

NONLINEAR THERMO-MECHANICAL STABILITY OF SHEAR DEFORMABLE FGM SANDWICH SHALLOW SPHERICAL SHELLS WITH TANGENTIAL EDGE CONSTRAINTS

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Abstract. This paper presents an analytical approach to investigate the nonlinear axisymmetric response of moderately thick FGM sandwich shallow spherical shells resting on elastic foundations, exposed to thermal environments and subjected to uniform external pressure. Material properties are assumed to be temperature independent, and effective properties of FGM layer are graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. Formulations are based on first order shear deformation shell theory taking geometrical nonlinearity, initial geometrical imperfection, Pasternak type elastic foundations and varying degree of tangential constraint of boundary edge into consideration. Approximate solutions are assumed to satisfy clamped boundary condition and Galerkin method is applied to derive closed-form expressions of critical buckling loads and nonlinear load-deflection relation. Effects of geometrical parameters, thickness of face sheets, foundation stiffness, imperfection, thermal environments and degree of tangential edge constraints on the nonlinear stability of FGM sandwich shallow spherical shells are analyzed and discussed.

Keywords: Functionally graded material, shallow spherical shell, sandwich shell, nonlinear stability, tangential edge constraint.

1. INTRODUCTION

Structural elements in the form of spherical shells are widely used in many engineering structures. Since these shells are usually exposed to complex loading conditions, their static and dynamic responses are important problems and received considerable attention. Static and dynamic responses of isotropic spherical shells have been investigated in works [1–6] using analytical and semi-analytical approaches. Subsequently, axisymmetric buckling and post-buckling behavior of moderately thick shallow spherical shells

made of symmetrically laminated composites and subjected to uniform external pressure have been analyzed by Xu [7] utilizing a Fourier-Bessel series solution and by Nath and Sandeep [8] using an iterative Chebyshev series solution technique. Also, Muc [9] made use of trigonometric series method to consider unsymmetrical buckling and post-buckling behavior of laminated shallow spherical shells subjected to external pressure.

The appearance of functionally graded materials (FGMs) necessitates more studies on static and dynamic responses of shallow spherical shells made of these novel materials. Shahsiah and his co-workers [10, 11] used an analytical method on the basis of adjacent equilibrium criterion to investigate linear asymmetric buckling of simply supported FGM shallow and deep spherical shells under thermal loads. Based on an alternative approach, Bich and Tung [12] investigated the nonlinear axisymmetric response of clamped thin FGM shallow spherical shells under uniform external pressure taking the effects of pre-existent thermal loads and initial imperfection into consideration. They made use of the classical shell theory to establish equilibrium and compatibility equations in terms of deflection and stress functions and these equations are solved by applying Galerkin method. This work is then extended by Boroujerdy and Eslami [13] for piezo-FGM shallow spherical shells, and by Duc et al. [14] for FGM shallow spherical shells on elastic foundations. Basing on an analytical method, nonlinear unsymmetrical static and dynamic buckling behavior of FGM shallow spherical shells have been analyzed by Bich et al. [15]. Recently, Tung [16] used equilibrium equations in terms of displacements and rotation on the basis of the first order shear deformation shell theory to study the nonlinear thermo-mechanical response of FGM shallow spherical shells. Actually, foregoing works only considered two extreme cases of in-plane boundary conditions as boundary edge is assumed to be unrestrained (movable edge) or fully restrained (immovable edge). In practical situations, edges of plate and shell may be partially movable only, and tangential edge constraints have considerable and important effects on the nonlinear stability and load carrying capacity of plate and shell structures [17]. Recently, Tung [18, 19] considered the effects of tangential edge constraints on the post-buckling of thin FGM cylindrical panels and shear deformable FGM sandwich plates under thermal and mechanical loads. More recently, Tung analyzed separate and simultaneous influences of elastic foundations and tangential constraints of edges on the nonlinear stability of FGM shallow spherical shells [20]. Some studies relating to stability of FGM plate and cylindrical shells on elastic foundations, buckling of FGM conical panel and postbuckling of FGM shallow spherical shell panel under nonuniform thermal environment have been addressed in works [21–24] basing on some approaches.

Motivated by previous works [18–20] and from lack of results for FGM sandwich spherical shells, this paper presents an analytical approach to investigate the nonlinear axisymmetric response of shear deformable FGM sandwich shallow spherical shells resting on elastic foundations, exposed to thermal environments and mechanically loaded by uniform external pressure. Approximate solutions are assumed to satisfy clamped boundary condition and Galerkin method is applied to obtain closed-form expressions of critical buckling loads and load-deflection relation. Numerical illustrations show the

effects of geometrical parameters, thickness of face sheets, foundation stiffness, geometrical imperfection, thermal environments and degree of tangential edge constraint on the nonlinear stability of FGM sandwich shallow spherical shells.

2. FGM SANDWICH SHALLOW SPHERICAL SHELL (FGM SSSS) ON AN ELASTIC FOUNDATION

Consider a Sandwich Shallow Spherical Shell (SSSS) of radius of curvature R , base radius a , uniform total thickness h and rise of shell H . The shell is clamped at boundary edge, rested on a Pasternak elastic foundation and is defined in a coordinate system (φ, θ, z) whose origin is located on the middle surface of the shell, φ and θ are in the meridional and circumferential directions, respectively, and z is perpendicular to the middle surface and points inwards $(-h/2 \leq z \leq h/2)$ as shown in Fig. 1.

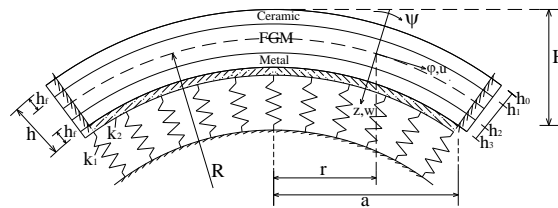


Fig. 1. Configuration and coordinate system of a sandwich shallow spherical shell (SSSS) on an elastic foundation

The SSSS is constructed from two isotropic face sheets (i.e. skins) separated by a thicker core layer made of FGMs. It is assumed that core layer and face sheets are perfectly bonded and the thickness of each face sheet is h_f . The top skin is isotropic homogeneous (fully ceramic) and the bottom skin is also isotropic homogeneous (fully metal), whereas FGM core layer is graded from a ceramic-rich interface to a metal-rich interface. This type of FGM SSSS is asymmetric about the mid-plane and the volume fraction of metal phase is expressed as

$$\begin{aligned}
 V_m(z) &= 0, z \in [h_0, h_1], \text{ top skin} \\
 V_m(z) &= \left(\frac{z - h_1}{h_2 - h_1} \right)^n, z \in [h_1, h_2], \text{ core layer} \\
 V_m(z) &= 1, z \in [h_2, h_3], \text{ bottom skin}
 \end{aligned} \tag{1}$$

Herein $h_0 = -\frac{h}{2}$, $h_1 = -\frac{h}{2} + h_f$, $h_2 = \frac{h}{2} - h_f$, $h_3 = \frac{h}{2}$ and $n \geq 0$ is volume fraction index that defines the gradation of material properties across the thickness direction. Moreover, the volume fraction of the ceramic constituent is given as $V_c(z) = 1 - V_m(z)$.

Effective properties of FGM SSSS such as modulus of elasticity E and coefficient of thermal expansion α can be determined by the linear rule of mixture as

$$\begin{aligned}
 E(z) &= E_c + E_{mc} V_m(z), \\
 \alpha(z) &= \alpha_c + \alpha_{mc} V_m(z),
 \end{aligned} \tag{2}$$

where

$$E_{mc} = E_m - E_c, \quad \alpha_{mc} = \alpha_m - \alpha_c \quad (3)$$

and $E_m, \alpha_m (E_c, \alpha_c)$ are temperature independent properties of metal (ceramic) constituents. Poisson's ratio ν is assumed to be constant in the present study.

3. FORMULATIONS

In the present study, the first order shear deformation shell theory is used for formulation and the FGM SSSS is assumed to be under axisymmetric deformation. According to this theory, displacement components $\bar{u}, \bar{v}, \bar{w}$ in φ, θ, z directions, respectively, at a distance z from the middle surface are represented as [7]

$$\bar{u}(r, z) = u(r) + z\psi(r), \quad \bar{v}(r, z) = 0, \quad \bar{w}(r, z) = w(r), \quad (4)$$

in which $r = R \sin \varphi$, u is displacement in the meridional direction at the middle surface, w is the deflection of the shell, and ψ is the rotation of a normal to the middle surface. Due to shallowness of the shell, it is approximately assumed that $\cos \varphi = 1$, $Rd\varphi = dr$ and $R = a^2/(2H)$. The non-zero strain components of the shell are defined as

$$\varepsilon_r = \varepsilon_{r0} + z\chi_r, \quad \varepsilon_\theta = \varepsilon_{\theta0} + z\chi_\theta, \quad \varepsilon_{rz} = \psi + w_{,r}, \quad (5)$$

where a comma denotes differentiation with respect to the corresponding variable, i.e. $(\cdot)_{,r} = d(\cdot)/dr$, and the strains at the middle surface $\varepsilon_{r0}, \varepsilon_{\theta0}$ and curvatures χ_r, χ_θ are related to the displacements and rotation in the form

$$\varepsilon_{r0} = u_{,r} - w/R + w_r^2/2, \quad \varepsilon_{\theta0} = u/r - w/R, \quad \chi_r = \psi_{,r}, \quad \chi_\theta = \psi/r. \quad (6)$$

Based on Hooke's law, stress-strain relations for an FGM SSSS including temperature effects are

$$\begin{aligned} \sigma_r &= \frac{E(z)}{1-\nu^2} [\varepsilon_r + \nu\varepsilon_\theta - (1+\nu)\alpha(z)\Delta T], \\ \sigma_\theta &= \frac{E(z)}{1-\nu^2} [\varepsilon_\theta + \nu\varepsilon_r - (1+\nu)\alpha(z)\Delta T], \\ \sigma_{rz} &= \frac{E(z)}{2(1+\nu)} \varepsilon_{rz}, \end{aligned} \quad (7)$$

where ΔT denotes the change of environment temperature from thermal stress free initial state. The force and moment resultants are expressed in terms of the stress components through the thickness as

$$(N_r, N_\theta) = \int_{-h/2}^{h/2} (\sigma_r, \sigma_\theta) dz, \quad (M_r, M_\theta) = \int_{-h/2}^{h/2} (\sigma_r, \sigma_\theta) z dz, \quad Q_r = K_S \int_{-h/2}^{h/2} \sigma_{rz} dz, \quad (8)$$

where K_S is shear correction coefficient.

Introduction of Eqs. (5) and (7) into Eqs. (8), the force and moment resultants are rewritten in the form

$$\begin{aligned}
 [N_r, M_r] &= \frac{[E_1, E_2]}{1-\nu^2} (\varepsilon_{r0} + \nu\varepsilon_{\theta0}) + \frac{[E_2, E_3]}{1-\nu^2} (\chi_r + \nu\chi_\theta) - \frac{[\Phi_0, \Phi_1]}{1-\nu}, \\
 [N_\theta, M_\theta] &= \frac{[E_1, E_2]}{1-\nu^2} (\varepsilon_{\theta0} + \nu\varepsilon_{r0}) + \frac{[E_2, E_3]}{1-\nu^2} (\chi_\theta + \nu\chi_r) - \frac{[\Phi_0, \Phi_1]}{1-\nu}, \\
 Q_r &= \frac{K_S E_1}{2(1+\nu)} (\psi + w_{,r}),
 \end{aligned}
 \tag{9}$$

where

$$[\Phi_0, \Phi_1] = \int_{-h/2}^{h/2} [1, z] E(z)\alpha(z)\Delta T dz,
 \tag{10}$$

and

$$\begin{aligned}
 E_1 &= E_c h + E_{mc} \left(h_f + \frac{h-2h_f}{n+1} \right), \\
 E_2 &= \frac{E_{mc}}{2} \left[hh_f - h_f^2 + \frac{n}{(n+1)(n+2)} (h-2h_f)^2 \right], \\
 E_3 &= \frac{E_c h^3}{12} + E_{mc} \left[\frac{1}{4} h^2 h_f - \frac{1}{2} h h_f^2 + \frac{1}{3} h_f^3 + \frac{(h-2h_f)^3}{n+1} \left(\frac{1}{4} - \frac{1}{n+2} + \frac{2}{(n+2)(n+3)} \right) \right].
 \end{aligned}
 \tag{11}$$

The nonlinear equilibrium equations of the geometrically imperfect FGM SSSS resting on elastic foundations are [20]

$$\begin{aligned}
 D \left(r\psi_{,rrr} + 2\psi_{,rr} - \frac{\psi_{,r}}{r} + \frac{\psi}{r^2} \right) + \frac{r}{R} \Delta_s f + f_{,rr} (w_{,r} + w_{,r}^*) \\
 + f_{,r} (w_{,rr} + w_{,rr}^*) + r (q - q_f) = 0,
 \end{aligned}
 \tag{12a}$$

$$D \left(r\psi_{,rr} + \psi_{,r} - \frac{\psi}{r} \right) - \frac{K_S E_1}{2(1+\nu)} r (\psi + w_{,r}) = 0.
 \tag{12b}$$

The strain compatibility equation of the geometrically imperfect FGM SSSS is [20]

$$\frac{1}{E_1} \Delta_s^2 f = -\frac{1}{R} \Delta_s w - \frac{1}{r} w_{,r} w_{,rr} - \frac{1}{r} w_{,r} w_{,rr}^* - \frac{1}{r} w_{,r}^* w_{,rr}.
 \tag{13}$$

These equations (12a), (12b) and (13) are expressed in terms of variable functions including the deflection $w(r)$, rotation $\psi(r)$ and stress function $f(r)$ defined as

$$N_r = \frac{f_{,r}}{r}, \quad N_\theta = f_{,rr}.
 \tag{14}$$

In addition, D is flexural rigidity, Δ_s is Laplace operator in case of axisymmetry

$$D = \frac{E_1 E_3 - E_2^2}{E_1 (1-\nu^2)}, \quad \Delta_s = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}.
 \tag{15}$$

Furthermore, $w^*(r)$ is a known function representing initial geometrical imperfection, q is uniform external pressure and q_f is shell-foundation interaction represented by Pasternak model as

$$q_f = k_1 w - k_2 \Delta_s w, \quad (16)$$

in which k_1 is Winkler foundation modulus and k_2 is the shear layer foundation stiffness of Pasternak model.

The FGM SSSS is assumed to be clamped on the boundary edge and under axisymmetric deformation. The symmetry condition at the center $r = 0$ together out-of-plane and in-plane boundary conditions at $r = a$ are expressed in the form [4,7,8]

$$\begin{aligned} \psi = 0, N_r = f_{,r}/r \text{ is finite} & \quad \text{at } r = 0, \\ w = 0, \psi = 0, N_r = N_{r0} & \quad \text{at } r = a, \end{aligned} \quad (17)$$

where N_{r0} is fictitious compressive stress due to tangential edge restraint, and $N_{r0} = 0$ for movable clamped edge.

To satisfy boundary conditions (17), approximate mode shapes for the deflection and rotation are assumed as [1]

$$(w, w^*) = (W, \mu h) \frac{(a^2 - r^2)^2}{a^4}, \quad \psi = \Psi \frac{r(a^2 - r^2)}{a^3}, \quad (18)$$

where W is the amplitude of the deflection, and imperfection function w^* is assumed to be in the form of deflection for the sake of simplicity. Also, non-dimensional parameter μ represents imperfection size and Ψ is a coefficient to be determined.

Substitution of w and w^* from Eqs. (18) into Eq. (13) and integrating of the resulting equation, at which constants of integration are determined from finiteness condition at the center $r = 0$ and in-plane boundary condition $N_r(r = a) = N_{r0}$, yield the stress function as

$$f_{,r} = \frac{E_1}{Ra^4} \left(\frac{a^2 r^3}{2} - \frac{r^5}{6} - \frac{a^4 r}{3} \right) W - \frac{E_1}{a^8} \left(a^4 r^3 - \frac{2a^2 r^5}{3} + \frac{r^7}{6} - \frac{a^6 r}{2} \right) W (W + 2\mu h) + N_{r0} r. \quad (19)$$

Subsequently, introduction of Eqs. (18) into Eq. (12b) and applying Galerkin method for the resulting equation

$$\int_0^a L_1 \times \frac{r(a^2 - r^2)}{a^3} dr = 0, \quad (20a)$$

where L_1 is the resulting expression received after substituting w and ψ from Eqs. (18) into the left-hand side of Eq. (12b). Implementing integration in the Eq. (20a) gives the coefficient Ψ as

$$\Psi = \frac{4K_S E_1 a}{32(1 + \nu) D + K_S E_1 a^2} W. \quad (20b)$$

Now, introduction of Eqs. (18) and (19) into the equilibrium equation (12a) and applying Galerkin method for obtained equation

$$\int_0^a L_2 \times \frac{(a^2 - r^2)^2}{a^4} dr = 0, \quad (21a)$$

where L_2 is the resulting expression received after substituting w , w^* and ψ from Eqs. (18) into the left-hand side of Eq. (12a). Performing integration in the Eq. (21a) yields the following relation

$$\begin{aligned}
 q = & \left[\frac{64\bar{D}\bar{E}_1K_S}{R_a^2R_h^2(32(1+\nu)\bar{D} + \bar{E}_1K_S R_a^2R_h^2)} + \frac{3\bar{E}_1}{7R_h^2} + \frac{\bar{D}}{7R_a^4R_h^4} \left(\frac{16}{3}K_1 + 40K_2 \right) \right] \bar{W} \\
 & - \frac{976\bar{E}_1}{693R_a^2R_h^3} \bar{W} (\bar{W} + \mu) - \frac{409\bar{E}_1}{693R_a^2R_h^3} \bar{W} (\bar{W} + 2\mu) \\
 & + \frac{848\bar{E}_1}{429R_a^4R_h^4} \bar{W} (\bar{W} + \mu) (\bar{W} + 2\mu) + \left[\frac{40}{7R_a^2R_h^2} (\bar{W} + \mu) - \frac{2}{R_h} \right] \bar{N}_{r0},
 \end{aligned} \tag{21b}$$

where

$$\begin{aligned}
 \bar{D} = D/h^3, \quad \bar{E}_1 = E_1/h, \quad \bar{E}_2 = E_2/h^2, \quad \bar{W} = W/h, \quad \bar{N}_{r0} = N_{r0}/h, \\
 R_a = a/R, \quad R_h = R/h, \quad K_1 = k_1a^4/D, \quad K_2 = k_2a^2/D.
 \end{aligned} \tag{22}$$

If the transverse shear deformation and foundation interaction are neglected, the expression (21b) leads to result which is similar to relation obtained in the work [12] for thin FGM shallow spherical shell without elastic foundation.

In the present study, the boundary edge of the shell is assumed to be clamped and with varying degrees of tangential restraint. The average end-shortening displacement Δ_r at the boundary edge $r = a$ is related to the corresponding compressive edge load N_{r0} by

$$\Delta_r c = N_{r0}, \tag{23}$$

where c is the average tangential stiffness in the meridional direction.

Eq. (23) indicates that value of $\Delta_r = 0$, enforced by selecting $c \rightarrow \infty$, corresponds to immovable edge, whereas condition of movable edge corresponds to $c = 0$ and $N_{r0} = 0$. Also, intermediate degrees of in-plane edge constraint correspond to values of c such that $0 < c < \infty$. The expression for the average end-shortening displacement is given by

$$\Delta_r = -\frac{1}{2\pi a^2} \int_0^{2\pi} \int_0^a u_{,r} r dr d\theta. \tag{24}$$

By virtue of Eqs. (6), (9) and (14), one can obtain the following relation for imperfect FGM SSSS

$$u_{,r} = \frac{1}{E_1} \left(\frac{f_{,r}}{r} - \nu f_{,rr} \right) - \frac{E_2}{E_1} \psi_{,r} - \frac{1}{2} w_{,r}^2 - w_{,r} w_r^* + \frac{w}{R} + \frac{\Phi_0}{E_1}. \tag{25}$$

Introduction of Eqs. (18), (19) into Eq. (25) and then substituting the resulting expression into Eq. (24) yield the following relation of the average end-shortening displacement

$$\begin{aligned}
 \Delta_r = & \left[\frac{5\nu - 7}{72R} - \frac{K_S E_2}{32(1+\nu)D + K_S E_1 a^2} \right] W \\
 & + \frac{35 - 13\nu}{144a^2} W (W + 2\mu h) - \frac{(1-\nu)}{2E_1} N_{r0} - \frac{\Phi_0}{2E_1}.
 \end{aligned} \tag{26}$$

In this study, thermal load is considered for case of uniform temperature rise and temperature change ΔT is independent of thickness variable. From Eq. (10), thermal parameter Φ_0 can be expressed as

$$\Phi_0 = P\Delta Th, \quad (27)$$

where

$$P = E_c\alpha_c + (E_c\alpha_{mc} + E_{mc}\alpha_c) \left(\frac{1 - 2\bar{h}_f}{n + 1} + \bar{h}_f \right) + E_{mc}\alpha_{mc} \left(\frac{1 - 2\bar{h}_f}{2n + 1} + \bar{h}_f \right). \quad (28)$$

Herein, $\bar{h}_f = h_f/h$ is face sheet thickness to total thickness ratio.

The combination of Eqs. (23), (26) and (27) gives the fictitious compressive force resultant on tangentially restrained edge \bar{N}_{r0} . Next, substituting the expression of \bar{N}_{r0} into Eq. (21b) leads to the following explicit expression of nonlinear load-deflection relation

$$q = A_1\bar{W} - A_2\bar{W}(\bar{W} + \mu) - A_3\bar{W}(\bar{W} + 2\mu) + A_4\bar{W}(\bar{W} + \mu)(\bar{W} + 2\mu) + \left[\frac{2}{R_h} - \frac{40}{7R_a^2R_h^2}(\bar{W} + \mu) \right] eP\Delta T, \quad (29)$$

where

$$\begin{aligned} A_1 &= \frac{64K_S\bar{D}\bar{E}_1}{R_a^2R_h^2(32(1+\nu)\bar{D} + K_S\bar{E}_1R_a^2R_h^2)} + \frac{4eK_S\bar{E}_1\bar{E}_2}{32(1+\nu)\bar{D}R_h + K_S\bar{E}_1R_a^2R_h^3} \\ &\quad + \frac{3\bar{E}_1}{7R_h^2} + \frac{(7-5\nu)e\bar{E}_1}{18R_h^2} + \frac{\bar{D}}{7R_a^4R_h^4} \left(\frac{16}{3}K_1 + 40K_2 \right), \\ A_2 &= \frac{976\bar{E}_1}{693R_a^2R_h^3} + \frac{10(7-5\nu)e\bar{E}_1}{63R_a^2R_h^3} + \frac{80K_S e\bar{E}_1\bar{E}_2}{7R_a^2R_h^2(32(1+\nu)\bar{D} + K_S\bar{E}_1R_a^2R_h^2)}, \\ A_3 &= \frac{409\bar{E}_1}{693R_a^2R_h^3} + \frac{(35-13\nu)e\bar{E}_1}{36R_a^2R_h^3}, \quad A_4 = \frac{848\bar{E}_1}{429R_a^4R_h^4} + \frac{5(35-13\nu)e\bar{E}_1}{63R_a^4R_h^4}, \\ e &= \frac{\bar{c}}{2\bar{E}_1 + \bar{c}(1-\nu)}, \quad \bar{c} = c/h. \end{aligned} \quad (30)$$

Specialization of Eq. (29) for case of $\Delta T = 0$ gives pressure-deflection relation in the form

$$q = B_1\bar{W} - B_2\bar{W}^2 + B_3\bar{W}^3, \quad (31)$$

where

$$B_1 = A_1 - \mu A_2 - 2\mu A_3 + 2\mu^2 A_4, \quad B_2 = A_2 + A_3 - 3\mu A_4, \quad B_3 = A_4. \quad (32)$$

From Eq. (31), values of deflection at extremum points on the pressure-deflection curves may be determined from condition $dq/d\bar{W} = 0$ yielding

$$\bar{W}_{1,2} = \frac{B_2 \mp \sqrt{B_2^2 - 3B_1B_3}}{3B_3}, \quad (33)$$

provided

$$B_2^2 - 3B_1B_3 \geq 0. \quad (34)$$

In case of material and geometrical properties of FGM SSSS, foundation stiffness and degree of in-plane edge constraint make the condition (34) satisfied, the shell experiences an extreme-type buckling behavior which upper and lower buckling pressures may be predicted as $q_{cr} = q(\bar{W}_1)$ and $q(\bar{W}_2)$, respectively. In contrast, extremum-type buckling will not occur and pressure-deflection paths are monotonically developed as $B_2^2 - 3B_1B_3 < 0$.

Furthermore, Eq. (29) indicates that pressure-loaded FGM SSSS in thermal environment can exhibit a bifurcation-type buckling behavior and corresponding bifurcation point pressure is predicted as

$$q_b^{\Delta T} = q(\bar{W} = 0) = \frac{2}{7R_a^2R_h^2} (7R_a^2R_h - 20\mu) eP\Delta T, \quad (35)$$

which is independent of stiffness of elastic foundation, as ΔT and tangential stiffness c are not zero-valued. It is interesting from Eq. (29) that for all values of temperature change ΔT , there is existence of an intersection point between load-deflection curves predicted as $(\bar{W}^* = 7R_a^2R_h/20 - \mu, q(\bar{W}^*))$.

4. RESULTS AND DISCUSSION

There are no results in the literature about the subject of the present study for direct comparison. Therefore, the nonlinear axisymmetric response of an isotropic homogeneous shallow spherical shell resting on an elastic foundation and subjected to uniform external pressure is considered as part of verification of the present approach.

Load-deflection curve of a geometrically perfect, immovable clamped shallow spherical shell is depicted in Fig. 2 in comparison with results reported by Civalek [6] using discrete singular convolution and differential quadrature methods. As can be seen, a good agreement is achieved in this comparison.

To illustrate the proposed approach, this section considers FGM SSSSs composed of aluminum (Al) and alumina (Al_2O_3) with the following properties [12]

$$\begin{aligned} E_m &= 70 \text{ GPa}, & \alpha_m &= 23 \times 10^{-6} \text{ }^\circ\text{C}^{-1}, \\ E_c &= 380 \text{ GPa}, & \alpha_c &= 7.4 \times 10^{-6} \text{ }^\circ\text{C}^{-1}, \end{aligned} \quad (36)$$

whereas Poisson's ratio is chosen to be $\nu = 0.3$ for both materials. The shear correction coefficient is assumed to be $K_S = 5/6$ in calculation. Moreover, to measure the degree of edge restraint in a convenient way, an alternate tangential stiffness parameter β is introduced such that $\beta = 0$ and $\beta = 1$ correspond to movable and immovable boundary edge $r = a$, respectively, and partially restrained edge is defined by $0 < \beta < 1$. In the present study, this alternate tangential stiffness parameter is given by

$$\beta = \frac{\bar{c}}{E_1 + \bar{c}}. \quad (37)$$

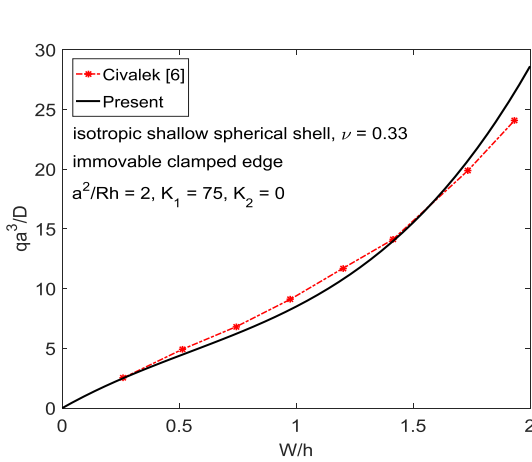


Fig. 2. Comparison of load-deflection response of an isotropic homogeneous shallow spherical shell

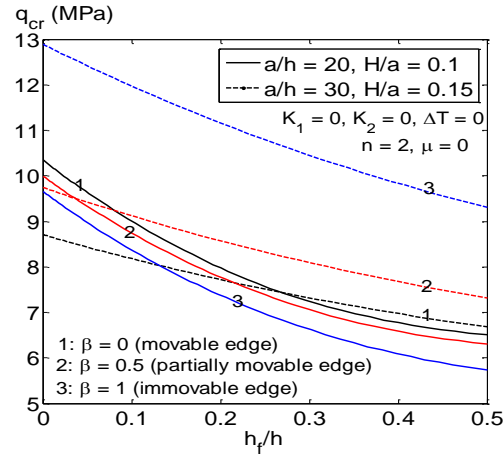


Fig. 3. Effects of h_f/h ratio and tangential edge constraints on the critical buckling pressure of FGM SSSSs

In characterizing the behavior of the FGM SSSSs, deformations in which the central region of a shell moves toward the center of respective sphere are referred to as (positive) inward deflections, whereas deformations in the opposite direction referred to as (negative) outward deflection. Since effects of volume fraction index have been mentioned in many studies, this section only considers FGM SSSSs with index $n = 2$.

Fig. 3 considers the effects of h_f/h ratio and β parameter on the critical pressures for (Al₂O₃/FGM/Al) FGM SSSSs under uniform external pressure only. It is evident that the critical buckling pressures are reduced as h_f/h ratio to be higher. In addition, the critical loads are enhanced as a/h and H/a ratios are increased and boundary edge is more rigorously restrained in tangential motion, i.e. higher values of β parameter.

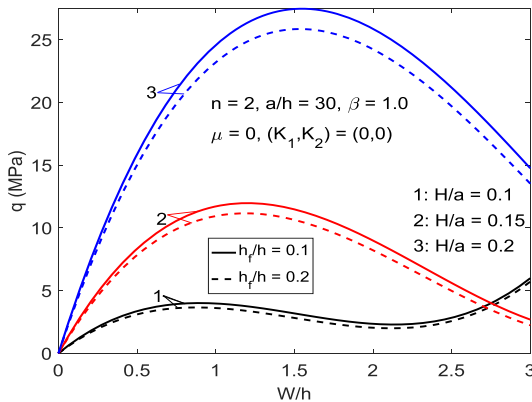


Fig. 4. Effects of h_f/h and H/a ratios on the nonlinear response of FGM SSSSs under uniform external pressure

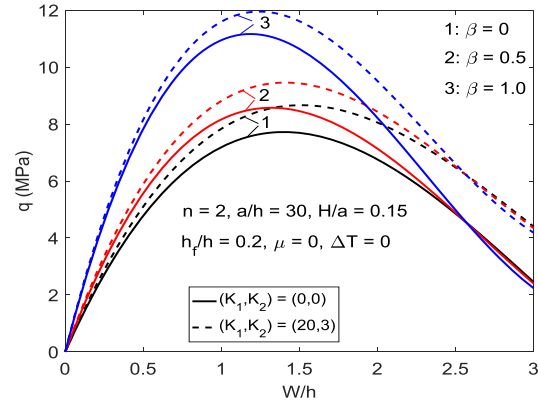


Fig. 5. Effects of tangential edge constraints (β) and elastic foundation on the nonlinear response of FGM SSSSs

The effects of h_f/h and H/a ratios on the nonlinear response of immovable clamped FGM SSSSs under uniform external pressure without elastic foundations and thermal environment are analyzed in Fig. 4. As can be seen, extreme type buckling pressures and severity of snap-through phenomenon are both increased as H/a ratio become larger (i.e. deeper shell). Furthermore, the pressure-deflection curves become lower with increase in h_f/h ratio. Next, Fig. 5 considers the effects of varying degree of tangential edge constraints (i.e. β parameter) and elastic foundation on the nonlinear response of FGM SSSSs under uniform external pressure only. Obviously, the increase in value of β , representing more rigorous constraint of in-plane motion of boundary edge, results in higher buckling loads and more intense snap-through response. In addition, pressure-deflection curves are enhanced and snap-through instability become more benign due to the presence of elastic foundation. Fig. 6 indicates that initial geometrical imperfection has significant influences on the nonlinear response of FGM SSSSs subjected to external pressure. It seems that the effective curvature of the spherical shell is changed due to the presence of initial geometrical imperfection and, as a result, the load carrying capacity of FGM SSSSs is reduced as μ parameter is increased from -0.1 to 0.1 .

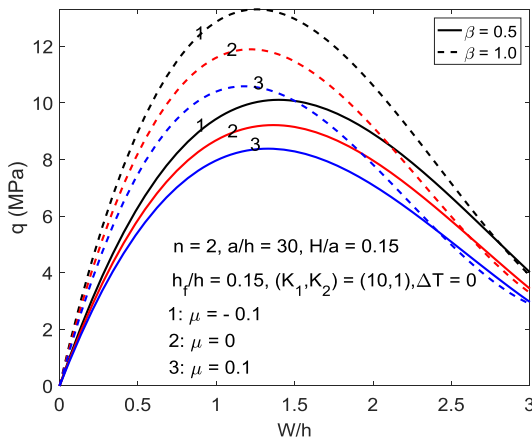


Fig. 6. Effects of tangential edge constraints (β) and imperfection (μ) on the nonlinear response of FGM SSSSs

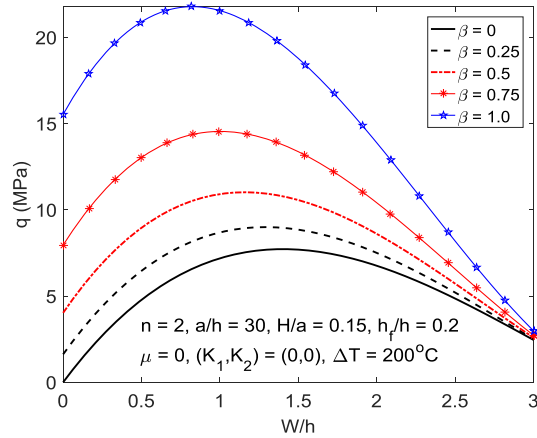


Fig. 7. Effects of tangential edge constraints and thermal environment on nonlinear thermo-mechanical response of FGM SSSSs

The effects of degree of tangential edge constraint and thermal environment on the nonlinear thermo-mechanical response of FGM SSSSs are analyzed in Fig. 7. It is evident that, externally pressurized FGM SSSSs exposed to a thermal environment will exhibit a bifurcation type buckling behavior as boundary edge is tangentially restrained. Moreover, bifurcation point pressure is enhanced by increase in degree of tangential edge constraint. This may be explained that the spherical shell is deflected outward (negative deflection) and, as a result, the shell is more curved due to pre-existent thermal stress, especially as boundary edge is rigorously restrained. Therefore, external pressure must reach a bifurcation point value at which the shell surface returns reference state prior deflecting inward as external pressure exceeds bifurcation point value.

Subsequently, the effects of thermal environments on the nonlinear thermo-mechanical response of FGM SSSSs on Pasternak foundations are analyzed in Fig. 8. This figure indicates that bifurcation point pressures are enhanced as β parameter and/or temperature change ΔT are increased. Furthermore, difference between pressure-deflection curves in the two cases of $\beta = 0.5$ and $\beta = 1.0$ is substantially bigger as FGM SSSSs are exposed to higher temperature fields.

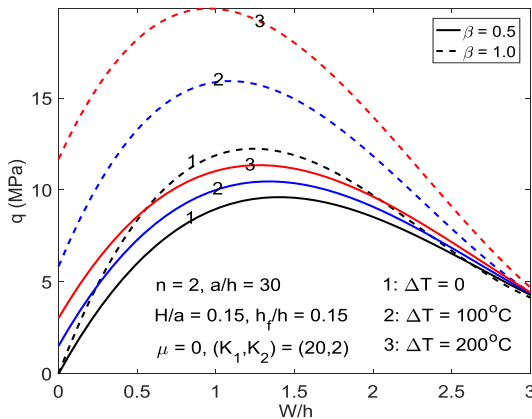


Fig. 8. Effects of tangential stiffness β and thermal load on nonlinear thermo-mechanical response of FGM SSSSs

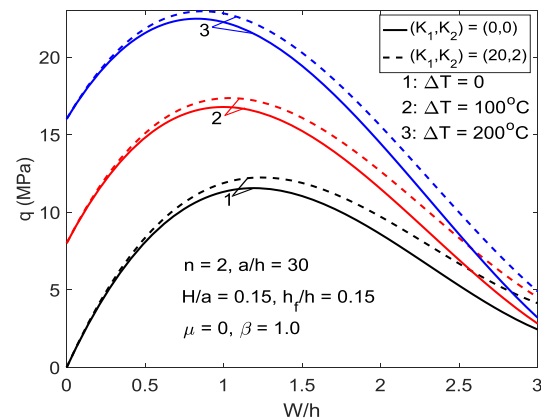


Fig. 9. Effects of elastic foundations and thermal environments on nonlinear thermo-mechanical response of FGM SSSSs

Finally, Fig. 9 examines the effects of elastic foundations on the thermo-mechanical response of the FGM SSSSs with different values of environment temperature. It is evident that load-deflection curves are enhanced and snap-through phenomenon becomes more benign, i.e. postbuckling response is more stable, when the FGM SSSSs are supported by Pasternak elastic foundations

5. CONCLUDING REMARKS

Snap-through instability and the nonlinear response of pressure-loaded FGM SSSSs in thermal environments have been investigated. Formulations are based on the first order shear deformation shell theory and closed-form expressions are derived by applying Galerkin method. The study reveals that thickness of face sheets, geometrical parameters and degree of tangential edge constraint have sensitive effects on the behavior and load carrying capacity of the FGM SSSSs. The shell has higher critical buckling load and more severe snap-through intensity as boundary edge is more rigorously restrained in tangential motion. The results also indicate that elastic foundations have beneficial influences on the load carrying capacity (i.e. higher load-deflection curves) and stabilization (i.e. more benign snap-through response) of the FGM SSSSs. Furthermore, initial geometrical imperfection changes the curvature of spherical shell and has significant effects on the nonlinear stability of the shell. As a final remark, buckling pattern of pressure-loaded FGM SSSS is changed from extremum type to bifurcation type as environment

temperature is elevated and bifurcation point pressure is enhanced with increasing in temperature change ΔT and/or degree of tangential edge constraint β .

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