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TWO-DIMENSIONAL OPTIMIZATION PROBLEM OF PLANT LOCATION

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ABSTRACT. In this paper, the following matters are presented: the adjoint problem of the two-dimensional matter propagation problem; the algorithm for determination of a domain in which a plant can be located so that the values of the pollution-level reflecting functional does not exceed a given value at considered sensitive areas; application of this algorithm for numerical experiments to a typical problem.

1. Equation of the suspended matter propagation and its adjoint equation (see [1])

The equation of the suspended matter propagation, i.e. the matter transport and diffusion equation in the horizontal 2D case has the following form:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \sigma C = f + \gamma \Delta C, \quad (x,y) \in G,\ 0 \le t \le T \tag{1.1}$$

with the initial and boundary conditions:

$$C\big|_{t=0} = C^0, \quad C\big|_{\Gamma^-} = \varphi, \quad \frac{\partial C}{\partial n}\Big|_{\Gamma^+} = 0, \tag{1.2}$$

where x, y, t are the space and time variables; C is the matter concentration; σ is the decay coefficient; f is the source intensity; γ is the diffusion coefficient; u, v are respectively velocity components in the x and y directions, and satisfy the following equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1.3}$$

$\Gamma = \Gamma^+ + \Gamma^-$ with Γ^+ is the boundary part, at which $u_n \ge 0$; Γ^- is the boundary part, at which $u_n < 0$; $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$-Laplace operator; u_n is the projection of the velocity on the external normal vector $\vec{n}$.

Using (1.3), the equation (1.1) can be rewritten as follows:

$$\frac{\partial C}{\partial t} + \frac{\partial uC}{\partial x} + \frac{\partial vC}{\partial y} + \sigma C = f + \gamma \Delta C. \tag{1.4}$$

Solution of the equation (1.1) may be determined under the form: $C = C_1 + C_2$ where, C_1 and C_2 are the solutions of the following equation:

$$\frac{\partial C_1}{\partial t} + u\frac{\partial C_1}{\partial x} + v\frac{\partial C_1}{\partial y} + \sigma C_1 = \gamma \Delta C_1 \tag{1.5}$$

with the initial and boundary conditions:

$$C_1\big|_{t=0} = C^0, \quad C_1\big|_{\Gamma-} = \varphi, \quad \frac{\partial C_1}{\partial n}\Big|_{\Gamma+} = 0$$

and

$$\frac{\partial C_2}{\partial t} + \frac{\partial uC_2}{\partial x} + \frac{\partial vC_2}{\partial y} + \sigma C_2 = \gamma \Delta C_2 + f \tag{1.6}$$

with the initial and boundary conditions:

$$C_2\big|_{t=0} = 0, \quad C_2\big|_{\Gamma-} = 0, \quad \frac{\partial C_2}{\partial n}\Big|_{\Gamma+} = 0. \tag{1.7}$$

We now establish the adjoint equation of the equation (1.6). By multiplying both sides of the equation (1.6) by a some function C_2^* and integrating the equation obtained on the area $G \times [0, T]$, we get:

$$\int_0^T dt \int_G C_2^* \frac{\partial C_2}{\partial t} dG + \int_0^T dt \int_G C_2^* \operatorname{div}(\vec{u}C_2) dG$$
$$+ \int_0^T dt \int_G \sigma C_2 C_2^* dG - \gamma \int_0^T dt \int_G C_2^* \Delta C_2 dG = \int_0^T dt \int_G C_2^* f dG. \tag{1.8}$$

Let $\gamma = \text{const}$, using the partial integration technique, the Green formula and the condition (1.3), we have:

$$\int_0^T dt \int_G C_2^* \frac{\partial C_2}{\partial t} dG = \int_G C_2^* C_2\big|_0^T dG - \int_0^T dt \int_G C_2 \frac{\partial C_2^*}{\partial t} dG,$$

$$\int_0^t dt \int_G C_2^* \operatorname{div}(\vec{u}C_2) dG = \int_0^T dt \int_\Gamma u_n C_2^* C_2 d\Gamma - \int_0^T dt \int_G C_2 \operatorname{div}(\vec{u}C_2^*) dG,$$

$$\gamma \int_0^T dt \int_G C_2^* \Delta C_2 dG = \gamma \int_0^T dt \int_\Gamma \Big(C_2^* \frac{\partial C_2}{\partial n} - C_2 \frac{\partial C_2^*}{\partial n}\Big) d\Gamma + \gamma \int_0^T dt \int_G C_2 \Delta C_2^* dG.$$

Putting these expressions into (1.8), one deduces:

$$\int_0^T dt \int_G C_2\Big(-\frac{\partial C_2^*}{\partial t} - \operatorname{div}(\vec{u}C_2^*) + \sigma C_2^* - \gamma\Delta C_2^*\Big)dG =$$

$$= \int_0^T dt \int_G C_2^* f dG - \int_G C_2^* C_2\big|_{t=T} dG + \int_G C_2^* C_2\big|_{t=0} dG - \int_0^T dt \int_\Gamma u_n C_2 C_2^* d\Gamma$$

$$+ \gamma \int_0^T dt \int_\Gamma \Big(C_2^*\frac{\partial C_2}{\partial n} - C_2\frac{\partial C_2^*}{\partial n}\Big)d\Gamma. \tag{1.9}$$

Let the function C_2^* satisfy the following equation:

$$-\frac{\partial C_2^*}{\partial t} - \operatorname{div}(\vec{u}C_2^*) + \sigma C_2^* - \gamma\Delta C_2^* = p. \tag{1.10}$$

From the initial and the boundary conditions (1.7), one yields:

$$\int_G C_2 C_2^*\big|_{t=0} dG = 0,$$

$$\int_0^T dt \int_\Gamma u_n C_2 C_2^* d\Gamma = \int_0^T dt \int_{\Gamma+} u_n C_2 C_2^* d\Gamma,$$

$$\gamma \int_0^T dt \int_\Gamma \Big(C_2^*\frac{\partial C_2}{\partial n} - C_2\frac{\partial C_2^*}{\partial n}\Big)d\Gamma = \gamma \int_0^T dt \int_{\Gamma+} \Big(-C_2\frac{\partial C_2^*}{\partial n}\Big)d\Gamma + \gamma \int_0^T dt \int_{\Gamma-} C_2^*\frac{\partial C_2}{\partial n}d\Gamma.$$

From the above expressions and (1.10), the equation (1.9) can be rewritten under the form:

$$\int_0^T dt \int_G pC_2 dG = \int_0^T dt \int_G f C_2^* dG + \int_G C_2 C_2^*\big|_{t=T} dG$$

$$+ \gamma \int_0^T dt \int_{\Gamma-} C_2^*\frac{\partial C_2}{\partial n}d\Gamma - \int_0^T dt \int_{\Gamma+} C_2\big(\gamma\frac{\partial C_2^*}{\partial n} + u_n C_2^*\big)d\Gamma. \tag{1.11}$$

Let the initial and boundary conditions of the equation (1.10) be chosen as follows:

$$C_2^*\big|_{t=T} = 0, \quad C_2^*\big|_{\Gamma-} = 0, \quad \Big(\gamma\frac{\partial C_2^*}{\partial n} + u_n C_2^*\Big)\Big|_{\Gamma+} = 0. \tag{1.12}$$

Then, from (1.11) and (1.12) we get the dual form:

$$\int_0^T dt \int_G pC_2 dG = \int_0^T dt \int_G fC_2^* dG. \tag{1.13}$$

It is easy to verify that the problem (1.10), (1.12) is the adjoint problem of the (1.6), (1.7). Indeed, with the notation:

$$A = \frac{\partial}{\partial t} + \frac{\partial u\cdot}{\partial x} + \frac{\partial v\cdot}{\partial y} + \sigma - \gamma\Delta \quad A^* = -\frac{\partial}{\partial t} - \frac{\partial u\cdot}{\partial x} - \frac{\partial v\cdot}{\partial y} + \sigma - \gamma\Delta,$$

we have:

$$AC_2 = f, \quad A^*C_2^* = p,$$

$$(AC_2, C_2^*) = (f, C_2^*) = \int_0^T dt \int_G fC_2^* dG = \int_0^T dt \int_G pC_2 dG = (C_2, p) = (C_2, A^*C_2^*).$$

Use of the variable transformation $t_1 = T - t$, the equation (1.10) becomes:

$$\frac{\partial C_2^*}{\partial t_1} - \operatorname{div}(\vec{u}C_2^*) + \sigma C_2^* - \gamma\Delta C_2^* = p, \tag{1.14}$$

$$C_2^*\big|_{t_1=0} = 0, \quad C_2^*\big|_{\Gamma-} = 0, \quad \left(\gamma\frac{\partial C_2^*}{\partial n} + u_n C_2^*\right)\Big|_{\Gamma+} = 0.$$

For simplicity, by using (1.3), we obtain an another form of the adjoint equation (1.14):

$$\frac{\partial C_2^*}{\partial t_1} - u\frac{\partial C_2^*}{\partial x} - v\frac{\partial C_2^*}{\partial y} + \sigma C_2^* - \gamma\Delta C_2^* = p. \tag{1.15}$$

2. Pollution-level reflecting functionals (see [1])

Assume that the suspended matter concentration C is calculated from the equation (1.1). We consider the following functionals:

a. The time-averaged amount of the matter concentration C on a sensitive area $G_k \subset G$ for the period T: $J_k^A = \frac{1}{T}\int_0^T dt \int_{G_k} C dG$.

b. The total amount of settling matter in the same area $G_k \subset G$: $J_k^B = \int_0^T dt \int_{G_k} aC dG$, where, the constant a represents portion of matter which settles down, that are mainly the heavy matters and partly the suspended matters settling down by downward diffusion.

c. Generalized functional:

$$J_k = \int_0^T dt \int_{G_k} pCdG \quad \text{where} \quad p = \begin{cases} \frac{1}{T} + a, & (x,y) \in G_k \\ 0, & (x,y) \notin G_k \end{cases} \tag{2.1}$$

and p is a function referring to the economic, sanitary, ecological, health standards and so on.

d. Global functional:

$$Y_p = \int_0^T dt \int_G pCdG \quad \text{where,} \quad p = \begin{cases} \frac{1}{T} + G_k, & (x,y) \in G_k, \ k = 1,2,\dots,m \\ 0, & (x,y) \notin \bigcup_{k=1}^{m} G_k. \end{cases}$$

3. Optimization problem of plant location (see [1])

Let G_k $(k = 1,2,\dots,m)$ be considered areas, recreation zones or other environmentally sensitive areas on the region G. Our problem is to determine the domain $\Omega_k \subset G$ so that the pollution matter from a plant located in this domain Ω_k satisfies the following condition for the sensitive area G_k:

$$Y_k = \int_0^T dt \int_{G_k} pCdG \leq \bar{c}_k, \quad \text{where} \quad p = \begin{cases} \frac{1}{T} + a_k, & (x,y) \in G_k \\ 0, & (x,y) \notin G_k \end{cases} \tag{3.1}$$

and $\bar{c}_k$ is a given figure.

If the determination of domain Ω_k is impossible on the G, the reduction of rate of the pollution emission Q, will make the determination of the plant location possible.

Assume that on the region G there are m sensitive areas G_k $(k = 1,\dots,m)$ and the source of matter emission is located at a point $r_0 = (x_0, y_0)$. Then, the source intensity can be described by the function: $f(x,y) = Q\delta(r - r_0)$, $\quad Q = \text{const}$

where, $\delta(r) = \begin{cases} \infty, & r = r_0 \\ 0, & r \neq r_0 \end{cases}$ is Dirac function, and from (1.1), we get:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \sigma C = Q\delta(r - r_0) + \gamma \Delta C$$

with the conditions: $C\big|_{t=0} = C^0, \quad C\big|_{\Gamma^-} = \varphi, \quad \left.\frac{\partial C}{\partial n}\right|_{\Gamma^+} = 0.$

In order to determine the domain Ω, in which the plant can be located so that in all sensitive areas G_k, the generalized functional Y_k satisfies the condition (3.1), we do as follows:

a. Calculation of concentration C from the equation (1.5):

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \sigma C = \gamma \Delta C$$

with the initial and boundary conditions $C\big|_{t=0} = C^0$, $\quad C\big|_{\Gamma^-} = \varphi$, $\quad \frac{\partial C}{\partial n}\big|_{\Gamma^+} = 0$

and generalized functional (2.1): $J_k = \int_0^T dt \int_{G_k} pCdG = \widetilde{c}_k$.

b. Solving m adjoint equations:

$$\frac{\partial C_k^*}{\partial t_1} - u\frac{\partial C_k^*}{\partial x} - v\frac{\partial C_k^*}{\partial y} + \sigma C_k^* - \gamma \Delta C_k^* = p_k$$

where, $p_k = \begin{cases} \frac{1}{T} + a_k, & (x,y) \in G_k \\ 0, & (x,y) \notin G_k \end{cases}$ with the conditions:

$$C_k^*\big|_{t_1=0} = 0, \quad C_k^*\big|_{\Gamma^-} = 0, \quad \left(v\frac{\partial C_k^*}{\partial n} + u_n C_k^*\right)\Big|_{\Gamma^+} = 0,$$

we obtain the solutions C_k^* $(k = 1, 2, \ldots, m)$.

From the dual form (1.13), we get:

$$Y_k^* = \int_0^T dt \int_G p_k CdG = \int_0^T dt \int_G Q\delta(r - r_0)C_k^* dG$$
$$= \int_0^T QC_k^*(r_0, t)dt = \int_0^T QC_k^*(r_0, T - t_1)dt_1$$

which must satisfy the condition: $Y_k^* \leq \overline{c}_k - \widetilde{c}_k = \overline{\overline{c}}_k$.

Now we consider the function: $Y_k^*(r) = Q\int_0^T C_k^*(r,t)dt$ and draw the iso-grams of $Y_k^*(r) = \text{const}$. Then, Ω_k in which the functional $Y_k^*(r) \leq \overline{\overline{c}}_k$ are found out. If there is perchance no area Ω_k inside G, it may be re-established anyway by reducing the discharge intensity Q.

c. Overlaying all the areas Ω_k $(k = 1, \ldots, m)$, we obtain the domain Ω, $(\Omega = \bigcap_{k=1}^{m} \Omega_k)$. Ω will be the domain in which the plant can be located so that pollution standards will be met in all the areas $G_k \subset G$, $(k = 1, 2, \ldots, m)$.

4. Algorithm (see [2]-[4])

The equation (1.5) and the adjoint equation (1.15) may be rewritten in a common form:

$$\frac{\partial C}{\partial t} + \Lambda C = f \tag{4.1}$$

where, $\Lambda = \Lambda_1 + \Lambda_2$, $\Lambda_1 = \pm u\frac{\partial}{\partial x} - \gamma\frac{\partial^2}{\partial x^2} + \frac{\sigma}{2}$, $\Lambda_2 = \pm v\frac{\partial}{\partial y} - \gamma\frac{\partial^2}{\partial y^2} + \frac{\sigma}{2}$.

Equation (4.1) is solved by the method of the directional decomposition (splitting method):

$$\frac{C^{k+1} - C^k}{dt} + \Lambda\big[\theta C^{k+1} + (1-\theta)C^k\big] = f^{k+1}$$

$$\text{or} \quad (I + dt\theta\Lambda)C^{k+1} = \big[I - dt(1-\theta)\Lambda\big]C^k + dtf^{k+1}, \tag{4.2}$$

where $0 \leq \theta \leq 1$, I is the unique operator.

Using approximation:

$$\big[I + dt\theta(\Lambda_1 + \Lambda_2)\big] = (I + dt\theta\Lambda_1)(I + dt\theta\Lambda_2) + 0(dt^2)$$

from (4.2), one deduces:

$$(I + dt\theta\Lambda_1)(I + dt\theta\Lambda_2)C^{k+1} = dtf^{k+1} + [I - dt(1-\theta)\Lambda]C^k.$$

The computational process contains two steps:

$$(I + dt\theta\Lambda_1)C^{k+1/2} = [I - dt(1-\theta)\Lambda]C^k + dtf^{k+1} \tag{4.3}$$

$$(I + dt\theta\Lambda_2)C^{k+1} = C^{k+1/2}. \tag{4.4}$$

a. Discretizing the equation (4.3) by an implicit finite difference scheme:

$$\Lambda_1 C^{k+1/2} = \frac{\big(\pm u + |u|\big)^{k+1/2}_{m,n}}{2}\frac{\big(C^{k+1/2}_{m,n} - C^{k+1/2}_{m-1,n}\big)}{\Delta x}$$

$$+ \frac{\big(\pm u - |u|\big)^{k+1/2}_{m,n}}{2}\frac{\big(C^{k+1/2}_{m+1,n} - C^{k+1/2}_{m,n}\big)}{\Delta x} - \gamma\frac{\big(C^{k+1/2}_{m+1,n} - 2C^{k+1/2}_{m,n} + C^{k+1/2}_{m-1,n}\big)}{\Delta x^2} + \frac{\sigma}{2},$$

$$\Lambda C^k = \pm u^{k+1/2}_{m,n}\frac{(C^k_{m+1,n} - C^k_{m-1,n})}{2\Delta x} - \gamma\frac{C^k_{m+1,n} - 2C^k_{m,n} + C^k_{m-1,n}}{\Delta x^2}$$

$$\pm v^{k+1/2}_{m,n}\frac{(C^k_{m,n+1} - C^k_{m,n-1})}{2\Delta y} - \gamma\frac{C^k_{m,n+1} - 2C^k_{m,n} + C^k_{m,n-1}}{\Delta y^2} + \sigma,$$

we obtain:

$$a_m C^{k+1/2}_{m+1,n} + b_m C^{k+1/2}_{m,n} + c_m C^{k+1/2}_{m-1,n} = d_m, \tag{4.5}$$

where,

$$a_m = \frac{\big(\pm u - |u|\big)_{m,n}^{k+1/2}\theta dt}{2\Delta x} - \frac{\gamma\theta dt}{(\Delta x)^2}, \quad b_m = 1 + \frac{\theta |u|_{m,n}^{k+1/2} dt}{\Delta x} + 2\frac{\gamma\theta dt}{(\Delta x)^2} + \frac{\sigma dt}{2},$$

$$c_m = -\frac{\big(\pm u + |u|\big)_{m,n}^{k+1/2}\theta dt}{2\Delta x} - \frac{\gamma\theta dt}{(\Delta x)^2}, \quad d_m = dt f_{m,n}^{k+1} + [I - dt(1-\theta)\Lambda]C_{m,n}^{k}.$$

It is easy to verify that: $b_m > 0$, $a_m < 0$, $c_m < 0$ and $|b_m| \geq |a_m| + |c_m| + \delta$, $\delta > 0$.

So, the linear equation system (4.5) has the unique solution and the computational error of the following double sweep method

$$C_{m,n}^{k+1} = L_m C_{m+1,n}^{k+1} + K_m, \tag{4.6}$$

where, $L_m = \dfrac{-a_m}{b_m + c_m L_{m-1}}$, $K_m = \dfrac{d_m - c_m K_{m-1}}{b_m + c_m L_{m-1}}$, is not accumulated (see [5]).

b. Discretizing the equation (4.4) by a difference scheme:

$$\Lambda_2 C^{k+1} = \frac{\big(\pm v + |v|\big)_{m,n}^{k+1}}{2}\frac{(C_{m,n}^{k+1} - C_{m,n-1}^{k+1})}{\Delta y} + \frac{\big(\pm v - |v|\big)_{m,n}^{k+1}}{2}\frac{(C_{m,n+1}^{k+1} - C_{m,n}^{k+1})}{\Delta y}$$
$$- \gamma\frac{(C_{m,n+1}^{k+1} - 2C_{m,n}^{k+1} + C_{m,n-1}^{k+1})}{\Delta y^2} + \frac{\sigma}{2}C_{m,n}^{k+1}$$

we also get:

$$\tilde{a}_n C_{m,n+1}^{k+1} + \tilde{b}_n C_{m,n}^{k+1} + \tilde{c}_n C_{m,n-1}^{k+1} = \tilde{d}_n, \tag{4.7}$$

where,

$$\tilde{a}_m = \frac{\big(\pm v - |v|\big)_{m,n}^{k+1}\theta dt}{2\Delta y} - \frac{\gamma\theta dt}{(\Delta y)^2}, \quad \tilde{b}_m = 1 + \frac{\theta |v|_{m,n}^{k+1} dt}{\Delta y} + 2\frac{\gamma\theta dt}{(\Delta y)^2} + \frac{dt\sigma}{2},$$

$$\tilde{c}_m = -\frac{\big(\pm v + |v|\big)_{m,n}^{k+1} dt}{2\Delta y} - \frac{\gamma\theta dt}{(\Delta y)^2}, \quad \tilde{d}_m = C_{m,n}^{k+1/2}.$$

Obviously: $\tilde{b}_m > 0$, $\tilde{a}_m < 0$, $\tilde{c}_m < 0$ and $|\tilde{b}_m| \geq |\tilde{a}_m| + |\tilde{c}_m| + \delta$, $\delta > 0$.

Also, the equation system (4.7) has the unique solution and the double sweep method (4.6) does not produce an accumulated computational error.

5. Numerical experiments

The mentioned-above algorithm is applied to solve the following optimization problem of plant location:

- The computed rectangular region $G = 1000\,m \times 1000\,m$ is covered by a uniform grid 51×51 with spacing steps: $dx = 20\,m$, $dy = 20\,m$.

- A constant velocity field (u, v): $u = 0.5\,\mathrm{m/s}$, $v = -0.5\,\mathrm{m/s}$.
- Diffusion coefficient : $\gamma = 0.5\,\mathrm{m^2/s}$.
- Decay coefficient: $\sigma = 0.0005\,\mathrm{s}^{-1}$.
- Time step: $dt = 5\,\mathrm{s}$.
- Time simulation: $T = 20000\,\mathrm{s}$.
- 3 considered sensitive rectangular areas G_k inside G $(k = 1, 2, 3)$ with the left-bottom corner coordinates and the right-top corner coordinates are as follows:

  $+G_1 = [(24.5, 8.5), (25.5, 9.5)]$, $+G_2 = [(37.5, 12.5), (39.5, 14.5)]$,

  $+G_3 = [(29.5, 33.5), (30.5, 34.5)]$.
- Standard concentration: $\bar{\bar{c}}_k = 10\,mg/l$ $(k = 1, 2, 3)$.

The numerical results are illustrated in Fig. 1. In this figure, the figure on the

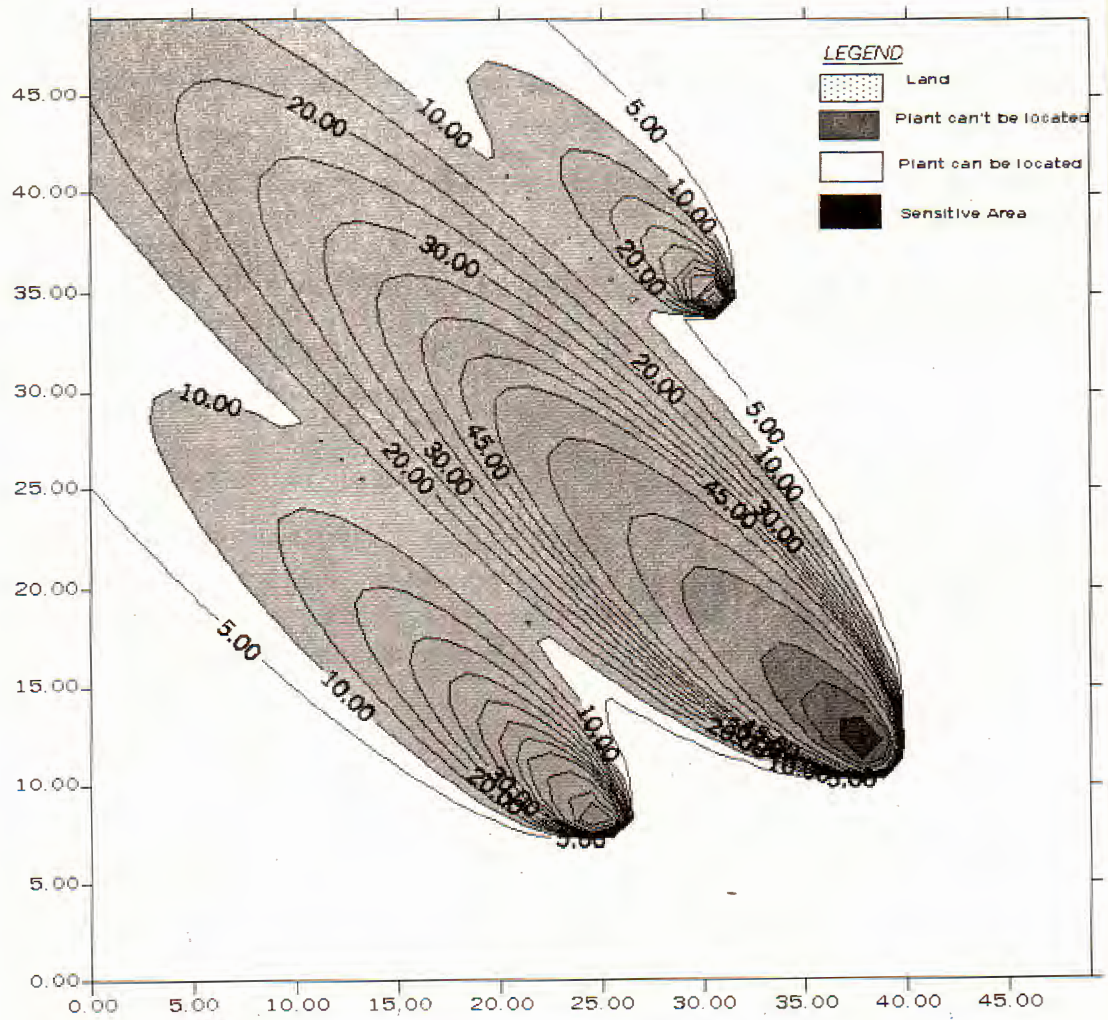

Fig. 1. Distribution of value of the pollution level-reflecting functionals Y_k^* for problem 1

contour lines indicates value of the pollution-level reflecting functionals Y_k^*. As a result, the domain Ω where the plant can be located so that the sanitary condition in the all areas G_k are satisfied (that means $Y_k^* \leq \bar{\bar{c}}_k$) is in white.

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BÀI TOÁN 2 CHIỀU TỐI ƯU XÁC ĐỊNH VỊ TRÍ NGUỒN THẢI

Bài báo trình bày các vấn đề sau: Bài toán liên hợp với bài toán lan truyền vật chất 2 chiều. Thuật toán xác định miền có thể đặt xí nghiệp sao cho phiếm hàm biểu thị mức độ ô nhiễm không vượt quá mức độ cho phép ở các vùng nhạy cảm quan tâm. Đã áp dụng thuật toán này để tính toán cho một bài toán mẫu.