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ON SOME NUMERICAL METHODS FOR SOLVING THE 1-D SAINT-VENANT EQUATIONS OF GENERAL FLOW REGIME
Part 1: Numerical methods

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ABSTRACT. Development of methods for numerical simulation of dike- or dam-break flood is one of essential problems of Fluid Mechanics at the present time. Many numerical methods for solving the 1-D Saint-Venant equations have been proposed. However, the analysis, the evaluation and the selection of appropriate and efficient methods are interest of many research groups and institutions in the world.

The purpose of this paper is to introduce and to evaluate four numerical methods for solving the 1-D homogenous Saint-Venant equations in combination with three approaches of processing source terms. The evaluation is based on the test problems, proposed by European Hydraulic Research Laboratories.

The Part 1 of the paper presents some modern numerical methods for solving the 1-D Saint-Venant equations of general flow regime, where the flow may be mixed between subcritical and supercritical. The homogenous part of the system of equations is numerically solved by "shock capturing methods" for conservation laws: the Lax-Friedrichs, the Self adjusting hybrid, the Roe's approximation and the Nessyahu-Tedmor methods. The source terms play an important role and are discretized by the pointwise, upwind or mixed approaches. In the second part of this paper the above methods are verified by a set of test problems, covering all of three flow regimes: subcritical, supercritical, transcritical. The results show that the mixed approach of processing source terms is better than the pointwise one. The Roe approximation method with the mixed discretization of source terms is then applied for a preliminary evaluation of the Son La - Hoa Binh dam-break problem.

1. Introduction

Free surface flow in a channel system can be simulated by solving the one- and two- dimensional Saint-Venant equations. Computational models of river flows, based on these equations, are well-established tools in engineering practice.

Many finite difference methods, including Preissman scheme, have been developed to solve Saint-Venant equations for subcritical flows. However, these schemes do not perform well in the presence of flow discontinuities. In such cases, one can use either empirical formula at the part of the river where super-critical flows occur, or impose large amounts of artificial viscosity to dam the resulting numerical oscillations. The inclusion of non-physical artificial viscosity significantly reduces the accuracy of the solution in areas with high velocity gradients. In some cases, even very high artificial viscosity cannot stabilize the model. As much of the numerical stability theory is based on linear equations, implicit schemes, that are believed to be 'unconditionally' stable, may become unstable in transcritical flow conditions.

For these reasons, numerical methods, those are capable to simulate discontinuities such as hydraulic jumps and bores in channels, extensively developed in the present. Many techniques come from aerodynamics and are known as numerical methods for conservation laws.

In that context, this paper represents some numerical methods for solving the 1-D Saint-Venant equations of general flow regime, where the flow is mixed between sub- and super-critical. The homogenous part of the equations is solving by following efficient methods for conservations laws: the Lax-Friedrichs, the Self-adjusting hybrid, and the Roe's approximation of Riemann solver, the Nessyahu-Tedmor. The source terms, that play a very important role, are discretized by pointwise, upwind or mixed between these approaches. In part 2, these developed methods will be verified by various test problems, covering all of three flow regimes: sub-, trans-, and super-critical. The Roe's approximation of Riemann solver with the mixed approach of source terms will be used for a preliminary evaluation of the Son La - Hoa Binh dambreak problem.

2. Equations and numerical methods

2.1. The Saint-Venant Equations

Let us consider the Saint-Venant equations of the form [1]

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q, \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\Big(\frac{Q^2}{A} + gI_1\Big) = S_d \tag{2.1}$$

where

$$S_d = gA(S_0 - S_f) + gI_2 \tag{2.2}$$

and

t is time

x - space coordinate

A - wet cross sectional area

Q - discharge

g - gravity

$I_1 = \int_0^h (h-\eta)b(x,\eta)d\eta$; $I_2 = \int_0^h (h-\eta)\frac{\partial b(x,\eta)}{\partial x}$ - account for pressure forces

S_0 - bed slope

$S_f = \frac{|Q|Q}{K^2}$ - bed friction

K - conveyance

q - literal unit discharge.

By using the change rule of derivatives for $P(x, A) = gI_1(x, A)$ ans considering $A = A(x, z)$ as a function of space coordinate x and water level z, one gets another forms of source terms [1, 2]:

$$S_d = \frac{\partial P}{\partial x}\Big|_{A=const} + \frac{\partial P}{\partial A}\frac{\partial A}{\partial x}\Big|_{z=const} - gAS_f \tag{2.3}$$

$$S_d = gA(S_0 - S_f) + g\Big(\frac{\partial I_1}{\partial x} - A\frac{\partial h}{\partial x}\Big) \tag{2.4}$$

$$S_d = -gAS_f + g\Big(\frac{\partial I_1}{\partial x} - A\frac{\partial z}{\partial x}\Big) \tag{2.5}$$

where z is the water level.

In the vector form one can write:

$$\frac{\partial U}{\partial t} + \frac{\partial F(x, U)}{\partial x} = S(x, U) \tag{2.6}$$

where

$$U = U(x,t) = \begin{pmatrix} A \\ Q \end{pmatrix}, \quad F(x,U) = \begin{pmatrix} Q \\ \dfrac{Q^2}{A} + gI_1 \end{pmatrix}, \quad S(x,U) = \begin{pmatrix} q \\ S_d \end{pmatrix}.$$

2.2. Numerical methods for the Saint-Venant equations

Numerical methods for conservation laws are mainly developed for the homogenous equations, where the source terms are identical to zero. For the non-homogenous equations, like the Saint Venant equations, one uses numerical methods for conservation laws to solve the homogenous part, then uses the pointwise [3] or upwind approaches to include the source terms [4, 5].

2.2.1. Numerical methods for homogenous Saint-Venant equations

For all numerical methods $\lambda = \dfrac{\Delta t}{\Delta x}$ is the relation of the time increment to the space increment, CFL stands for the Courant-Friedrichs-Levy number and TVD stands for the total variation diminishing.

2.2.1.1. The Lax-Friedrichs Method

In this method the following formula is used:

$$U_i^{n+1} = \frac{U_{i-1}^n + U_{i+1}^n}{2} - \frac{1}{2}\lambda(F_{i+1}^n - F_{i-1}^n) \tag{2.7}$$

where Δt and Δx are time and space steps, $F_i^n = F(U_i^n)$ is the numerical flux. The scheme is conservative, consistent, L^∞-stable and TVD if it fulfills the CFL condition.

2.2.1.2. The Self-Adjusting Hybrid Method

The idea of this method was presented by Harten and Zwas in [6] and by Sod in

[7] by using a combination of a first and a higher order schemes:

$$U_i^{n+1} = \left(L_1 U^n\right)_i = U_i^n - \lambda\left(f_{i+\frac{1}{2}}^1 - f_{i-\frac{1}{2}}^1\right),$$
$$U_i^{n+1} = \left(L_k U^n\right)_i = U_i^n - \lambda\left(f_{i+\frac{1}{2}}^k - f_{i-\frac{1}{2}}^k\right).$$

So that it remains conservative property. Here L_1, L_k denote the first and the higher order schemes and $f_{i+\frac{1}{2}}^1$, $f_{i+\frac{1}{2}}^k$ are their numerical fluxes. The new scheme has the form:

$$U_i^{n+1} = (LU^n)_i = U_i - \lambda\left(f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}\right), \quad f_{i+\frac{1}{2}} = \theta_{i+\frac{1}{2}} f_{i+\frac{1}{2}}^1 + \left(1 - \theta_{i+\frac{1}{2}}\right) f_{i+\frac{1}{2}}^k$$

with $\theta_{i+\frac{1}{2}} \in [0,1]$. At discontinuities the automatic switch is such that $\theta \approx 1$. Hence at the discontinuities the hybrid scheme is essentially the non-oscillatory first-order scheme.

According to [7] this method has the following formula:

$$\overline{U_i^{n+1}} = U_i^n - \lambda(F_{i+1}^n - F_i^n)$$
$$U_i^{n+1} = \frac{1}{2}\left(\overline{U_i^{n+1}} - U_i^n\right) - \frac{\lambda}{2}\left(\overline{F_i^{n+1}} - \overline{F_{i-1}^{n+1}}\right) + \frac{1}{8}\left(\theta_{i+\frac{1}{2}}^n (U_{i+1}^n - U_i^n) - \theta_{i-\frac{1}{2}}^n (U_i^n - U_{i-1}^n)\right)$$
$$\theta_{i+\frac{1}{2}}^n = \max(\theta_i', \theta_{i+1}')$$
$$\theta_i' = \begin{cases} \left| \dfrac{|A_{i+1} - A_i| - |A_i - A_{i-1}|}{|_{i+1} - A_i| + |A_i - A_{i-1}|} \right|, & |A_{i+1} - A_i| + |A_i - A_{i-1}| > \varepsilon \\ 0 \quad , & |A_{i+1} - A_i| + |A_i - A_{i-1}| \le \varepsilon \end{cases}$$

where $F_i^n = F(x_i, U_i^n)$, $\overline{F_i^n} = F(x_i, \overline{U_i^n})$, A_i is the wet cross sectional area, ε is a small positive number which is a measure of negligible variation of the wet cross sectional area.

2.2.1.3. The Roe's Approximate Riemann Solvers

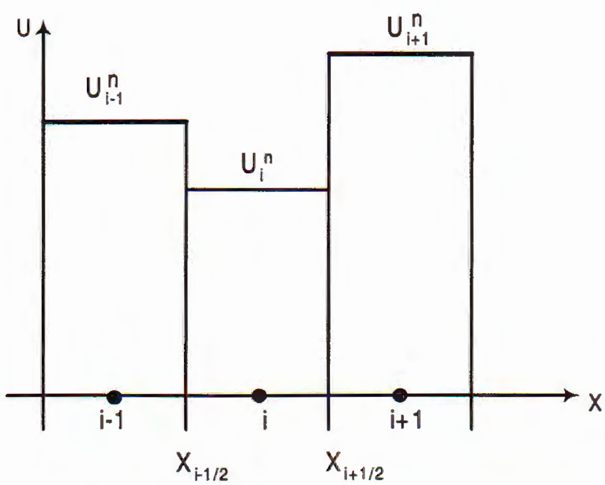

Fig. 1. Cell average for the Godunov method

Integrating equations (2.1) in control volume $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}] \times [t_n, t_{n+1}]$ (Fig. 1), it is follows

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x_i}\left(F_{i+\frac{1}{2}}^n - F_{i-\frac{1}{2}}^n\right) \tag{2.8}$$

where $\Delta x_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$ and

$$U_i^n = \frac{1}{\Delta x_i}\int\limits_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} U(x,t_n)dx, \quad U_i^{n+1} = \frac{1}{\Delta x_i}\int\limits_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} U(x,t_{n+1})dx$$

are cell average values at time t_n and t_{n+1}.

$$F_{i+\frac{1}{2}}^n = \frac{1}{\Delta t}\int\limits_{t_n}^{t_{n+1}} F\big(U(x_{i+\frac{1}{2}},t)\big)dt, \quad F_{i-\frac{1}{2}}^n = \frac{1}{\Delta t}\int\limits_{t_n}^{t_{n+1}} F\big(U(x_{i-\frac{1}{2}},t)\big)dt$$

are numerical fluxes.

Taking cell average values for $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$ to be the initial values at time t_n and solving a series of Riemann problems:

$$\frac{\partial W}{\partial t} + \frac{\partial F(W)}{\partial x} = 0$$

$$W(x,t=t_n) = \begin{cases} U_i^n, & x < 0 \\ U_{i+1}^n, & x \geq 0 \end{cases}$$

one can get solution value at the interface $U\big(x_{i+\frac{1}{2}},t\big) = W(0,t)$ and after that the cell average value at time t_{n+1}. That is the Godunov method. However, it is difficult to obtain the exact solution of the Riemann problem, especially in the case of nonlinearity, so we apply the Roe's approximation of the Riemann solver [8].

The Roe's flux function has the form:

$$F_{i+\frac{1}{2}} = F(U_i) + \sum_{\tilde{\lambda}_p \leq 0} \tilde{\lambda}_p \tilde{\alpha}_p \tilde{r}_p = F(U_{i+1}) - \sum_{\tilde{\lambda}_p \geq 0} \tilde{\lambda}_p \tilde{\alpha}_p \tilde{r}_p$$

or

$$F_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{1}{2}\sum |\tilde{\lambda}_p| \tilde{\alpha}_p \tilde{r}_p$$

where $\tilde{\lambda}_p$, $\tilde{r}_p$ are eigenvalues and their corresponding eigenvectors of the Roe matrix and $\tilde{\alpha}_p$ are wave strengths:

$$U_{i+1} - U_i = \sum \tilde{\alpha}_p \tilde{r}_p.$$

However, this method has a limitation that cannot recognize the reafaction wave at sonic points. To overcome this disadvantage it is nessasary to correct the numerical flux at points where the eigeinvalue $\tilde{\lambda}_p$ is near 0. There exist different ways. Here is one which is presented in [9]:

$$\tilde{\lambda}_p^m = \lambda_p(U_L)\Big(\frac{\tilde{\lambda}_p - \lambda_p(U_R)}{\lambda_p(U_L) - \lambda_p(U_R)}\Big).$$

For the Saint-Venant equations the Roe's Matrix is obtained according to Roe [3, 8] as done for the Euler equation.

It requires to select a vector variable $\overline{W}$ so that U and $F(U)$ are homogenous functions of the second order of $\overline{W}$. Here the vector variable takes the form:

$$\overline{W} = \begin{pmatrix} \sqrt{A} \\ u\sqrt{A} \end{pmatrix}, \quad u = \frac{Q}{A}$$

and one can derive the Roe's matrix:

$$\tilde{A}(U_L, U_R) = \begin{pmatrix} 0 & 1 \\ -\tilde{u}^2 + \tilde{c}^2 & 2\tilde{u} \end{pmatrix}$$

where

$$\tilde{u} = \frac{u_L\sqrt{A_L} + u_R\sqrt{A_R}}{\sqrt{A_L} + \sqrt{A_R}} \quad \text{and} \quad \tilde{c} = \sqrt{\frac{\partial P}{\partial A}} = \sqrt{\frac{P(x_{i+\frac{1}{2}}, A_R) - P(x_{i+\frac{1}{2}}, A_L)}{A_R - A_L}}.$$

The Roe's matrix has two eigenvalues:

$$\tilde{\lambda}_1 = \tilde{u} - \tilde{c}, \quad \tilde{\lambda}_2 = \tilde{u} + \tilde{c},$$

and two corresponding eigenvectors:

$$\tilde{r}_1 = \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} 1 \\ \tilde{u} - \tilde{c} \end{pmatrix}, \quad \tilde{r}_2 = \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \tilde{u} + \tilde{c} \end{pmatrix}$$

and wave strengths

$$\tilde{\alpha}_1 = \frac{\tilde{\lambda}_2 \Delta A - \Delta Q}{2\tilde{c}}, \quad \tilde{\alpha}_2 = \frac{-\tilde{\lambda}_1 \Delta A + \Delta Q}{2\tilde{c}}.$$

Then the numerical flux has the form:

$$F^n_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{1}{2}\sum |\tilde{\lambda}_p| \tilde{\alpha}_p \tilde{r}_p.$$

2.2.1.4. The Nessyahu-Tedmor Method

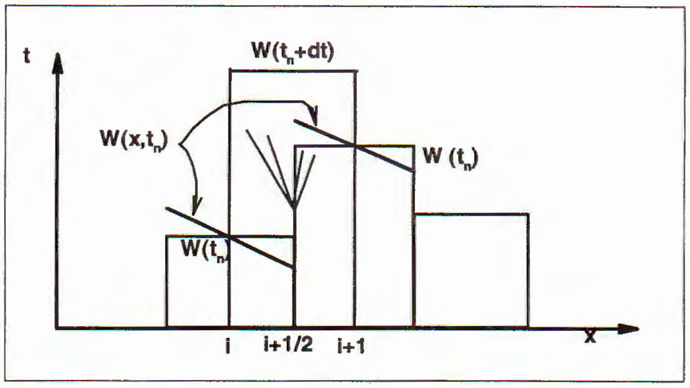

Fig. 2. Nessyahu-Tedmor integration

This method is presented in [10] with the stagged grid and is an extension of the Godunov idea. But, unlike the Godunov method, where the average takes by cell $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}] \times [t_n, t_{n+1}]$, this method uses the integral in the region $[x_i, x_{i+1}] \times [t_n, t_{n+1}]$. Let us consider the generalized Riemann problem on a stagged grid with equal space increment $\Delta x_i = \Delta x$ and the initial condition of piecewise linear functions:

$$U(x, t_n) = U_i^n + \frac{x - x_i}{\Delta x_i}\sigma_i^j \quad \text{on} \quad [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}],$$

where σ_i^j is the slope of the cell, one can get the following scheme:

$$\begin{aligned}
&U_{i+\frac{1}{2}}^{n+1} = \frac{1}{2}(U_i^n + U_{i+1}^n) - \lambda(g_{i+1} - g_i) \\
&g_i = F\big(U_i^{n+\frac{1}{2}}\big) + \frac{1}{8\lambda}u_i'^n, \quad U_i^{n+\frac{1}{2}} = U_i^n - \frac{\lambda}{2}F_j'^n \\
&U_i'^n = \min\operatorname{mod}\big(u_{i+1}^n - U_i^n, U_i^n - U_{i-1}^n\big) \\
&F_i'^n = \min\operatorname{mod}\big(F(U_{i+1}^n) - F(U_i^n), F(U_i^n) - F(U_{i-1}^n)\big) \\
&\min\operatorname{mod}(x, y) = \frac{1}{2}(\operatorname{sign}(x) + \operatorname{sign}(y))\min(|x|, |y|) \\
&\operatorname{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}
\end{aligned}$$

The scheme is second order of accuracy, conservative, consistent, TVD and stable if $CFL \leq 0.32$.

The best advantage of this method is that it does not require solving the Riemann problem, but the processing grids are more expensive.

2.2.2. Source terms

Source terms can be discretized by different ways: pointwise, upwind or mixed. The upwind and the mixed approaches are better than the pointwise one.

2.2.2.1 Pointwise approach for the source terms

This approach uses the splitting method [3, 9] to solve the non-homogenous equations:

Firstly, solve the initial value problem for the homogenous equations to find U^*

$$\frac{\partial U^*}{\partial t} + \frac{\partial F(U^*)}{\partial x} = 0. \tag{2.10}$$

Secondly, solve the ordinary differential equation with the initial condition U^* $(t = t^n : U^{n+1} = u^*)$:

$$\frac{\partial U^{n+1}}{\partial t} = S(x, U^{n+1}) \tag{2.11}$$

to find U^{n+1}. In this case, the numerical scheme has the following form:

$$u_i^{n+1} = U_i^* + \Delta t S(x_i, U)i^*).$$

2.2.2.2. The Roe's approximation with upwinding and mixed technique for the source terms

Following the cell average approach and applying the Roe's approximation to the homogenous part of equations, one can get:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta X_i}(F^{*n}_{i+\frac{1}{2}} - F^{*n}_{i-\frac{1}{2}}) + \frac{\Delta t}{\Delta x_i} S_i^*, \tag{2.12}$$

$$S_i^* = \frac{1}{\Delta t} \int\limits_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \int\limits_{t_n}^{t_{n+1}} S(x, U) dx dt. \tag{2.13}$$

Substitute the following formulas for numerical flux into (2.12)

$$F^{*n}_{i+\frac{1}{2}} = F_i + (\tilde{R}\tilde{\Lambda}^- \tilde{R}^{-1}\Delta U)_{i+\frac{1}{2}}, \quad F^{*n}_{i-\frac{1}{2}} = F_i - (\tilde{R}\tilde{\Lambda}^+ \tilde{R}^{-1}\Delta U)_{i-\frac{1}{2}}$$

one gets:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x_i}\Big(\big(\tilde{R}\tilde{\Lambda}^- \tilde{R}^{-1}\Delta U\big)_{i+\frac{1}{2}} + \big(\tilde{R}\tilde{\Lambda}^+ \tilde{R}^{-1}\Delta U\big)_{i-\frac{1}{2}} \Big) + \frac{\Delta t}{\Delta x_i} S_i^* \tag{2.14}$$

where

$$\hat{A} = \tilde{R}\tilde{\Lambda}\tilde{R}^{-1}, \quad \tilde{\Lambda}^{\pm} = \frac{1}{2}(\tilde{\Lambda} \pm |\tilde{\Lambda}|) \tag{2.15}$$

$|\tilde{\Lambda}|$ is getting from $\tilde{\Lambda}$ by applying the pointwise module operator to the eigenvalues.

$$\Delta F_{i+\frac{1}{2}} = \big(\hat{A}\Delta U\big)_{i+\frac{1}{2}} = \big(\tilde{R}\tilde{\Lambda}\tilde{R}^{-1}\Delta U\big)_{i+\frac{1}{2}} = \Big(\sum \tilde{\lambda}_k \tilde{\alpha}_k \tilde{r}_k\Big)_{i+\frac{1}{2}}. \qquad (2.16)$$

The pointwise approximation of the source term integral like this

$$S_i^* = \frac{1}{\Delta t} \int\limits_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \int\limits_{t_n}^{t_{n+1}} S(x_i, W) dx dt = \Delta x S(x_i, W) \qquad (2.17)$$

leads to the pointwise approach, presented above. Here the upwinding technique for source terms are applied as in [4], [5], [11], [12], [13].

The idea is to project the source term integral onto eigenvectors of the Roe's matrix $\hat{A}$ so that its linearized form can be expressed as follows:

$$\int\limits_{x_i}^{x_{i+1}} S dx \approx \tilde{S}_{i+\frac{1}{2}} = \big(\tilde{R}\tilde{R}^{-1}\tilde{S}\big)_{i+\frac{1}{2}} = \Big(\sum_{k=1}^{N_w} \tilde{\beta}_k \tilde{r}_k\Big)_{i+\frac{1}{2}} \qquad (2.18)$$

where $\big(\tilde{\beta}_k\big)_{i+\frac{1}{2}}$ are coefficients and equal to components of vector $\big(\tilde{R}^{-1}\tilde{S}\big)_{i+\frac{1}{2}}$. It is easy to recognize that integral (2.13) consists of parts of successive cells with common interface $i+\dfrac{1}{2}$ and can be expressed like the numerical flux form in (2.14). S_i^* is constructed from components that come from both sides of the cells (fig. 2)

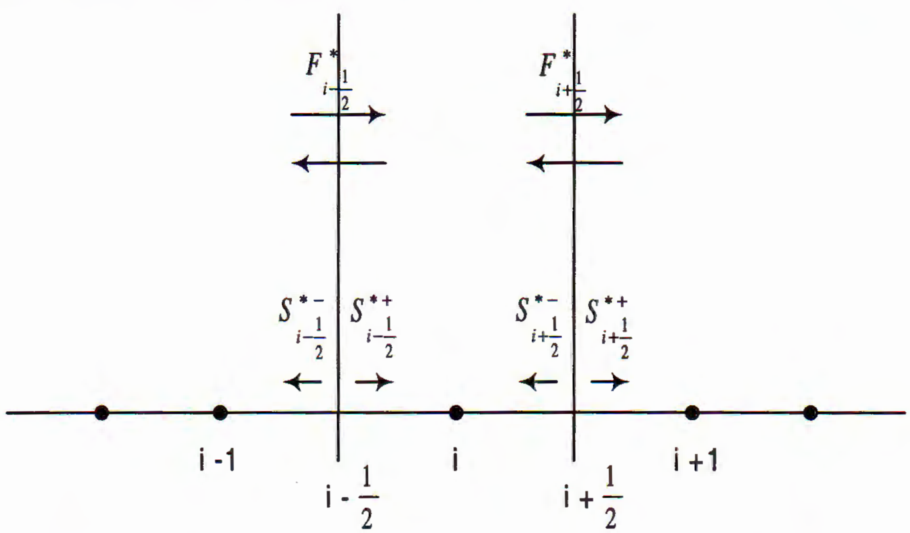

Fig. 3. Upwind decomposition of fluxes and source terms

Using upwinded source terms, one gets the following scheme for cell i:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x_i}\Big(\tilde{R}\big(\tilde{\Lambda}^- \tilde{R}^{-1}\Delta U - I^- \tilde{R}^{-1}\tilde{S}\big)_{i+\frac{1}{2}} + \tilde{R}\big(\tilde{\Lambda}^+ \tilde{R}^{-1}\Delta U - I^+ \tilde{R}^{-1}\tilde{S}\big)_{i-\frac{1}{2}}\Big)$$

where

$$I^{\pm}\tilde{\Lambda}^{-1}\tilde{\Lambda}^{\pm} = \frac{1}{2}\begin{pmatrix} 1 \pm \dfrac{|\lambda_1|}{\lambda_1} & 0 \\ 0 & 1 \pm \dfrac{|\lambda_2|}{\lambda_2} \end{pmatrix}$$

or by the average flux formula:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x_i}(F^{*n}_{i+\frac{1}{2}} - F^{*n}_{i-\frac{1}{2}}) + \frac{\Delta t}{\Delta x_i}((\tilde{R}I^-\tilde{R}^{-1}\tilde{S})_{i+\frac{1}{2}} + (\tilde{I}^+\tilde{R}^{-1}\tilde{S})_{i-\frac{1}{2}}),$$

$$F^{*n}_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{1}{2}\sum|\tilde{\lambda}_p|\tilde{\alpha}_p\tilde{r}_p.$$

2.2.2.3. Discretization of source terms for the Saint-Venant equations

For the Saint-Venant equation source terms play a very important role, so the discretization of these terms must be done carefully to avoid large errors.

Using the momentum equation of forms (2.2), (2.3) the Saint-Venant equation has two components of source terms:

* A source terms, caused by the channel geometry, including the bed slope and the width variation:

$$\frac{\partial P(x,A)}{\partial x}\Big|_{z=\text{const}} = \frac{\partial P}{\partial x}\Big|_{A=\text{const}} + \frac{\partial P}{\partial A}\frac{\partial A}{\partial x}\Big|_{z=\text{const}}.$$

* The friction term: $-gAS_f$.

Here the mixed discretization has been used.

Component $S_1 = \left(\frac{\partial P}{\partial x}\right)_{A=\text{const}}$ is approximated by the central scheme. The two other components $S_2 = \frac{\partial P}{\partial A}\left(\frac{\partial A}{\partial x}\right)_{z=\text{const}}$ and $S_3 = -gAS_f$ are approximated by the upwind technique. The formulas are as follows:

For the first pressure component:

$$S^*_{1,i} = \frac{1}{\Delta t}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}\int_{t_n}^{t_{n+1}} S_1(x,W)dxdt = \Delta x_i \begin{pmatrix} 0 \\ P_{i+\frac{1}{2}}(A_i) - P_{i-\frac{1}{2}}(A_i) \end{pmatrix}.$$

For the second pressure component:

$$S^*_{2,i} = S^{*+}_{2,i-\frac{1}{2}} + S^{*-}_{2,i+\frac{1}{2}}$$

where

$$S^{*-}_{2,i+\frac{1}{2}} = \frac{1}{\Delta t}\int_{x_i}^{x_{i+1}}\int_{t_n}^{t_{n+1}} S_2(x,W)dxdt = (\tilde{R}I^-\tilde{R}^{-1})_{i+\frac{1}{2}}\begin{pmatrix} 0 \\ \tilde{c}^2_{i+\frac{1}{2}}(A_{i+1} - A_i)_{z=0.5*(z_i+z_{i+1})} \end{pmatrix}$$

$$S^{*+}_{2,i-\frac{1}{2}} = \frac{1}{\Delta t}\int_{x_{i-1}}^{x_i}\int_{t_n}^{t_{n+1}} S_2(x,W)dxdt = (\tilde{R}I^+\tilde{R}^{-1})_{i-\frac{1}{2}}\begin{pmatrix} 0 \\ \tilde{c}^2_{i-\frac{1}{2}}(A_i - A_{i-1})_{z=0.5*(z_i+z_{i-1})} \end{pmatrix}$$

For the friction term

$$S^{*}_{3,i} = S^{*+}_{3,i-\frac{1}{2}} + S^{*-}_{3,i+\frac{1}{2}}$$

where

$$S^{*-}_{3,i+\frac{1}{2}} = \frac{1}{\Delta t} \int\limits_{x_i}^{x_{i+1}} \int\limits_{t_n}^{t_{n+1}} S_3(x,W)dxdt = -g\Delta x_i(\tilde{R}I^{-}\tilde{R}^{-1})_{i+\frac{1}{2}} \begin{pmatrix} 0 \\ \tilde{A}\dfrac{\tilde{Q}|\tilde{Q}|}{\tilde{K}^2} \end{pmatrix}_{i+\frac{1}{2}}$$

$$S^{*+}_{3,i+\frac{1}{2}} = \frac{1}{\Delta t} \int\limits_{x_{i-1}}^{x_i} \int\limits_{t_n}^{t_{n+1}} S_3(x,W)dxdt = -g\Delta x_{i-1}(\tilde{R}I^{+}\tilde{R}^{-1})_{i-\frac{1}{2}} \begin{pmatrix} 0 \\ \tilde{A}\dfrac{\tilde{Q}|\tilde{Q}|}{\tilde{K}^2} \end{pmatrix}_{i-\frac{1}{2}}$$

The average state of A, Q, K, that are $\tilde{A}$, $\tilde{Q}$, $\tilde{K}$ at interfaces, may be evaluated by one of three following ways:

a. By average values of two neighboring cells

$$\tilde{\Diamond}_{i\pm\frac{1}{2}} = \frac{1}{2}\Big(\tilde{\Diamond}_i + \tilde{\Diamond}_{i\pm\frac{1}{2}}\Big), \quad \text{with} \quad \tilde{\Diamond} = \tilde{A},\ \tilde{Q},\ \tilde{K}$$

b. By Roe's average state

$$\tilde{h} = \frac{\tilde{c}^2}{g}, \quad \tilde{A} = A_{i\pm\frac{1}{2}}(\tilde{h}), \quad \tilde{Q} = \tilde{u}\tilde{A}, \quad \tilde{K} = K_{i\pm\frac{1}{2}}(\tilde{h})$$

c. By average of depths of two neighboring cells

$$\tilde{h} = \frac{1}{2}(h_i + h_{i\pm\frac{1}{2}}), \quad \tilde{A} = A_{i\pm\frac{1}{2}}(\tilde{h}), \quad \tilde{Q} = \frac{1}{2}(Q_i \pm Q_{i\pm\frac{1}{2}}(\tilde{h}).$$

3. Conclusion

In this paper, some numerical methods for solving the 1-D Saint-Venant equations of general flow regime are presented. The homogenous part of the equations is solved by numerical methods for conservation laws: the Lax-Friedrichs, the Self-adjusting Hybrid, the Nessyahu-Tedmor, and the Roe's approximation methods. The source terms can be discretized following the pointwise, upwind or mixed approaches. The mixed approach for discretization of source terms is recommended for balancing the flux and source terms.

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VỀ MỘT SỐ PHƯƠNG PHÁP GIẢI SỐ
HỆ PHƯƠNG TRÌNH SAINT-VENANT MỘT CHIỀU
TRONG CHẾ ĐỘ DÒNG CHẢY TỔNG QUÁT.
PHẦN 1: CÁC PHƯƠNG PHÁP SỐ

Một trong các vấn đề thời sự của Cơ học Chất lỏng hiện nay là việc xây dựng các phương pháp mô phỏng số bài toán ngập lụt khi xẩy ra vỡ đê, vỡ đập. Nhiều phương pháp số tìm lời giải gián đoạn của hệ phương trình Saint-Venant một chiều đã được đề xuất. Tuy nhiên, việc phân tích, đánh giá và lựa chọn phương pháp thích hợp và hiệu quả đang thu hút sự quan tâm giải quyết của nhiều tập thể nghiên cứu trên thế giới.

Mục đích của bài báo này là, dựa trên các bài toán kiểm tra do các Phòng nghiên cứu Thủy lực châu Âu mới đề xuất, trình bày và đánh giá 4 phương pháp số giải hệ phương trình Saint-Venant một chiều thuần nhất trong tổ hợp với 3 phương pháp xử lý các thành phần nguồn.

Phần 1 của bài báo trình bày một số phương pháp số hiện đại để tính toán lời giải số hệ phương trình Saint-Venant một chiều trong chế độ dòng chảy tổng quát, khi dòng chảy có thể là hỗn hợp giữa chảy êm và chảy xiết. Phần thuần nhất của hệ phương trình được giải số theo các phương pháp số "bắt sóng" cho các định luật bảo toàn: phương pháp Lax-Friedrichs, phương pháp kết hợp tự chỉnh, phương pháp xấp xỉ Roe, phương pháp Nessyahu-Tedmor. Thành phần nguồn đóng một vai trò quan trọng và được rời rạc theo điểm, hay theo dạng hỗn hợp giữa dạng điểm và dạng theo hướng. Trong phần 2 của bài báo, các phương pháp trên sẽ được kiểm tra bằng một loạt các bài toán mẫu, bao gồm cả 3 chế độ dòng chảy: êm, xiết, chuyển ngưỡng. Các kết quả cho thấy việc xử lý hỗn hợp thành phần nguồn tốt hơn so với cách xử lý theo dạng điểm. Phương pháp xấp xỉ Roe với cách xử lý hỗn hợp thành phần nguồn sau đó được áp dụng đánh giá thử nghiệm bài toán vỡ đập các đập Sơn La và Hòa Bình.