u. Because $A = \sigma_u/s$, $P = \Phi'(s)$, $\sigma_{11} = \sigma_{11}(y)$, $\sigma_{22} = \sigma_{22}(x)$, $\sigma_u = \sigma_u(x, y)$, where s implicitly depends on x, y, so R_j also implicitly depend on x, y. Because of this, a direct solution to the above stability equation is complicated.

To reach the critical forces according to Bubnov-Galerkin method, we need to fulfill the following steps.

a) Approximating the expression of δw in series

$$\delta w = \sum_{k=1}^{N} B_k \delta w_k, \tag{2.12}$$

where N is the number of terms of the series, B_k are coefficients different from 0, δw_k are functions being linearly independent and satisfying the boundary conditions.

In fact, δw_k are chosen as the product of two functions

$$\delta w_k = X_k \cdot Y_k = X_k(x) \cdot Y_k(y)$$

where $X_k = X_k(x)$ - a function of x only, $Y_k = Y_k(y)$ - a function of y only.

b) Putting the expression (2.12) of δw into the stability equation (2.10) and using notation $\Omega(\delta w)$ for the left side of the received equation. The result is

$$\Omega(\delta w) = 0 \tag{2.13}$$

Because of (2.12) we can write

$$\Omega(\delta w) = \sum_{k=1}^{N} B_k \Omega(\delta w_k) \tag{2.14}$$

where $\Omega(\delta w_k)$ is the expression one can get by putting δw_k into left side of the stability equation (2.10), instead of putting δw. It is clear that both $\Omega(\delta w_k)$ and $\Omega(\delta w)$ depend on x, y,

c) Multiplying both side of (2.13) by δw_i $(i = 1, 2, \ldots, N)$ and integrating both sides of the received equation all over the volume of the plate. The result is

$$\int_0^a \int_0^b \delta w_i \Omega(\delta w) dxdy = 0 \Leftrightarrow \sum_{k=1}^{N} B_k \int_0^a \int_0^b \delta w_i \Omega(\delta w_k) dxdy = 0. \tag{2.15}$$

d) Letting i be equal to $1, 2, \ldots, N$ in turn, we get N equations of the form (2.15), i.e, a system of N linear algebraic equations with the unknowns $B_1, B_2, \ldots, B_N$. This system has the form

$$(C_{ik})\{B_k\} = 0 \tag{2.16}$$

where

$\{B_k\}^T = (B_1, B_2, \ldots, B_N)$

(C_{ik}) - an N-order square matrix,

$$C_{ik} = \int_0^a \int_0^b \delta w_i \Omega(\delta w_k) dx dy. \qquad (2.17)$$

Because $B_1, B_2, \ldots, B_N$ must be non-trivial solutions of the system (2.16), the condition for this is

$$\det(C_{ik}) = 0. \qquad (2.18)$$

In the process of solving the problem to determine strain - stress state of the plate, the process parameter t is monotonely increased, the condition (2.18) allows to find out the moment at which the instability starts to appear. The values of the external forces at this moment are critical forces.

2.2.2 Determining C_{ik}

The main difficulty in the approach to the cristal forces belongs to how to calculate the elements of matrix (C_{ik}) according to (2.17). Because the coefficients R_j in (2.11) depend implicity on x and y, so $\Omega(\delta w_k)$ and $\Omega(\delta w)$ depend implicitly on x and y as well.

To overcome this difficulty, the author would propose an approximate method with the following operations:

a) Dividing the plane of the plate into M rectangular pieces by *nodal lines* parallel with the edges, respectively (figure 4).

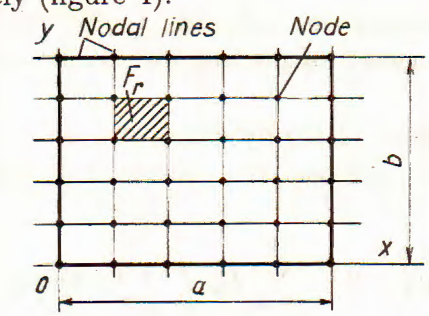

Fig. 4

b) At every vertex (Node) of piece $F_r (r = 1, 2, \ldots, M)$ whose coordinates are known, it is possible to calculate the values of quantities φ_A, φ_P, $\hbar$, ρ, λ at any moment t.

c) On each piece F_r we suppose that φ_A, φ_P, $\hbar$, ρ, λ depend linearly on x, y

$$\varphi_A = r_{A1}x + r_{A2}y + r_{A3}, \quad \varphi_p = r_{P1}x + r_{P2}y + r_{P3}$$
$$\hbar = r_{H1}x + r_{H2}y + r_{H3}, \quad \rho = r_{R1}x + r_{R2}y + r_{R3}, \quad \lambda = r_{L1}x + r_{L2}y + r_{L3} \qquad (2.19)$$

where the coefficients r_{A1}, r_{A2}, r_{A3}, r_{P1}, r_{P2}, r_{P3}, r_{H1}, r_{H2}, r_{H_3}, r_{L1}, r_{L2}, r_{L3} are determined in accordance with the minimum square method using the known values of the quantities φ_A, φ_P, $\hbar$, ρ, λ and the coordinates x, y at the four vertexes of the corresponding piece F_r.

The assumption (2.19) can be acceptable because of the continuity of the material used and the continuous distribution of the external forces with respect to the coordinates.

d) Putting the determined axpressions of φ_A, φ_P, $\hbar$, ρ, λ according to (2.19) into (2.9) to determine R_j. Afterwards, using (2.11) to determine the coefficients α_k $(k = 1 \div 10)$ of stability equation (2.10).

e) In equation (2.17) to calculate C_{ik}, because δw_i, δw_k depend on x and y, so it is possible to denote $\delta w_i \Omega(\delta w_k) = \Theta_{ik}(x, y)$. Combining with dividing the plane of the plate into M rectangular pieces, we can write

$$C_{ik} = \sum_{j=1}^{M} \int_{F_j} \Theta_{ik}(x, y) dx dy. \qquad (2.10)$$

f) To implement the integrals in (2.20), we apply the Gaussian quadric method presented in [6]. This method supplies high accuracy results whereas reduces the time expense on PC.

3. Some results of numerical calculation

Now we consider a plate made of the stell 30XГCA which has the shear modulus $G = 0.8667 \cdot 10^6$ kG/cm^2 and the corresponding material function $\Phi = \Phi(s)$ presented in [1]. Let's accept the geometrical relations as $a/h = 55$, $b/h = 45$, and the compressive forces p, q of the form (1.3), where: $p_1(t) = 2000t$, $q_1(t) = 1500t^2$.

Suppose the plate are simply supported along the four edges, the expression of δw which satisfies the boundary conditions is chosen as

$$\delta w = \sum_{k=1}^{4} B_k \sin k\pi x a \sin k\pi y b \cdot \qquad (3.1)$$

Some concrete results are given out in following:

a) If $a_1 = a_2 = 0$, $b_1 = b_2 = 1$ then

$$p \equiv p_1(t) = 2000t, \quad q \equiv q_1(t) = 1500t^2$$

i.e. the plate is in the pure loading. In this case, we can determine the critical forces in two ways: one belongs to that of pure loading, the other belongs to the method of the impure loading which is presented in this paper and concerned with dividing the plane of the plate into pieces. Theoretically, the two ways should give the same results.

The performances of the two calculation programmes give us the same results

$$p^* = 2452 kG/cm^2, \quad q^* = 2255 kG/cm^2.$$

This proves the soundness of the method proposed in this paper for calculating the elements of matrix (C_{ik}).

b) If assigning $a_1 = 0$, $b_1 = 1$, $a_2 = 1/a$, $b_2 = 1$ then we have

$$p = p_1(t) = 2000t, \quad q = 1500t^2\Big(\frac{x}{a} + 1\Big).$$

The results in this case are $p_1^* = 2104kG/cm^2$, $q_1^* = 1660kG/cm^2$.

c) If accepting $a_1 = 0$, $b_1 = 1$, $a_2 = 1/a$, $b_2 = 0$ then we get

$$p = p_1(t) = 2000t, \quad q = 1500t^2\Big(\frac{x}{a}\Big).$$

The performance of the programme gives $p_1^* = 3104kG/cm^2$, $q_1^* = 3613kg/cm^2$.

d) If $a_1 = 1/b$, $b_1 = 1$, $a_2 = 1/a$, $b_2 = 0$ then

$$p = 2000t\Big(\frac{y}{b} + 1\Big), \quad q = 1500t^2\Big(\frac{x}{a}\Big)$$

We receive $p_1^* = 2380kG/cm^2$, $q_1^* = 2124kG/cm^2$.

4. Conclusion

In the paper, the author applies the elastoplastic process theory to formulate the governing equation of the stability problem of thin rectangular plates under complex and impure loadings, and gives out some concrete result concerned with calculating the values of critical forces.

When applying the Bubnov-Galerkin method to determining the values of critical forces, the author proposes a possible method to calculate the elements of matrix (C_{ik}), where the minimum square method and the Gaussian quadric are used.

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ỔN ĐỊNH ĐÀN DẺO CỦA TẤM CHỮ NHẬT CHỊU TẢI TRỌNG PHỨC TẠP, KHÔNG THUẦN NHẤT

Bài báo liên quan đến ổn định đàn dẻo của tấm mỏng chữ nhật chịu tải phức tạp, không thuần nhất. Tải trọng tác dụng lên tấm trong trường hợp đang khảo sát bao gồm các lực nén theo hai phương phân bố theo quy luật bậc nhất dọc theo các cạnh của tấm.

Các phương trình giải của bài toán được thiết lập trên cơ sở áp dụng lý thuyết quá trình đàn dẻo. Giá trị tới hạn của các lực ngoài được xác định bằng phương pháp Bubnov - Galerkin.

Để tính các phần tử của ma trận có liên quan đến điều kiện mất ổn định của tấm, tác giả đề xuất phương pháp chia nhỏ mặt phẳng của tấm bằng các đường nút và khảo sát trạng thái ứng suất - biến dạng tại các điểm nút tại những thời điểm khác nhau của quá trình. Việc tính các tích phân xuất hiện trong quá trình giải toán được áp dụng phương pháp cầu phương Gauss. Các kết quả tính toán cụ thể bằng số cũng được đưa ra ở trong bài báo.