Vietnam Journal of Mechanics, NCST of Vietnam Vol. **23**, 2001, No 1 (17 – 24)

CONTROL OF A POINTER LOCATED ON A VIBRATING SYSTEM

F. Casciati[1], L. Faraveli[1], Nguyen Dong Anh[2]
[1] *Dept. of Structural Mechanics University of Pavia, Italy*
[2] *Institute of Mechanics, NCST of Vietnam*

ABSTRACT. In this paper the problem of controlling a bar supporting, at one end, a pointer and hinged, at the other end, to a structural system vibrating under any external excitation is investigated. The problem is to maintain the pointer toward the target with a prescribed tolerance. Emphasis is put on the uncertainty, which characterises the problem and a simple fuzzy controller approach is discussed. Some numerical simulations are presented.

Keywords. Active control, fuzzy control, linear electromagnetic motor, pointer.

1. Introduction

The control of a structure can pursue two different objectives. [Cassiati et al. (1998), Kobori et al. (1999)]:

- To reduce the response to extreme external excitation in order to reduce the stresses and hence to increase the system reliability;

- To respect service ability constraints even when external disturbances would prevent from it.

This paper approaches the latter problem in which the control of a bar supporting, at one end, a pointer and hinged, at the other end, to a structural system vibrating under any external excitation is considered. The problem is to maintain the pointer toward the target with a prescribed tolerance. Further, since fuzzy control is a recognised alternative to standard control tools allowing the resolution of imprecise or uncertain information, [Casciati and Faravelli (1991)], a fuzzy-chip controller is introduced in order to incorporate uncertainty and to ensure robustness.

2. Problem formulation

With reference to figure 1, consider the bar OO_1 of length L and mass m. It is supported by a hinge O while $PQ = u(t)$ is regarded as a control distance. The control action is provided by a linear electromagnetic engine. The supporting hinge O and the point P are kept in a table which moves along the horizontal direction OR.

The motion of the bar is controlled by the control distance $PQ = u(t)$. The initial value of $u(t)$ is $u(t_0)$. Let $x(t)$ be the displacement time history with the initial values $x(t_0) = x_0$, $\dot{x}(t_0) = \dot{x}_0$.

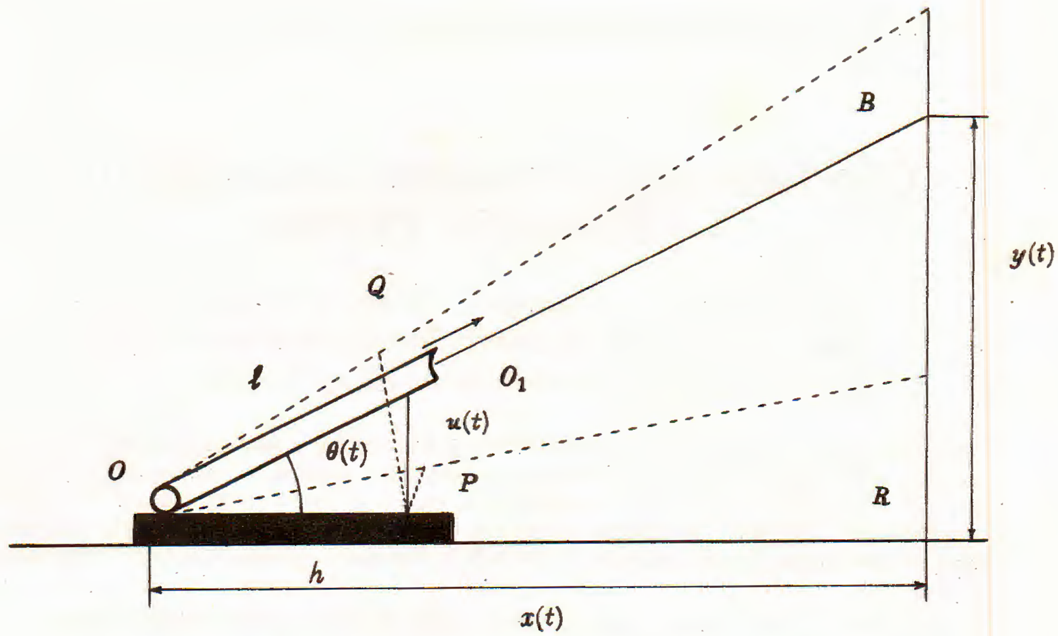

Fig. 1. The bar supporting a pointer on a vibrating table

The goal is that during the motion the line OO_1 always goes into a ε-neighborhood of the point B_0 of position y_0, ε being a given positive number. It can be expressed as

$$y_0 - \varepsilon \le y(t) \le y_0 + \varepsilon, \tag{2.1}$$

where $y(t)$ denotes the position of point B. From the triangle OPQ one has

$$PQ^2 = OP^2 + OQ^2 - 2OP \cdot PQ \cos\theta(t)$$

or

$$u^2(t) = h^2 + \ell^2 - 2h\ell \cos\theta(t). \tag{2.2}$$

Now the control problem can be formulated as follows: find the control law $u(t)$ such that condition (2.1) is satisfied during the motion, $x(t)$ being measured by an adequate sensor.

3. Fully deterministic case (Exact measurement and perfect control)

Suppose that the measurement of the horizontal motion $x(t)$ can be obtained exactly and the control process can be operated perfectly. In this case it is possible to find a control law in such a way that the bar OO_1 always goes into the point B_0. In fact, one gets from the triangle ORB: $\tan\theta(t) = RB_0/OR$, or

$$\tan\theta(t) = \frac{y_0}{x(t)} \cdot \tag{3.1}$$

Substituting (3.1) into (2.2) one can express the control distance in terms of the horizontal motion $x(t)$:

$$u^2(x) = h^2 + \ell^2 - 2h\ell \frac{x}{\sqrt{y_0 + x^2}} \cdot \tag{3.2}$$

Thus, at every value x, control law (3.2) imposes on the control distance $u(t)$ to keep the straight line OO_1 in B_0. It is seen from (3.2) that the control law is a non-linear function of the input $x(t)$.

4. The role of uncertainty

Case 1. Exact measurement and time-delay control

Suppose that the measurement of the horizontal motion $x(t)$ can be obtained exactly and the control process operates with a time delay τ. So, at the time instant t, instead of the required controlled distance $u(t)$ one can only produce $u(t-\tau)$ which will cause a deviation from the point B_0 denoted as $\Delta_D y_0$. The deviation depends on the time t and the time delay τ and can be calculated as follows. At time t one should have $y_0 = x(t)\tan\theta(t)$, but due to time-delay in the control operation, one gets

$$y_0 + \Delta_D y_0 = x(t)\tan\theta(t-\tau). \tag{4.1}$$

Putting in (3.1) t by $t - \tau$ yields

$$y_0 = x(t-\tau)\tan\theta(t-\tau). \tag{4.2}$$

Using (4.1) and (4.2) gives

$$\Delta_D y_0(t) = \frac{\{x(t) - x(t-\tau)\}}{x(t-\tau)} y_0. \tag{4.3}$$

Case 2. Noised measurement and perfect control

Suppose that one can measure only the acceleration and the horizontal motion $x(t)$ is obtained by a double integration. Thus, there is some error or noise $\varepsilon_N(t)$ in the data of $x(t)$. So, instead of the exact $x(t)$ one has a noised input $z(t)$ for the control procedure,

$$z(t) = x(t) + \varepsilon_N(t). \tag{4.4}$$

At the time instant t, according to the input $z(t)$ and the control law (3.1), the control distance $u_N(t)$ will produce angle $\theta_N(t)$ such that one has

$$z(t)\tan\theta_N(t) = y_0. \tag{4.5}$$

But the exact position at t is $x(t)$. So, the control $u_N(t)$ will cause a deviation from the point B_0 denoted as $\Delta_N y_0$ and one has

$$x(t)\tan\theta_N(t) = y_0 + \Delta_N y_0. \tag{4.6}$$

Using (4.5) and (4.6) yields

$$\Delta_N y_0(t) = \frac{-\varepsilon_N(t)}{x(t) + \varepsilon_N(t)} y_0. \tag{4.7}$$

Case 3. Noised measurement and time-delay control

The present case considers some error or noise $\varepsilon_N(t)$ in the data of $x(t)$ and together a time-delay control. The total deviation from the point B_0 is due to both the noise and the time delay. At time t, according to the input $z(t)$ and the control law (3.1), the control distance $u_N(t)$ will produce angle $\theta_N(t)$ satisfying condition (4.5). But due to the time delay in the control procedure, instead of $\theta_N(t)$ one can only produce $\theta_N(t-\tau)$. On the other hand, at time t, the exact position is $x(t)$. Thus, with the exact position $x(t)$ and the controlled angle $\theta_N(t-\tau)$ the bar OO_1 will go into the point B_e with the notation $RB_e = y_e(t)$. One has

$$y_e = x(t)\tan\theta_N(t-\tau). \tag{4.8}$$

Entering $t-\tau$ instead of t in (4.5) yields

$$y_0 = z(t-\tau)\tan\theta_N(t-\tau). \tag{4.9}$$

Denoting the deviation of the point B_e and B_0 by $\Delta y_0 = y_e - y_0$ and using (4.8), (4.9) one gets

$$\begin{aligned}\Delta y_0(t) &= \frac{x(t) - z(t-\tau)}{z(t-\tau)} y_0 = \frac{x(t) - x(t-\tau)}{x(t-\tau)} y_0 \\ &+ \frac{x(t) - x(t-\tau)}{x(t-\tau)} y_0 \left\{ \frac{x(t-\tau) - z(t-\tau)}{z(t-\tau)} \right\} + \frac{x(t-\tau) - z(t-\tau)}{z(t-\tau)} y_0.\end{aligned}$$

Alternatively, using (4.3) and (4.7) yields

$$\Delta y_0(t) = \Delta_D y_0(t) + \Delta_N y_0(t-\tau) + \frac{1}{y_0}\Delta_D y_0(t)\Delta_N y_0(t-\tau). \tag{4.10}$$

It is seen from (4.10) that in the case of noised measurement and time-delay control the total deviation is not a simple sum of deviation (4.3) and (4.7) but there are interactions between them.

5. Numerical simulations

For the numerical simulation the following data are assigned:

$$OQ = 60\,cm \quad PR = 300\,cm \qquad x(0) = 380\,cm \quad u(0) = 45\,cm$$
$$OP = 80\,cm \quad RB_0 = 241\,cm \qquad \cos\theta_0 = 0.83.$$

The horizontal motion is taken as

$$x(t) = 380 + 7.5(\sin 2t + 0.3\sin 3t + 0.1\sin 6t)\ (cm)$$

The measurement noise is proposed as

$$\varepsilon_m(t) = 0.75(0.1\sin 12t + 0.08\sin 18t).$$

The time history of the horizontal motion $x(t)$ is shown in the Fig. 2. Plots of $\Delta_D y_0(t)$ versus t are given in Fig. 3 for different values of time-delay τ. It is seen that the deviation from the point B_0 may be large when the time-delay increases. Plots of the deviation from y_0 due to noise $\Delta_N y_0(t)$ versus t are shown in Fig. 4 while Figure 5 presents the deviation for both cases of time-delay control and measurement noise considered together.

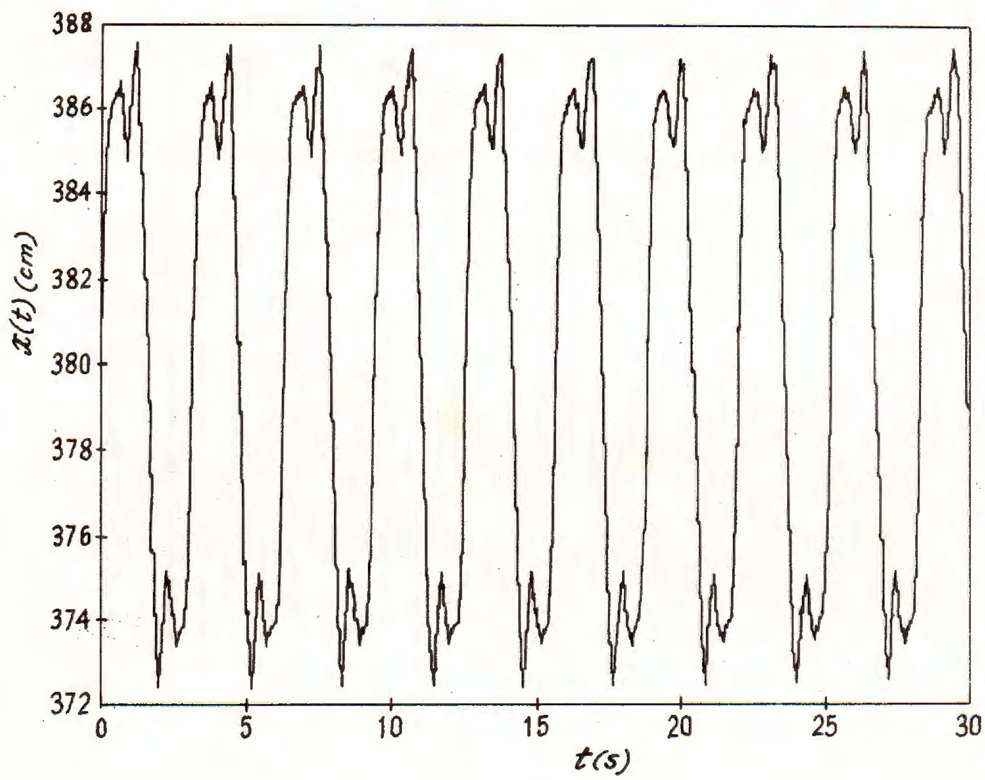

Fig. 2. Horizontal motion $x(t)$

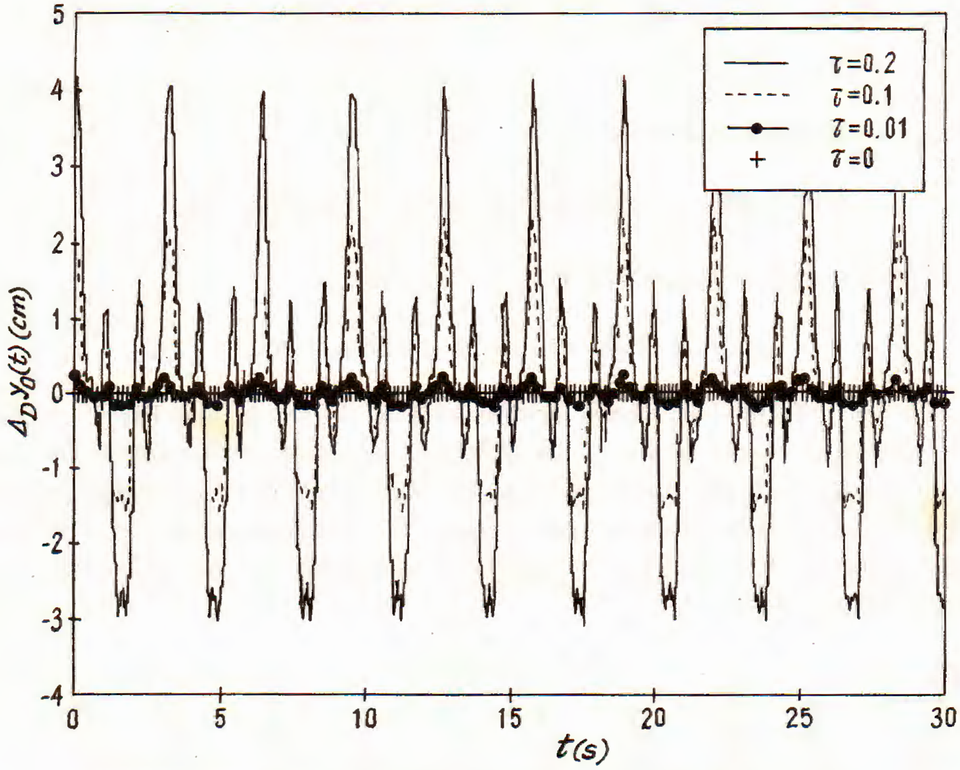

Fig. 3. Deviation from y_0 when there is a time-delay in the control action

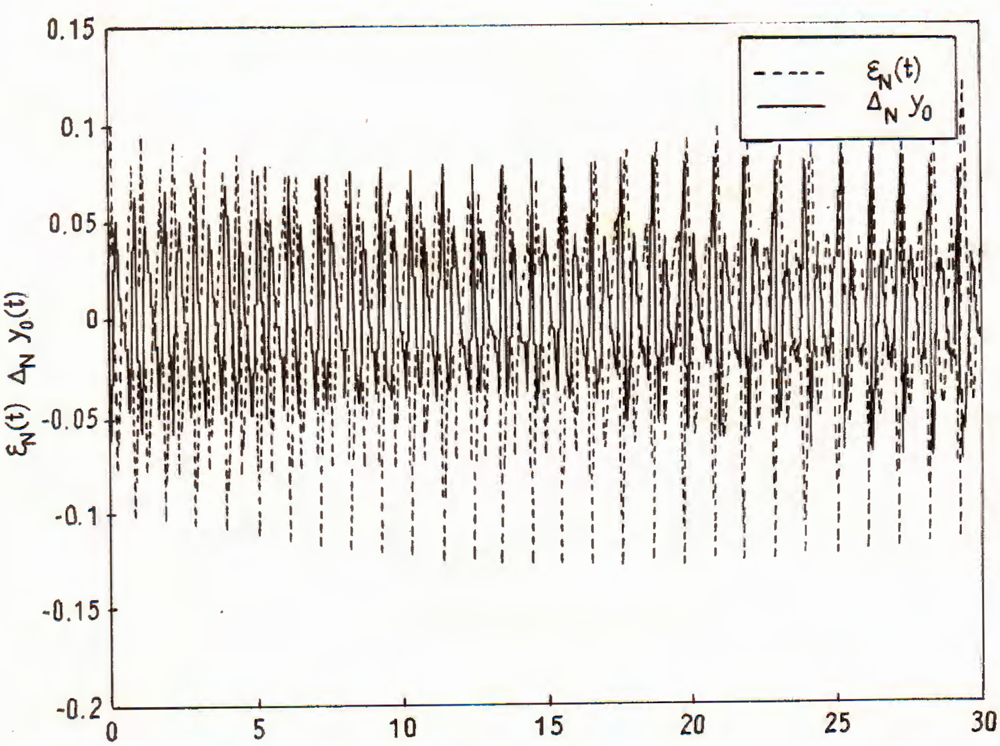

Fig. 4. Measurement noise and deviation from y_0 due to noise

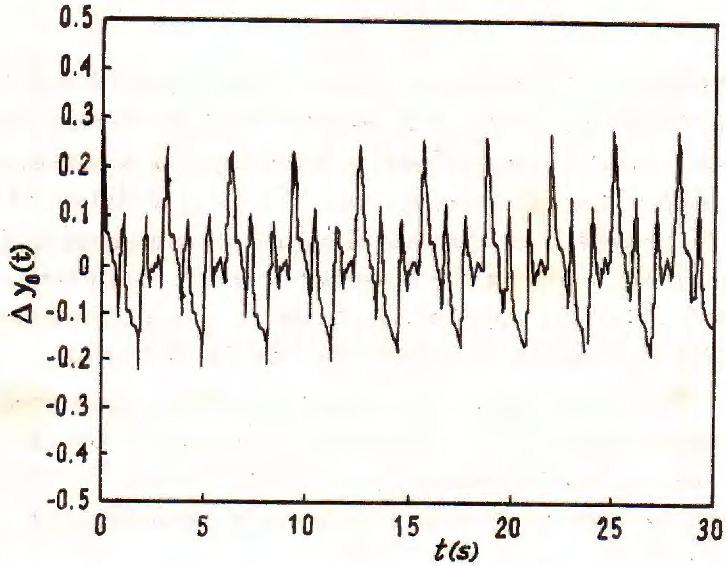

Fig. 5. Deviation from y_0 due to time-delay in control and measurement noise

6. Adopting a fuzzy controller

Previous sections show that if there is a time-delay τ in the control operation and the measurement noise $\varepsilon_N(t)$ is known, some relations between those values and the deviation $\Delta y_0(t)$ can be established. In practice, however, τ and $\varepsilon_N(t)$ are unknown and random in nature. In this context the fuzzy control theory is an alternative tool to treat uncertain information [Casciati et al. (1996), Casciati et al (1999)]. First of all, since we are interested in the relative motion of $x(t)$, so the point with zero coordinate is located now at the initial value x_0. The fuzzy universe of the relative horizontal motion, rx can be devised, for example, into 5 fuzzy subsets:

$M_1\{rx \mid rx \text{ is large positive}\}$
$M_2\{rx \mid rx \text{ is positive}\}$
$M_3\{rx \mid rx \text{ is zero}\}$
$M_4\{rx \mid rx \text{ is negative}\}$
$M_5\{rx \mid rx \text{ is large negative}\}$

Corresponding to 5 fuzzy subsets $M_i\{rx\}$, $i = 1, 2, 3, 4, 5$ one can define 5 fuzzy subsets of the fuzzy universe of the control action, ca:

$A_1\{ca \mid ca \text{ is large positive}\}$
$A_2\{ca \mid ca \text{ is positive}\}$
$A_3\{ca \mid ca \text{ is zero}\}$
$A_4\{ca \mid ca \text{ is negative}\}$
$A_5\{ca \mid ca \text{ is large negative}\}$

The membership functions of these fuzzy subsets are then selected. The next step is to find the fuzzy rules determining the control. For this purpose a control strategy may be adopted as follows: The relative motion is positive (negative) large, a positive (negative) large control action is required. The problem in Figure 1 is presently being tested. Therefore, the details of the rule table and the fuzzy controller are demanded to another paper.

7. Conclusions

This paper provides the mathematical model, which governs the motion of a pointer located on a vibrating system. It is obtained that in the case of exact measurement and perfect control the control law is determined as a non-linear function of the horizontal motion of the vibrating system. The total deviation of the pointer from the target is also presented for the case of noised measurement and time delay control. Further, emphasis is put on the uncertainty which characterizes the problem and a simple fuzzy controller approach is discussed. The optimal design of such a controler is presently in progress, supported by laboratory testing.

Acknowledgment. The paper reports on a research activity supported by a grant of ASI (the Italian space Agency), ASI-ARS-99.50. The stay of the third author at the University of Pavia was made possible by a CICOPS award for 1999-2000. The support of the council for natural science of Vietnam is acknowledged.

REFERENCES

1. Casciati F., Faravelli L. Fragility analysis of complex structural systems, Research Studies Press, Tauton, UK, 1991.

2. Casciati F, Faravelli L., Yao T. Control of nonlinear structures using the fuzzy control approach, Nonlinear Dynamics, **11** (1996), 171-187.

3. Casciati F., Maceri F., Singh M. P., Spanos P. (Eds.), Civil infrastructure systems: Intelligent Renewal, World Scientific, Singapore 1998.

4. Casciati F., Faravelli L., Torelli G. A fuzzy chip controller for nonlinear vibrations, Nonlinear Dynamics **20** (1999), 95-98.

5. Faravelli L., Yao T. Use of adaptive network in fuzzy control of civil structures, Microcomputers in Civil Engineering **11** (1) (1996), 67-76.

6. Kobori T., Inoue Y., Seto K., Iemura H., Nishitani A. (Eds.) Proccedings of the Second World Conference on Structural Control, Kyoto, Japan, John Wiley & Sons 1999.

Received July 30, 2000

ĐIỀU KHIỂN ĐẦU NGẮM NẰM TRÊN MỘT HỆ DAO ĐỘNG

Bài báo trình bày vấn đề điều khiển một dầm mang một đầu ngắm và được đỡ bằng một khớp quay trên một hệ dao động dưới kích động của ngoại lực. Vấn đề điều khiển được đặt ra, theo đó cần giữ cho đầu ngắm luôn luôn ngắm vào một mục tiêu cho trước với một độ chính xác nào đó. Các yếu tố ngẫu nhiên đặc trưng cho bài toán đã được phân tích kỹ và một bộ điều khiển mờ đơn giản đã được đề nghị ứng dụng. Các thí nghiệm kiểm tra đang được thực hiện nhằm đánh giá độ chính xác của vấn đề trên cơ sở một số tính toán mô phỏng số.