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SOLVING METHOD FOR STABILITY PROBLEM OF ELASTOPLASTIC CYLINDRICAL SHELLS WITH COMPRESSIBLE MATERIAL SUBJECTED TO COMPLEX LOADING PROCESSES

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ABSTRACT. The system of stability equations of elasto plastic cylindrical shell made of compressible material was established in work [3]. In the present paper, we study the solution of the problems and methods for determining the critical load. The obtained results describe the influence of the compressibility of material on the stability of the shell. When a material is incompressible, these results reduce to the previous well-known ones (see [1, 2, 4, 5]).

1. Stability problem

Let's consider a cylindrical shell of length L, radius R and thickness h. We choose x_1 lying along the generatrix of the shell, $x_2 = R\theta_1$ with θ_1 - the angle of circular arc and z in the direction of the normal to the middle surface.

Assume that a material is compressible. We consider the shell being acted upon by the external forces p_{11}, p_{12}, p_{22} which depend arbitrarily on a loading parameter t. One of the main aims of the stability problem is to find the moment t_* when the instability of structure happens and respectively the critical loads, $p_{ij}^* = p_{ij}(t_*)$. Suppose that the unloading does not happen in the structure. We use the criterion of bifurcation of equilibrium state to study the proposed problem.

An analysis of the elastoplastic stability problem is always made in two parts: pre-buckling process and post-buckling process.

1.1. Pre-buckling process

Suppose that at any moment t there exists a membrane plane stress state in the shell

$$\sigma_{11} = -p_{11},\ \sigma_{12} = -p_{12},\ \sigma_{22} = -p_{22},\ \sigma_{13} = \sigma_{23} = \sigma_{33} = 0. \tag{1.1}$$

So that

$$\sigma = \frac{\sigma_{11} + \sigma_{22}}{3} = -\frac{p_{11} + p_{22}}{3},$$
$$\sigma_u = \sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2 + 3\sigma_{12}^2} = \sqrt{p_{11}^2 - p_{11}p_{22} + p_{22}^2 + 3p_{12}^2}\,.$$

Components of the strain velocity tensor determined according to the theory of elastoplastic processes [1] are of the form

$$\begin{aligned}
\dot{\varepsilon}_{11} &= \frac{1}{N}\big(-\dot{p}_{11}+\frac{1}{2}\dot{p}_{22}\big) - Q(s,t)\big(p_{11}-\frac{1}{2}p_{22}\big) - \frac{1}{9K}(\dot{p}_{11}+\dot{p}_{22}),\\
\dot{\varepsilon}_{22} &= \frac{1}{N}\big(-\dot{p}_{22}+\frac{1}{2}\dot{p}_{11}\big) - Q(s,t)\big(p_{22}-\frac{1}{2}p_{11}\big) - \frac{1}{9K}(\dot{p}_{11}+\dot{p}_{22}),\\
\dot{\varepsilon}_{33} &= \frac{1}{2N}(\dot{p}_{11}+\dot{p}_{22}) + \frac{1}{2}Q(s,t)(p_{11}+p_{22}) - \frac{1}{9K}(\dot{p}_{11}+\dot{p}_{22}),\\
\dot{\varepsilon}_{12} &= -\frac{3\dot{p}_{12}}{2N} - \frac{3}{2}Q(s,t)p_{12}.
\end{aligned} \tag{1.2}$$

where

$$\begin{aligned}
Q(s,t) &= \Big(\frac{1}{\phi'}-\frac{1}{N}\Big)\frac{1}{\sigma_u^2}\big(p_{11}\dot{p}_{11}+p_{22}\dot{p}_{22}-\frac{1}{2}p_{11}\dot{p}_{22}-\frac{1}{2}p_{22}\dot{p}_{11}+3p_{12}\dot{p}_{12}\big),\\
\phi' &= \phi'(s), \quad N=\frac{\sigma_u}{s}.
\end{aligned}$$

The arc-length of the strain trajectory is given respectively by the formula

$$\frac{ds}{dt} = \frac{\sqrt{2}}{3}\Big[(\dot{\varepsilon}_{11}-\dot{\varepsilon}_{22})^2+(\dot{\varepsilon}_{22}-\dot{\varepsilon}_{33})^2+(\dot{\varepsilon}_{33}-\dot{\varepsilon}_{11})^2+6\dot{\varepsilon}_{12}^2\Big]^{1/2} \equiv F(s,t). \tag{1.3}$$

So, we can determine, from obtained equations, stress and strain states of the cylindrical shell in the pre-buckling process.

1.2. Post - buckling process

As shown in [3], the system of stability equations of the cylindrical shell is written in the form

$$\begin{aligned}
&\beta_1\frac{\partial^4\varphi}{\partial x_1^4}+\beta_2\frac{\partial^4\varphi}{\partial x_1^3\partial x_2}+\beta_3\frac{\partial^4\varphi}{\partial x_1^2\partial x_2^2}+\beta_4\frac{\partial^4\varphi}{\partial x_1\partial x_2^3}\\
&\quad+\beta_5\frac{\partial^4\varphi}{\partial x_2^4}+\frac{N}{R}\frac{\partial^2\delta w}{\partial x_1^2}=0,
\end{aligned} \tag{1.4}$$

$$\begin{aligned}
&\alpha_1\frac{\partial^4\delta w}{\partial x_1^4}+\alpha_2\frac{\partial^4\delta w}{\partial x_1^3\partial x_2}+\alpha_3\frac{\partial^4\delta w}{\partial x_1^2\partial x_2^2}+\alpha_4\frac{\partial^4\delta w}{\partial x_1\partial x_2^3}+\alpha_5\frac{\partial^4\delta w}{\partial x_2^4}\\
&\quad+\frac{9}{Nh^2}\Big(p_{11}\frac{\partial^2\delta w}{\partial x_1^2}+2p_{12}\frac{\partial^2\delta w}{\partial x_1\partial x_2}+p_{22}\frac{\partial^2\delta w}{\partial x_2^2}-\frac{1}{R}\frac{\partial^2\varphi}{\partial x_1^2}\Big)=0.
\end{aligned} \tag{1.5}$$

where

$$\begin{aligned}
\beta_1 &= 1+\frac{1}{4}\Big(\frac{N}{\phi'}-1\Big)\Big(\frac{2\sigma_{22}-\sigma_{11}}{\sigma_u}\Big)^2+\frac{N}{9K},\\
\beta_2 &= -3\Big(\frac{N}{\phi'}-1\Big)\frac{\sigma_{12}(2\sigma_{22}-\sigma_{11})}{\sigma_u^2},
\end{aligned}$$

$$\beta_3 = 2\Big(1+\frac{N}{9K}\Big) + \frac{1}{2}\Big(\frac{N}{\phi'}-1\Big)\frac{(2\sigma_{22}-\sigma_{11})(2\sigma_{11}-\sigma_{22})}{\sigma_u^2} + 9\Big(\frac{N}{\phi'}-1\Big)\Big(\frac{\sigma_{12}}{\sigma_u}\Big)^2,$$

$$\beta_4 = -3\Big(\frac{N}{\phi'}-1\Big)\frac{\sigma_{12}(2\sigma_{11}-\sigma_{22})}{\sigma_u^2},$$

$$\beta_5 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'}-1\Big)\Big(\frac{2\sigma_{11}-\sigma_{22}}{\sigma_u}\Big)^2 + \frac{N}{9K},$$

$$\alpha_1 = \frac{3}{4N}\Big[(D_{11}\sigma_{11} - D_{12}\sigma_{22})\frac{\sigma_{11}}{\sigma_u^2} + 2D_{13} - D_{14}\Big],$$

$$\alpha_2 = \frac{3}{4N}\Big[2(D_{11}\sigma_{11} - D_{12}\sigma_{22})\frac{\sigma_{12}}{\sigma_u^2} + 2D_{31}\frac{\sigma_{12}\sigma_{11}}{\sigma_u^2} - 4D_{32} - 2D_{33}\Big],$$

$$\alpha_3 = \frac{3}{4N}\Big[(D_{11}\sigma_{11} - D_{12}\sigma_{22})\frac{\sigma_{22}}{\sigma_u^2} + D_{13} - 2D_{14} + (-D_{21}\sigma_{11} + D_{22}\sigma_{22})\frac{\sigma_{11}}{\sigma_u^2}$$
$$- 2D_{23} + D_{24} + 4D_{31}\frac{\sigma_{12}^2}{\sigma_u^2} + 2D_{34}\Big],$$

$$\alpha_4 = \frac{3}{4N}\Big[2(-D_{21}\sigma_{11} + D_{22}\sigma_{22})\frac{\sigma_{12}}{\sigma_u^2} + 2D_{31}\frac{\sigma_{12}\sigma_{22}}{\sigma_u^2} - 2D_{32} - 4D_{33}\Big],$$

$$\alpha_5 = \frac{3}{4N}\Big[(-D_{21}\sigma_{11} + D_{22}\sigma_{22})\frac{\sigma_{22}}{\sigma_u^2} - D_{23} + 2D_{24}\Big]. \tag{1.6}$$

The coefficients D_{ij} in (1.6) are calculated as follows [3]

$$D_{11} = \frac{A}{C}\Big(1+\frac{B}{3K}+A\frac{\sigma\sigma_{22}}{3K\sigma_u^2}\Big), \qquad D_{12} = \frac{A}{C}\Big(\frac{B}{3K}+A\frac{\sigma\sigma_{11}}{3K\sigma_u^2}\Big), \tag{1.7}$$

$$D_{13} = \frac{B}{C}\Big(1+\frac{B}{3K}+A\frac{\sigma\sigma_{22}}{3K\sigma_u^2}\Big), \qquad D_{14} = \frac{B}{C}\Big(\frac{B}{3K}+A\frac{\sigma\sigma_{11}}{3K\sigma_u^2}\Big),$$

$$D_{21} = \frac{A}{C}\Big(\frac{B}{3K}+A\frac{\sigma\sigma_{22}}{3K\sigma_u^2}\Big), \qquad D_{22} = \frac{A}{C}\Big(1+\frac{B}{3K}+A\frac{\sigma\sigma_{11}}{3K\sigma_u^2}\Big),$$

$$D_{23} = \frac{B}{C}\Big(\frac{B}{3K}+A\frac{\sigma\sigma_{22}}{3K\sigma_u^2}\Big), \qquad D_{24} = \frac{B}{C}\Big(1+\frac{B}{3K}+A\frac{\sigma\sigma_{11}}{3K\sigma_u^2}\Big),$$

$$D_{31} = A - \frac{A^2}{C}\cdot\frac{\sigma(\sigma_{11}+\sigma_{22})}{3K\sigma_u^2}, \qquad D_{32} = \frac{AB}{C}\frac{\sigma\sigma_{12}}{3K\sigma_u^2},\ D_{33} = \frac{AB}{C}\frac{\sigma\sigma_{12}}{3K\sigma_u^2},$$

$$D_{34} = B, \quad C = 1 + \frac{2B}{3K} + A\frac{\sigma(\sigma_{11}+\sigma_{22})}{3K\sigma_u^2}, \quad A = \phi' - N, \quad B = \frac{2}{3}N.$$

In order to solve the stability problem of shell, we suppose that the kinematic boundary condition is simply supported at the planes $x_1 = 0$ and $x_1 = L$.

2. Method of solution

We find the solution δw in the form

$$\delta w = A_{mn} \sin\left(\frac{m\pi x_1}{L} + \frac{n x_2}{R}\right). \tag{2.1}$$

It is easy to see that this solution satisfies the kinematic boundary condition in the sense of Saint-Venant.

Using the expression of δw and the equation (1.4), we obtain a relation for determining the function φ. The particular solution of this equation is of the form

$$\varphi = B_{mn} \sin\left(\frac{m\pi x_1}{L} + \frac{n x_2}{R}\right), \tag{2.2}$$

where

$$B_{mn} = \frac{N}{R} A_{mn} \Big[\beta_1\Big(\frac{m\pi}{L}\Big)^2 + \beta_2\Big(\frac{m\pi}{L}\Big)\Big(\frac{n}{R}\Big) + \beta_3\Big(\frac{n}{R}\Big)^2 + \beta_4\Big(\frac{L}{m\pi}\Big)\Big(\frac{n}{R}\Big)^3 + \beta_5\Big(\frac{L}{m\pi}\Big)^2\Big(\frac{n}{R}\Big)^4\Big]^{-1}. \tag{2.3}$$

Substituting (2.1) and (2.2) into the stability equation (1.5) and taking into account the existence of non-trivial solution i.e. $A_{mn} \neq 0$, we receive the expression for defining critical loads

$$\begin{aligned} p_{11}\Big(\frac{m\pi}{L}\Big)^2 + 2p_{12}\Big(\frac{m\pi}{L}\Big)\Big(\frac{n}{R}\Big) + p_{22}\Big(\frac{n}{R}\Big)^2 = \\ = N\Big\{\frac{h^2}{9}\Big[\alpha_1\Big(\frac{m\pi}{L}\Big)^4 + \alpha_2\Big(\frac{m\pi}{L}\Big)^3\Big(\frac{n}{R}\Big) + \alpha_3\Big(\frac{m\pi}{L}\Big)^2\Big(\frac{n}{R}\Big)^2 \\ + \alpha_4\Big(\frac{m\pi}{L}\Big)\Big(\frac{n}{R}\Big)^3 + \alpha_5\Big(\frac{n}{R}\Big)^4\Big] + \Big(\frac{m\pi}{L}\Big)^2\Big[\beta_1\Big(\frac{m\pi R}{L}\Big)^2 \\ + \beta_2\Big(\frac{m\pi R}{L}\Big)n + \beta_3 n^2 + \beta_4\Big(\frac{L}{m\pi R}\Big)n^3 + \beta_5\Big(\frac{L}{m\pi R}\Big)^2 n^4\Big]^{-1}\Big\}. \end{aligned} \tag{2.4}$$

By putting $\psi = n^2$, $\beta = \dfrac{m\pi R}{nL}$, $i = \dfrac{3R}{h}$ (called the slenderness of the shell), the relation (2.4) is written in the form

$$i^2 = \frac{N}{p_{11}\beta^2 + 2p_{12}\beta + p_{22}}\Big[(\alpha_1\beta^4 + \alpha_2\beta^3 + \alpha_3\beta^2 + \alpha_4\beta + \alpha_5)\psi + \frac{i^2\beta^4}{(\beta_1\beta^4 + \beta_2\beta^3 + \beta_3\beta^2 + \beta_4\beta + \beta_5)\psi}\Big]. \tag{2.5}$$

From here, we get

$$i^2 = \frac{N\psi^2(\alpha_1\beta^4 + \alpha_2\beta^3 + \alpha_3\beta^2 + \alpha_4\beta + \alpha_5)(\beta_1\beta^4 + \beta_2\beta^3 + \beta_3\beta^2 + \beta_4\beta + \beta_5)}{(p_{11}\beta^2 + 2p_{12}\beta + p_{22})(\beta_1\beta^4 + \beta_2\beta^3 + \beta_3\beta^2 + \beta_4\beta + \beta_5)\psi - N\beta^4}. \tag{2.6}$$

Because of the complex loading process depending on a parameter t, the quantities in the expression (2.6) are functions of t and s. Besides, the arc-length of the strain trajectory s from (1.3) is a function of t, too. So, we have to solve simultaneously the equations (1.3) and (2.6) by the loading parameter method [1]. After determining the critical value t_*, we can find the critical loads as follows

$$p_{11}^* = p_{11}(t_*), \quad p_{12}^* = p_{12}(t_*), \quad p_{22}^* = p_{22}(t_*).$$

Hereafter, we will consider some concrete problems.

3. Some concrete problems

3.1. Cylindrical shell subjected to compression along the generatrix

In this case, we get

$$\begin{aligned}
&\sigma_{11} = -p, \quad \sigma_{12} = \sigma_{22} = 0, \quad \sigma_u = |\sigma_{11}| = p,\\
&\dot{\varepsilon}_{11} = -\frac{\dot{p}}{\phi'} - \frac{\dot{p}}{9K}, \quad \dot{\varepsilon}_{22} = \frac{\dot{p}}{2\phi'} - \frac{\dot{p}}{9K}, \quad \dot{\varepsilon}_{33} = \frac{\dot{p}}{2\phi'} - \frac{\dot{p}}{9K}, \quad \dot{\varepsilon}_{12} = 0,\\
&\frac{ds}{dt} = \frac{\dot{p}(t)}{\phi'(s)} \quad \text{or} \quad s = \phi^{-1}(\sigma_u) = \phi^{-1}(p).
\end{aligned} \tag{3.1}$$

The coefficients in (1.6) and (1.7) are of the form

$$\begin{aligned}
&\alpha_1 = \frac{1}{4C} + \frac{3\phi'}{4NC} + \frac{\phi'}{9KC}, && \alpha_2 = \alpha_4 = 0,\\
&\alpha_3 = 1 + \frac{1}{C} - \frac{2\phi'}{9KC}, && \alpha_5 = \frac{1}{C} + \frac{\phi'}{9KC},\\
&\beta_1 = \frac{3}{4} + \frac{N}{4\phi'} + \frac{N}{9K}, && \beta_2 = \beta_4 = 0,\\
&\beta_3 = 3 - \frac{N}{\phi'} + \frac{2N}{9K}, && \beta_5 = \frac{N}{\phi'} + \frac{N}{9K};\\
&C = 1 + \frac{3N + \phi'}{9K}.
\end{aligned} \tag{3.2}$$

The relation (2.6) becomes

$$i^2 = \frac{N\psi^2(\alpha_1\beta^4 + \alpha_3\beta^2 + \alpha_5)(\beta_1\beta^4 + \beta_3\beta^2 + \beta_5)}{p\psi\beta^2(\beta_1\beta^4 + \beta_3\beta^2 + \beta_5) - N\beta^4}.$$

Putting $\theta = \beta^2 = \left(\frac{m\pi R}{nL}\right)^2$ and varying i^2, we obtain

$$i^2 = \frac{N}{p}\,\frac{\psi^2\left(\alpha_1\theta + \alpha_3 + \frac{\alpha_5}{\theta}\right)\left(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\right)}{\psi\left(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\right) - \frac{N}{p}}. \tag{3.3}$$

Minimizing this expression (3.3), i.e. $\dfrac{\partial i^2}{\partial \psi} = 0$, $\dfrac{\partial i^2}{\partial \theta} = 0$, gives us

$$\psi = \frac{2N}{p(\beta_1\theta + \beta_3 + \dfrac{\beta_5}{\theta})},$$

$$(\alpha_1\beta_3 - \alpha_3\beta_1)\theta^2 + 2(\alpha_1\beta_5 - \alpha_5\beta_1)\theta + \alpha_3\beta_5 - \alpha_5\beta_3 = 0. \tag{3.4}$$

Substituting this value into (3.3), we get

$$i^2 = \frac{4N^2}{p^2}\,\frac{(\alpha_1\theta + \alpha_3 + \dfrac{\alpha_5}{\theta})}{\beta_1\theta + \beta_3 + \dfrac{\beta_5}{\theta}}.$$

By taking into account (3.2), from this expression, we obtain

$$i^2 = \frac{4N^2}{p^2}\Big\{\Big(\frac{1}{4C} + \frac{3\phi'}{4NC} + \frac{\phi'}{9KC}\Big)\theta^2 + \Big(1 + \frac{1}{C} - \frac{2\phi'}{9KC}\Big)\theta + \frac{1}{C} + \frac{\phi'}{9KC}\Big\}\cdot$$
$$\cdot\Big\{\Big(\frac{3}{4} + \frac{N}{4\phi'} + \frac{N}{9K}\Big)\theta^2 + \Big(3 - \frac{N}{\phi'} + \frac{2N}{9K}\Big)\theta + \frac{N}{\phi'} + \frac{N}{9K}\Big\}^{-1}. \tag{3.5}$$

where θ is a solution of the second equation of (3.4).

For the long cylindrical shell i.e., $\psi = 1$, $\theta \ll 1$, we deduce from (3.3)

$$i^2 = \frac{N\alpha_5\beta_5}{p\Big(\beta_5\theta - \dfrac{N}{p}\theta^2\Big)}.$$

The minimization of this expression i^2 yields

$$\frac{\partial i^2}{\partial \theta} = -\frac{N\alpha_5\beta_5(p\beta_5 - 2N\theta)}{(p\beta_5\theta - N\theta^2)^2} = 0,$$

from here, we obtain $\theta = \dfrac{p\beta_5}{2N} = \theta_*$.

Substituting this value into $\dfrac{\partial^2 i^2}{\partial \theta^2}$, we have the sufficient condition of minimum

$$\frac{\partial^2 i^2}{\partial \theta^2}\Big|_{\theta=\theta_*} = \frac{32N^4}{p^4}\,\frac{\alpha_5}{\beta_5^3} = \frac{32N^4}{p^4}\,\frac{\dfrac{1}{C} + \dfrac{\phi'}{9KC}}{\Big(\dfrac{N}{\phi'} + \dfrac{N}{9K}\Big)^3}.$$

Since $C = 1 + \dfrac{3N+\phi'}{9K} > 0$, then $\dfrac{\partial^2 i^2}{\partial\theta^2}\Big|_{\theta=\theta_*} > 0$. Finally, we receive the expression for finding the critical loads as follows

$$i^2 = \frac{4N^2}{p^2}\left(\frac{\dfrac{1}{C}+\dfrac{\phi'}{9KC}}{\dfrac{N}{\phi'}+\dfrac{N}{9K}}\right). \tag{3.6}$$

Remarks

* If the material is incompressible i.e. $K \to +\infty$, $C = 1$, then (3.5) and (3.6) return to the results in [1, 4, 5].

* The obtained result (3.5) coincides with the one given in [3].

3.2. Cylindrical shell subjected to external pressure

The pre-buckling process is of the form

$$\begin{aligned}
&\sigma_{22} = -\frac{R}{h}\tilde{q} \equiv -q, \quad \sigma_{11} = \sigma_{12} = 0, \quad \sigma_u = |\sigma_{22}| = q,\\
&\dot\varepsilon_{11} = \frac{\dot q}{2\phi'} - \frac{\dot q}{9K}, \quad \dot\varepsilon_{22} = -\frac{\dot q}{\phi'} - \frac{\dot q}{9K}, \quad \dot\varepsilon_{33} = \frac{\dot q}{2\phi'} - \frac{\dot q}{9K}, \quad \dot\varepsilon_{12} = 0,\\
&\frac{ds}{dt} = \frac{\dot q(t)}{\phi'(s)} \quad \text{or} \quad s = \phi^{-1}(\sigma_u) = \phi^{-1}(q).
\end{aligned} \tag{3.7}$$

We can determine, from (1.6) and (1.7), the coefficients α_i, β_i as follows

$$\begin{aligned}
&\alpha_1 = \frac{1}{C} + \frac{\phi'}{9KC}, && \alpha_2 = \alpha_4 = 0,\\
&\alpha_3 = 1 + \frac{1}{C} - \frac{2\phi'}{9KC}, && \alpha_5 = \frac{1}{4C} + \frac{3\phi'}{4NC} + \frac{\phi'}{9KC},\\
&\beta_1 = \frac{N}{\phi'} + \frac{N}{9K}, && \beta_2 = \beta_4 = 0,\\
&\beta_3 = 3 - \frac{N}{\phi'} + \frac{2N}{9K}, && \beta_5 = \frac{3}{4} + \frac{N}{4\phi'} + \frac{N}{9K},\\
&C = 1 + \frac{3N+\phi'}{9K}.
\end{aligned}$$

In this case, the expression (2.6) is given by

$$i^2 = \frac{N\psi^2(\alpha_1\beta^4 + \alpha_3\beta^2 + \alpha_5)(\beta_1\beta^4 + \beta_3\beta^2 + \beta_5)}{q(\beta_1\beta^4 + \beta_3\beta^2 + \beta_5)\psi - N\beta^4}. \tag{3.8}$$

By denoting $\theta = \beta^2 = \left(\dfrac{m\pi R}{nL}\right)^2$, we can rewrite the above equation in the form

$$i^2 = \frac{N\psi^2\left(\alpha_1\theta + \alpha_3 + \dfrac{\alpha_5}{\theta}\right)\left(\beta_1\theta + \beta_3 + \dfrac{\beta_5}{\theta}\right)}{\dfrac{q}{\theta}\left(\beta_1\theta + \beta_3 + \dfrac{\beta_5}{\theta}\right)\psi - N}. \tag{3.9}$$

Minimizing this relation i.e. $\dfrac{\partial i^2}{\partial\psi}=0,\ \dfrac{\partial i^2}{\partial\theta}=0$, we get an expression for finding critical load

$$i^2=\frac{4N^2}{q^2}\frac{\theta^2\left(\alpha_1\theta+\alpha_3+\dfrac{\alpha_5}{\theta}\right)}{\left(\beta_1\theta+\beta_3+\dfrac{\beta_5}{\theta}\right)},$$

$$\begin{aligned}&\Big(\alpha_1-\frac{\alpha_5}{\theta^2}\Big)\Big(\beta_1\theta+\beta_3+\frac{\beta_5}{\theta}\Big)-\Big(\beta_1-\frac{\beta_5}{\theta^2}\Big)\Big(\alpha_1\theta+\alpha_3+\frac{\alpha_5}{\theta}\Big)\\&\quad+\frac{2}{\theta}\Big(\alpha_1\theta+\alpha_3+\frac{\alpha_5}{\theta}\Big)\Big(\beta_1\theta+\beta_3+\frac{\beta_5}{\theta}\Big)=0.\end{aligned}\tag{3.10}$$

Substituting α_i, β_i $(i=1,2,3)$ into the first relation of (3.10), we obtain

$$\begin{aligned}i^2=&\frac{4N^2}{q^2}\theta^2\Big\{\Big(\frac{1}{C}+\frac{\phi'}{9KC}\Big)\theta^2+\Big(1+\frac{1}{C}-\frac{2\phi'}{9KC}\Big)\theta+\frac{1}{4C}+\frac{3\phi'}{4NC}\\&+\frac{\phi'}{9KC}\Big\}\cdot\Big\{\Big(\frac{N}{\phi'}+\frac{N}{9K}\Big)\theta^2+\Big(3-\frac{N}{\phi'}+\frac{2N}{9K}\Big)\theta+\frac{3}{4}+\frac{N}{4\phi'}+\frac{N}{9K}\Big\}^{-1},\end{aligned}\tag{3.11}$$

where θ is a solution of the second equation of (3.10).

For long cylindrical shells i.e. $\psi=1$, $\theta\ll1$, the expression (3.9) gives us

$$i^2=\frac{N}{q}\Big(\alpha_5+\frac{i^2\theta^2}{\psi\beta_5}\Big)\quad\text{or}\quad i^2=\frac{N\alpha_5\beta_5}{q\beta_5-N\theta^2}\cdot$$

By the same method, we get $\dfrac{\partial i^2}{\partial\theta}=0$ i.e. $\theta_*=0$; the sufficient condition of minimum is satisfied:

$$\frac{\partial^2 i^2}{\partial\theta^2}\Big|_{\theta=\theta_*}=\frac{2N^2}{q^2}\frac{\alpha_5}{\beta_5}=\frac{2N^2}{q^2}\frac{\dfrac{1}{4C}+\dfrac{3\phi'}{4NC}+\dfrac{\phi'}{9KC}}{\dfrac{3}{4}+\dfrac{N}{4\phi'}+\dfrac{N}{9K}}>0.$$

So the minimal value of i^2 leads to

$$i^2=\frac{N}{q}\alpha_5=\frac{N}{q}\Big(\frac{1}{4C}+\frac{3\phi'}{4NC}+\frac{\phi'}{9KC}\Big)\tag{3.12}$$

where $C=1+\dfrac{3N+\phi'}{9K}>0$.

Remarks

* If a shell material is incompressible, then (3.11) and (3.12) coincide with the results in [1, 4, 5].

* According to (3.7) N, $\phi'(s)$ can be represented as functions of q, from (3.12) we can determine critical q^*.

3.3. Cylindrical shell subjected to torsion at its butt-ends

Let us consider a cylindrical shell of radius R, length L and thickness h subjected to torsion by a pair of moments $M_k = 2\pi h R^2 \tau$ at the two butt-ends. Suppose that a material of shell is compressible and a kinematic boundary condition is simply supported at the planes $x_1 = 0$ and $x_1 = L$.

The pre-buckling process, in this case, is represented as

$$\begin{aligned}
&\sigma_{12} = -\tau, \quad \sigma_{11} = \sigma_{22} = 0, \quad \sigma_u = \sqrt{3}|\sigma_{12}|,\\
&\dot{\varepsilon}_{11} = \dot{\varepsilon}_{22} = \dot{\varepsilon}_{33} = 0, \quad \dot{\varepsilon}_{12} = -\frac{3\dot{\tau}}{2\phi'},\\
&\frac{ds}{dt} = \frac{\sqrt{3}\,\dot{\tau}}{\phi'(s)} \quad \text{or} \quad s = \phi^{-1}(\sigma_u) = \phi^{-1}(\sqrt{3}\,\tau).
\end{aligned} \tag{3.13}$$

The coefficients α_i, β_i are calculated by the formulae

$$\begin{aligned}
&\alpha_1 = \alpha_5 = \frac{1}{C} + \frac{N}{9KC}, \ \alpha_2 = \alpha_4 = 0, \ \alpha_3 = \frac{1}{C} + \frac{\phi'}{N} - \frac{2N}{9KC},\\
&\beta_1 = \beta_5 = 1 + \frac{N}{9K}, \ \beta_2 = \beta_4 = 0, \ \beta_3 = \frac{2N}{9K} + \frac{3N}{\phi'} - 1,\\
&C = 1 + \frac{4N}{9K}\cdot
\end{aligned} \tag{3.14}$$

Putting $i = \frac{3R}{h}$, $\psi = n^2$, $\beta = \frac{m\pi R}{nL}$, the relation (2.6) takes the form

$$i^2 = \frac{N}{2\tau}\Big[\big(\alpha_1\beta^3 + \alpha_3\beta + \frac{\alpha_5}{\beta}\big)\psi + \frac{i^2\beta^3}{\psi(\beta_1\beta^4 + \beta_3\beta + \beta_5)}\Big]\cdot$$

From here, one deduces

$$i^2 = \frac{N}{2\tau}\frac{\big(\alpha_1\beta^3 + \alpha_3\beta + \frac{\alpha_5}{\beta}\big)\big(\beta_1\beta^4 + \beta_3\beta + \beta_5\big)\psi^2}{\psi(\beta_1\beta^4 + \beta_3\beta + \beta_5) - \frac{N}{2\tau}\beta^3}\cdot$$

Minimizing this expression, after series of calculations, gives us

$$i^2 = \frac{N^2}{\tau^2}\,\frac{(\alpha_1\beta^6 + \alpha_3\beta^4 + \alpha_5\beta^2)}{(\beta_1\beta^4 + \beta_3\beta + \beta_5)}, \tag{3.15}$$

where β is a solution of the following equation:

$$\begin{aligned}
&\Big(3\alpha_1\beta^2 + \alpha_3 - \frac{\alpha_5}{\beta^2}\Big)(\beta_1\beta^4 + \beta_3\beta + \beta_5)\beta - (4\beta_1\beta^3 + \beta_3)\Big(\alpha_1\beta^3 + \alpha_3\beta + \frac{\alpha_5}{\beta}\Big)\beta\\
&+ 3\Big(\alpha_1\beta^3 + \alpha_3\beta + \frac{\alpha_5}{\beta}\Big)(\beta_1\beta^4 + \beta_3\beta + \beta_5) = 0.
\end{aligned} \tag{3.16}$$

Substituting α_i, β_i $(i = 1, 2, 3)$ from (3.14) into (3.16), we obtain

$$i^2 = \frac{N^2}{\tau^2}\Big[\Big(\frac{1}{C} + \frac{N}{9KC}\Big)\beta^6 + \Big(\frac{1}{C} + \frac{\phi'}{N} - \frac{2N}{9KC}\Big)\beta^4 + \Big(\frac{1}{C} + \frac{N}{9KC}\Big)\beta^2\Big] \cdot$$
$$\cdot \Big[\Big(1 + \frac{N}{9K}\Big)\beta^4 + \Big(\frac{2N}{9K} + \frac{3N}{\phi'} - 1\Big)\beta + 1 + \frac{N}{9K}\Big]^{-1}. \tag{3.17}$$

The relations (3.16), (3.17) let us define the critical loads.

For long cylindrical shells, i.e. $\psi = 1$, $\beta^2 \ll 1$, we receive from (3.15)

$$i^2 = \frac{N}{2\tau}\frac{\alpha_5\beta_5}{\beta\beta_5 - \dfrac{N}{2\tau}\beta^4}.$$

The minimization of i^2, i.e. $\dfrac{\partial i^2}{\partial \beta} = 0$, yields

$$\beta = \sqrt[3]{\frac{\tau\beta_5}{2N}} = \beta_*$$

Consider the sufficient condition of extremum

$$\frac{\partial^2 i^2}{\partial \beta^2}\Big|_{\beta=\beta_*} = \frac{16}{3}\frac{N^2}{\tau^2}\frac{\alpha_5}{\beta_5} = \frac{16}{3}\frac{N^2}{\tau^2}\cdot\frac{1}{C}.$$

Since $C = 1 + \dfrac{4N}{9K} > 0$, then $\dfrac{\partial^2 i^2}{\partial \beta^2}\Big|_{\beta=\beta_*} > 0$, this condition is satisfied.

Substituting β_* into the expression i^2, we have

$$i^2 = \Big(\frac{2N}{\tau}\Big)^{4/3}\frac{\Big(1 + \dfrac{N}{9K}\Big)^{2/3}}{1 + \dfrac{4N}{9K}} \tag{3.18}$$

It is clear that the result (3.17) returns to the one in [2, 4], if the material is incompressible.

3.4. Cylindrical shell simultaneously subjected to compression along the generatrix and external pressure

Assume that the structure is subjected to compression p and external pressure q. The material is compressible. In this case, the pre-buckling process is of the form

$$\sigma_{11} = -p, \quad \sigma_{22} = -q, \quad \sigma_{12} = 0, \quad \sigma_u^2 = p^2 - pq + q^2.$$

Components of the strain velocity tensor determined according to the theory of elastoplastic processes [1] are given by the formulae [3]

$$\begin{aligned}
&\dot{\varepsilon}_{11} = \frac{1}{N}\Big(-\dot{p}+\frac{1}{2}\dot{q}\Big) - Q(s,t)\Big(p-\frac{1}{2}q\Big) - \frac{\dot{p}+\dot{q}}{9K},\\
&\dot{\varepsilon}_{22} = \frac{1}{N}\Big(-\dot{q}+\frac{1}{2}\dot{p}\Big) - Q(s,t)\Big(q-\frac{1}{2}p\Big) - \frac{\dot{p}+\dot{q}}{9K},\\
&\dot{\varepsilon}_{33} = \frac{1}{2N}(\dot{p}+\dot{q}) + \frac{1}{2}Q(s,t)(p+q) - \frac{\dot{p}+\dot{q}}{9K},\\
&\dot{\varepsilon}_{12} = 0,\\
&Q(s,t) = \frac{1}{\sigma_u^2}\Big(\frac{1}{\phi'}-\frac{1}{N}\Big)\Big(p\dot{p}+q\dot{q}-\frac{1}{2}p\dot{q}-\frac{1}{2}q\dot{p}\Big),\\
&\phi' = \phi'(s), \quad N = \frac{\sigma_u}{s}.
\end{aligned} \tag{3.19}$$

The coefficients α_i, β_i are calculated as follows

$$\begin{aligned}
&\alpha_1 = \frac{1}{C} - \frac{3}{4C}\Big(1-\frac{\phi'}{N}\Big)\frac{p^2}{\sigma_u^2} + \frac{\phi'}{9KC}, \quad \alpha_2=\alpha_4=0,\\
&\alpha_3 = 1 + \frac{1}{C} - \frac{3}{2C}\Big(1-\frac{\phi'}{N}\Big)\frac{pq}{\sigma_u^2} - \frac{2\phi'}{9KC},\\
&\alpha_5 = \frac{1}{C} - \frac{3}{4C}\Big(1-\frac{\phi'}{N}\Big)\frac{q^2}{\sigma_u^2} + \frac{\phi'}{9KC},
\end{aligned}$$

$$\begin{aligned}
&\beta_1 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'}-1\Big)\Big(\frac{2q-p}{\sigma_u}\Big)^2 + \frac{N}{9K}, \quad \beta_2=\beta_4=0,\\
&\beta_3 = 2\Big(1+\frac{N}{9K}\Big) + \frac{1}{2}\Big(\frac{N}{\phi'}-1\Big)\frac{(2q-p)(2p-q)}{\sigma_u^2},\\
&\beta_5 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'}-1\Big)\Big(\frac{2p-q}{\sigma_u}\Big)^2 + \frac{N}{9K},\\
&C = 1 + \frac{4N}{9K} + \frac{(\phi'-N)}{9K}\frac{(p+q)^2}{p^2-pq+q^2}.
\end{aligned} \tag{3.20}$$

By the same method presented in the above part we obtain a relation for finding the critical load

$$i^2 = \frac{N}{p\beta^2+q}\Big[\psi(\alpha_1\beta^4+\alpha_3\beta^2+\alpha_5) + \frac{i^2\beta^4}{\psi(\beta_1\beta^4+\beta_3\beta^2+\beta_5)}\Big]$$

or

$$i^2 = \frac{N\psi^2(\alpha_1\beta^4+\alpha_3\beta^2+\alpha_5)(\beta_1\beta^4+\beta_3\beta^2+\beta_5)}{(p\beta^2+q)(\beta_1\beta^4+\beta_3\beta^2+\beta_5)\psi - N\beta^4}$$

where $i = \frac{3R}{h}$, $\psi = n^2$, $\beta = \frac{m\pi R}{nL}$.

For calculating usefully, we represent i^2 in the another form by denoting $\theta = \beta^2$.

$$i^2 = \frac{N\psi^2\big(\alpha_1\theta + \alpha_3 + \frac{\alpha_5}{\theta}\big)\big(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\big)}{\big(p + \frac{q}{\theta}\big)\big(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\big)\psi - N}. \tag{3.21}$$

Minimizing this expression i.e. $\frac{\partial i^2}{\partial \psi} = 0$, $\frac{\partial i^2}{\partial \theta} = 0$, after some calculations, gives us

$$\psi = \frac{2N}{\big(p + \frac{q}{\theta}\big)\big(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\big)} \tag{3.22}$$

$$\begin{aligned}&\Big(\alpha_1 - \frac{\alpha_5}{\theta^2}\Big)\Big(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\Big) - \Big(\beta_1 - \frac{\beta_5}{\theta^2}\Big)\Big(\alpha_1\theta + \alpha_3 + \frac{\alpha_5}{\theta}\Big)\\ &+ \frac{2q}{\theta^2\big(p + \frac{q}{\theta}\big)}\Big(\alpha_1\theta + \alpha_3 + \frac{\alpha_5}{\theta}\Big)\Big(\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}\Big) = 0.\end{aligned} \tag{3.23}$$

So, substituting (3.22) into (3.21), we get

$$i^2 = \frac{4N^2}{\big(p + \frac{q}{\theta}\big)^2}\left(\frac{\alpha_1\theta + \alpha_3 + \frac{\alpha_5}{\theta}}{\beta_1\theta + \beta_3 + \frac{\beta_5}{\theta}}\right). \tag{3.24}$$

By taking into account (3.20), from (3.24), we obtain

$$\begin{aligned}i^2 = \frac{4N^2\theta^2}{(p\theta + q)^2}&\Big\{\Big[\frac{1}{C} - \frac{3}{4C}\Big(1 - \frac{\phi'}{N}\Big)\frac{p^2}{p^2 - pq + q^2} + \frac{\phi'}{9KC}\Big]\theta^2\\ &+ \Big[1 - \frac{2\phi'}{9KC} + \frac{1}{C} - \frac{3}{2C}\Big(1 - \frac{\phi'}{N}\Big)\frac{pq}{p^2 - pq + q^2}\Big]\theta + \frac{1}{C}\\ &- \frac{3}{4C}\Big(1 - \frac{\phi'}{N}\Big)\frac{q^2}{p^2 - pq + q^2} + \frac{\phi'}{9KC}\Big\}\cdot\Big\{\Big[1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2q - p)^2}{p^2 - pq + q^2}\\ &+ \frac{N}{9K}\Big]\theta^2 + \Big[2\Big(1 + \frac{N}{9K}\Big) + \frac{1}{2}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2q - p)(2p - q)}{p^2 - pq + q^2}\Big]\theta\\ &+ 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2p - q)^2}{p^2 - pq + q^2} + \frac{N}{9K}\Big\}^{-1}.\end{aligned} \tag{3.25}$$

where θ is a solution of the equation of (3.23).

Applying the loading parameter method [1] we solve simultaneously the equations (1.3) and (3.25). After finding the critical value t_*, we can determine the critical loads as follows

$$p^* = p(t_*), \quad q^* = q(t_*).$$

For long cylindrical shells, see [1, 2], we get

$$\psi = 1, \quad \theta \ll 1, \quad i^2 = \frac{N\alpha_5\beta_5}{(p\theta + q)\beta_5 - N\theta^2}. \tag{3.26}$$

Minimizing this expression i.e. $\dfrac{\partial i^2}{\partial \theta} = 0$, gives us

$$\theta = \frac{p\beta_5}{2N} \equiv \theta_*.$$

Now consider the sufficient condition of extremum

$$\frac{\partial^2 i^2}{\partial \theta^2}\bigg|_{\theta=\theta_*} = \frac{2\alpha_5\beta_5 N^2}{\left(\dfrac{p^2\beta_5^2}{4N} + q\beta_5\right)^2}.$$

We will demonstrate $\alpha_5 > 0$, $\beta_5 > 0$. In reality, we have

$$\begin{aligned}
C &= 1 + \frac{4N}{9K} + \frac{\phi' - N}{9K}\frac{(p+q)^2}{\sigma_u^2} \\
&= 1 + \frac{\phi'}{9K}\frac{(p+q)^2}{\sigma_u^2} + \frac{N}{3K}\frac{(p-q)^2}{\sigma_u^2} > 0, \\
\alpha_5 &= \frac{1}{C}\Big[1 - \frac{3}{4}\Big(1 - \frac{\phi'}{N}\Big)\frac{q^2}{\sigma_u^2} + \frac{\phi'}{9K}\Big] \\
&= \frac{1}{C}\Big[\frac{(2p-q)^2}{4\sigma_u^2} + \frac{3}{4}\frac{\phi' q^2}{N\sigma_u^2} + \frac{\phi'}{9K}\Big] > 0, \\
\beta_5 &= 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2p-q)^2}{\sigma_u^2} + \frac{N}{9K} \\
&= \frac{3q^2}{4\sigma_u^2} + \frac{N}{4\phi'}\frac{(2p-q)^2}{\sigma_u^2} + \frac{N}{9K} > 0.
\end{aligned}$$

So $\dfrac{\partial^2 i^2}{\partial \theta^2}\bigg|_{\theta=\theta_*} > 0$, the sufficient condition of minimum is satisfied.

Substituting α_5, β_5 from (3.20) into (3.26), this relation takes the form

$$i^2 = \frac{4N^2\Big[\dfrac{1}{C} - \dfrac{3}{4C}\Big(1 - \dfrac{\phi'}{N}\Big)\dfrac{q^2}{p^2 - pq + q^2} + \dfrac{\phi'}{9KC}\Big]}{p^2\Big[1 + \dfrac{1}{4}\Big(\dfrac{N}{\phi'} - 1\Big)\dfrac{(2p-q)^2}{p^2 - pq + q^2} + \dfrac{N}{9K}\Big] + 4Nq}. \tag{3.27}$$

Expressing $p = p(t)$, $q = q(t)$ one can find the critical value of loading parameter t_* from (1.3) and (3.27). The critical loads will be given $p^* = p(t_*)$, $q^* = q(t_*)$.
Remarks
* If a shell material is incompressible, then (3.25) and (3.27) coincide with the results obtained in [1, 4].
* If the compression p is equal to zero, then (3.25) returns to (3.5).
* If the pressure q takes the value equal to zero, then we can obtain from (3.25) the result (3.11).

4. Numerical examples and discussion

Let us consider a long cylindrical shell subjected to compression p along the generatrix and external pressure q which depend on a loading parameter t as follows

$$p(t) = \frac{(q_0 + q_1 t)^2}{q_2}; \quad q(t) = q_0 + q_1 t, \tag{4.1}$$

where q_0, q_1, q_2 are known constants.

Substituting (4.1) into (3.27), we obtain

$$i \equiv \frac{3R}{h} = \frac{2q_2 N}{(q_0 + q_1 t)^2} \Big\{ \frac{1}{C} - \frac{3}{4C}\Big(1 - \frac{\phi'}{N}\Big) \frac{(q_0 + q_1 t)^2}{\sigma_u^2} + \frac{\phi'}{9KC} \Big\}^{\frac{1}{2}} \cdot \tag{4.2}$$
$$\cdot \Big\{ 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big) \Big[\frac{2(q_0 + q_1 t)^2}{q_2} - (q_0 + q_1 t) \Big]^2 \cdot \frac{1}{\sigma_u^2} + \frac{N}{9K} + \frac{4Nq_2^2}{(q_0 + q_1 t)^3} \Big\}^{-\frac{1}{2}}$$

where

$$\sigma_u = \sigma_u(t) = \Big[\frac{(q_0 + q_1 t)^4}{q_2^2} - \frac{(q_0 + q_1 t)^3}{q_2} + (q_0 + q_1 t)^2 \Big]^{\frac{1}{2}}; \ \phi' = \phi'(s); \ N = \frac{\sigma_u}{s}. \tag{4.3}$$

If the shell is incompressible, we have $K \to +\infty$, $C = 1$ and

$$i \equiv \frac{3R}{h} = \frac{2q_2 N}{(q_0 + q_1 t)^2} \Big\{ 1 - \frac{3}{4}\Big(1 - \frac{\phi'}{N}\Big) \frac{(q_0 + q_1 t)^2}{\sigma_u^2} \Big\}^{\frac{1}{2}} \cdot \tag{4.4}$$
$$\cdot \Big\{ 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big) \Big[\frac{2(q_0 + q_1 t)^2}{q_2} - (q_0 + q_1 t) \Big]^2 \cdot \frac{1}{\sigma_u^2} + \frac{4Nq_2^2}{(q_0 + q_1 t)^3} \Big\}^{-\frac{1}{2}}.$$

The arc-length of stain trajectory in (1.3) is calculated by the Euler iterative method

$$s(t_{n+1}) = s(t_n) + \tau F(t_n, s(t_n)); \quad \tau = t_{n+1} - t_n. \tag{4.5}$$

In order to investigate the influence of compressibility of material on the stability of cylindrical shell, we will calculate and compare the critical forces of the compressible shell with the ones of incompressible shell.

A numerical analysis is considered on the long cylindrical shell made of steel 30ХГСА with an elastic modulus $3G = 2.6 \cdot 10^5$ MPa, an yield point $\sigma_s = 400$ MPa, the constants $q_0 = 2$ MPa; $q_1 = 0.1$ MPa; $q_2 = 0.1$ MPa, the iterative step $\tau = 0.01$. The ratio $\frac{R}{h}$ varies from 22 to 49 and from 50 to 77 with the arithmetical ratio equal to 3. The coefficients K and ν are taken the value

$$K = \frac{E}{3(1-2\nu)}; \quad E = 2G(1+\nu);$$

$\nu = 0.310;\ 0.320;\ 0.330;\ 0.350;\ 0.400;\ 0.410;\ 0.420;\ 0.430;\ 0.440$

Hereafter, we give the numerical results in two cases

1. The Poisson coefficient ν varies, while the ratio $\frac{R}{h}$ is constant (see table 1)

2. The slenderness of shell $i = \frac{3R}{h}$ varies, while the Poisson coefficient ν is constant (see table 2)

Table 1

ν	t^*	$s \cdot 10^3$	p^* MPa	q^* MPa	σ_u^* MPa
Ratio $R/h = 22$; $i = 66$					
0.500	56.61	9.0148	586.8	7.66	583.0
0.310	51.92	7.8972	517.1	7.19	513.5
0.320	52.11	7.9699	519.8	7.21	516.3
0.330	52.30	8.0007	522.6	7.23	519.0
0.340	52.48	8.0252	525.2	7.25	521.6
0.350	52.69	8.0871	528.2	7.27	524.6
0.400	53.87	8.4108	545.5	7.39	541.9
0.410	54.11	8.4718	549.1	7.41	545.4
0.420	54.36	8.5386	552.8	7.44	549.1
0.430	54.63	8.5904	556.8	7.46	553.1
0.440	54.90	8.6479	560.9	7.49	557.1
Ratio $R/h = 40$; $i = 120$					
0.500	51.14	3.4970	505.9	7.11	502.4
0.310	46.57	2.7527	443.0	6.66	439.7
0.320	46.73	2.7857	445.2	6.67	441.9
0.330	46.92	2.8154	447.7	6.69	444.4
0.340	47.12	2.8550	450.4	6.71	447.1
0.350	47.29	2.8756	452.7	6.73	449.3
0.400	48.38	3.0512	467.4	6.84	464.1
0.410	48.62	3.0968	470.7	6.86	467.3
0.420	48.86	3.1383	474.0	6.89	470.6

0.430	49.14	3.1838	477.9	6.91	474.5
0.440	49.42	3.2317	481.8	6.94	478.3
Ratio $R/h = 62$; $i = 186$					
0.500	46.54	1.7579	442.6	6.65	439.3
0.310	28.41	1.0212	234.3	4.84	231.9
0.320	29.34	1.0530	243.3	4.93	240.9
0.330	30.29	1.0859	252.8	5.03	250.3
0.340	31.27	1.1204	262.8	5.13	260.2
0.350	32.28	1.1566	273.2	5.23	270.6
0.400	37.85	1.3669	334.5	5.78	331.7
0.410	39.08	1.4158	348.9	5.91	346.0
0.420	40.36	1.4676	364.2	6.04	361.2
0.430	41.68	1.5221	380.3	6.17	377.3
0.440	43.05	1.5797	397.4	6.30	394.3
Ratio $R/h = 77$; $i = 231$					
0.500	28.22	0.8848	232.4	4.82	230.0
0.310	11.71	0.4357	100.5	3.17	98.9
0.320	12.33	0.4497	104.5	3.23	102.9
0.330	12.97	0.4643	108.6	3.30	107.0
0.340	13.63	0.4796	113.0	3.36	111.4
0.350	14.31	0.4956	117.6	3.43	116.0
0.400	18.08	0.5896	144.9	3.81	143.1
0.410	18.92	0.6117	151.4	3.89	149.5
0.420	19.79	0.6350	158.2	3.98	156.3
0.430	20.70	0.6599	165.6	4.07	163.6
0.440	21.64	0.6861	173.3	4.16	171.3

Table 2

R/h	i	t^*	$s \cdot 10^3$	p^* MPa	q^* MPa	σ_u^* MPa
$\nu = 0.500$						
22	66	55.61	9.0148	586.8	7.66	583.0
25	75	55.19	7.2518	565.2	7.52	561.5
28	84	54.07	6.0078	548.5	7.41	544.8
31	93	53.14	5.0610	534.8	7.31	531.2
34	102	52.48	4.4502	525.2	7.25	521.6
37	111	51.88	3.9910	516.5	7.19	513.0
40	120	51.14	3.4970	505.9	7.11	502.4

43	129	50.42	3.0470	495.8	7.04	492.3
46	138	49.90	2.7508	488.5	6.99	485.0
49	147	49.43	2.5295	481.9	6.94	478.5
50	150	49.27	2.4634	479.7	6.93	476.3
53	159	48.70	2.2608	471.8	6.87	468.4
56	168	48.03	2.0577	462.7	6.80	459.3
59	177	47.46	1.9178	455.0	6.75	451.6
62	186	46.54	1.7579	442.6	6.65	439.3
65	195	45.19	1.6356	424.8	6.52	421.6
68	204	41.15	1.4261	373.8	6.11	370.8
71	213	36.35	1.2102	317.4	5.63	314.6
74	222	32.06	1.0321	270.9	5.21	268.4
77	231	28.22	0.8848	232.4	4.82	230.0
$\nu = 0.310$						
22	66	51.92	7.8972	517.1	7.19	513.5
25	75	50.67	6.3290	499.3	7.07	495.8
28	84	49.61	5.1344	484.4	6.96	481.0
31	93	48.96	4.3813	474.0	6.89	470.6
34	102	48.11	3.7855	463.8	6.81	460.4
37	111	47.23	3.1718	451.9	6.72	448.5
40	120	46.57	2.7527	443.0	6.66	439.7
43	129	46.00	2.4685	435.5	6.60	432.2
46	138	45.26	2.1900	425.8	6.53	422.5
49	147	44.43	1.9405	415.0	6.44	411.8
50	150	44.15	1.8786	411.4	6.41	408.2
53	159	43.51	1.7620	403.2	6.35	400.1
56	168	38.70	1.5042	344.5	5.87	341.6
59	177	33.22	1.2354	283.1	5.32	280.5
62	186	28.41	1.0212	234.3	4.84	231.9
65	195	24.20	0.8504	195.3	4.42	193.1
68	204	20.48	0.7126	163.8	4.05	161.8
71	213	17.20	0.6011	138.3	3.72	136.5
74	222	14.30	0.5104	117.6	3.43	115.9
77	231	11.71	0.4357	100.5	3.17	99.9

From the above results we can lead some conclusions

1. Theory of elastoplastic processes can be applied to the stability problem of cylindrical shell when both pre-buckling and post-buckling processes are complicated

2. The compressibility of material has an influence on the stability of structure. The more the Poisson coefficient ν decreases, the more the value of critical force

diminishes when the ratio $\frac{R}{h}$ is constant. This remark is deduced from the results on the table 1.

3. The more the shell is thin i.e. the slenderness i increases, the more influence of compressibility is great. This observation can be seen on the table 2.

4. For long shells we have shown the necessary and sufficient conditions of extremum. In general case, the sufficient condition, by reason of mathematical complication, will be verified in numerical examples.

5. When a material is incompressible, the obtained results return to the previous well-known ones (see [1, 2, 4, 5)

6. Associating with the loading parameter method, the proposed method gives a way to solve efficiently series of stability problems of cylindrical shells.

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PHƯƠNG PHÁP GIẢI BÀI TOÁN ỔN ĐỊNH ĐÀN DẺO CỦA VỎ TRỤ VỚI VẬT LIỆU NÉN ĐƯỢC CHỊU QUÁ TRÌNH ĐẶT TẢI PHỨC TẠP

Bài này nhằm nghiên cứu lời giải và phương pháp xác định tải tới hạn trong bài toán ổn định đàn dẻo của kết cấu chịu quá trình đặt tải phức tạp. Các kết quả thu được cho thấy ảnh hưởng của tính nén được của vật liệu lên sự ổn định của vỏ trụ. Khi vật liệu là không nén được, ta nhận được các kết quả đã có trước đây.

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