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NUMERICAL MODEL OF THREE PHASE-THREE DIMENSIONAL FLOW IN POROUS MEDIA FOR RESERVOIR SIMULATION

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ABSTRACT. The paper presents the results of simulation and study of mixture - oil, gas and water - flows in porous medium using three-phase, three dimensional "Black oil" model. For discretizing the system of mathematical equations, the finite volume difference method was used. To solve numerically the system of discretized equations the IMPES and iterative methods were chosen. The numerical results are compared with a number of analytical solution, experimental measurement and calculation results of the other authors and they shown a good agreement. The model can be developed, adapted and used for study and simulation of mixture - oil, gas and water - flow in different experimental and real field conditions.

1. Introduction

The simulation is a good way to investigate many processes and phenomena in the nature and industry. In the petroleum industry this method is applied intensively also (see, for example, Duong Ngoc Hai et al. 1988, 1989, 1992; Nigmatulin et al. 1991; Duong Ngoc Hai 1994, 1996, and etc.).

The primary objective of the reservoir study is to simulate the performance of reservoir and, based on this, predicting and possibly finding the ways and means for increasing ultimate oil recovery. The classical reservoir engineering deals with the reservoir based on gross average basis (tank model) and cannot take into account adequately the variations of reservoir and fluid characteristics in space and time. Reservoir simulation by computers allows more detailed study of reservoir by dividing it into a number of blocks and applying fundamental equations for flow in porous media to each block (Bear 1987; Khalid 1979; Calvin 1990).

In the world, reservoir simulators are developed and used widely (CMG-Canada, Eclipse-USA, Athos-France...) and using the adequate simulator is a good way to describe quantitatively the flow of multiple phases in a homogeneous or heterogeneous oil-field having a production schedule determined not only by the reservoir properties, but also by market demand, investment strategy, and government regulation.

Based on the most basic model of multiphase flow in reservoir simulation, known as the *black-oil model*, and the essential technique for solving the problem-IMPES method, the numerical model of three-phase, three-dimensional flow in porous medium

is constructed and developed for simulation of different situation occurred in experiment and real oil-field conditions. The other techniques for treatment of non-linearities, solving the matrix equation and improving the accuracy of numerical solution are described too.

2. Mathematical model

2.1. Fluid flow equation

It hypothetically proposes that the Darcy law may be extended to describe the simultaneous flow of more than one phase in porous media. In this case, to obtain the equations for determining phases saturation (S_i) and pressure (p_i), the mass conservation equations of phase with using Darcy law were rewritten in the following forms (see, for example, Aziz 1979; Calvin 1990)

$$\nabla\Big[\frac{kk_{ro}}{\mu_o B_o}(\nabla p_o - \rho_o g \nabla D)\Big] - q_o = \frac{\partial}{\partial t}\Big(\frac{\phi S_o}{B_o}\Big), \tag{2.1}$$

$$\nabla\Big[\frac{kk_{rw}}{\mu_w B_w}(\nabla p_w - \rho_w g \nabla D)\Big] - q_w = \frac{\partial}{\partial t}\Big(\frac{\phi S_w}{B_w}\Big), \tag{2.2}$$

$$\nabla\Big[\frac{kk_{rg}}{\mu_g B_g}(\nabla p_g - \rho_g g \nabla D)\Big] + \nabla\Big[\frac{kk_{ro}R_s}{\mu_o B_o}(\nabla p_o - \rho_o g \nabla D)\Big] -$$

$$- q_g - R_s q_o = \frac{\partial}{\partial t}\Big[\phi\Big(\frac{S_g}{B_g} + \frac{R_s S_o}{B_o}\Big)\Big]. \tag{2.3}$$

In many cases, the considered problems normally are non-isothermal and the model must have additionally the equations for heat transfer processes, and the properties of the medium and fluids should be the functions of pressure and temperature. However, in this paper, the reservoirs in the below presented test cases have constant temperature and they are considered as the isothermal problems.

2.2. The auxiliary relations

In addition to the differential equations (2.1)-(2.3), certain auxiliary relations must be provided to close the mathematical model. They include (see, for example Calvin 1970):

- In the system at any point, the sum of volumes occupied by three phases must be always equal the dynamic porous volume (PV):

$$S_g + S_o + S_w = 1; \tag{2.4}$$

- The rock-fluid interactive properties are the functions of saturations:

$$p_g = p_o + p_{c,go}(S_o, S_w), \quad p_w = p_o - p_{c,ow}(S_o, S_w), \quad k_{ri} = k_{ri}(S_o, S_w); \tag{2.5}$$

- The properties of porous medium are the functions of space coordinate and pressure:

$$\phi = \phi(x, y, z, p_o), \quad k = k(x, y, z, p_o); \tag{2.6}$$

- And the fluid properties B_i, μ_i, ρ_i, and gas solution R_s are the functions of pressure:

$$B_i = B_i(p_i); \quad \mu_i = \mu_i(p_i); \quad \rho_i = \rho_i(p_i); \quad R_s = R_s(p_o). \tag{2.7}$$

2.3. Initial and boundary conditions

Initially, it can be possible to specify pressures and saturations of the phases at any spacial position:

$$p_i(0, x, y, z) = p_i^0(x, y, z); \tag{2.8}$$
$$S_i(0, x, y, z) = S_i^0(x, y, z). \tag{2.9}$$

More commonly, however, phase pressures are specified only at a reference depth, and to compute the initial distribution of phase pressures and saturations, the capillary pressure-saturations and pressure-density relationships are used in combination with the following ordinary differential equation:

$$\frac{dp_i}{dD} = \rho_i(p_i)g. \tag{2.10}$$

Generally, the boundary conditions are used in hybrid form on the outer boundary of the reservoir:

$$-\lambda \frac{\partial p}{\partial n} = h(p - p_b) + q_b. \tag{2.11}$$

The most important part of the boundary conditions is the specification of fluid injection and production intensities at the well:

$$q_i = WI \sum_{\ell} \frac{2\pi k_\ell h_\ell}{\ln(r_e/r_w) - 3/4 + AS} \frac{k_{ri}}{\mu_i B_i} (p_{i,\ell} - p_{wf}). \tag{2.12}$$

The system of equations (2.1)-(2.12) is closed and it can be used for simulating and studying the behavior of simultaneous flow of oil, gas and water at different condition in porous media.

3. Numerical method

For solving numerically the system of mathematical equations (2.1)-(2.12), the method IMPES (IMplicit Pressure Explicit Saturation) is used. The basic idea of this method is to obtain a single pressure equation by combination of the equations (2.1)-(2.3). After the pressure has been advanced in time, the saturations are updated explicitly.

The finite-difference equations of (2.1)-(2.3) can be written in terms of p_o and saturations as following:

$$\Delta[T_w(\Delta p_o - \Delta p_{c,ow} - \gamma_w \Delta D)] = C_{1p}\Delta_t p_{c,ow} + C_{1w}\Delta_t S_w + q_w, \tag{3.1}$$

$$\Delta[T_o(\Delta p_o - \gamma_o \Delta D)] = C_{2p}\Delta_t p_o + C_{2o}\Delta_t S_o + q_o, \tag{3.2}$$

$$\Delta[T_g(\Delta p_o + \Delta p_{c,go} - \gamma_g \Delta D)] + \Delta[R_s T_o(\Delta p_o - \gamma_o \Delta D)] = C_{3p}\Delta_t p_o + \\ + C_{3p}\Delta_t p_{c,go} + C_{3o}\Delta_t S_o + C_{3g}\Delta_t S_g + R_s q_o + q_g. \tag{3.3}$$

The basic assumption of the IMPES method is that the capillary pressure in the flow terms does not change over a time step ($\Delta_t p_{c,ow} = \Delta_t p_{c,go} = 0$). Then the terms involving $p_{c,ow}$ and $p_{c,go}$ can be evaluated explicitly by using its values at the old time level and it leads to: $\Delta_t p_o = \Delta_t p_w = \Delta_t p_g$. For simplification, we can therefore denote p_o by p.

Three equations (3.1)-(3.3) are combined in a way that all terms contained $\Delta_t S_i$ disappear. This is achieved by multiplying the water equation (3.1) by A, gas equation (3.3) by B and adding all three received equations, then A and B are found by setting the coefficient of the terms $\Delta_t S_i$ $(i = w, g)$ to equal zero. After some arrangements, the equation for determining pressure has the form as follows:

$$B_o \Delta T_o \Delta p + B_w \Delta T_w \Delta p + B_g \Delta T_g \Delta p - R_s B_g \Delta T_o \Delta p + B_g \Delta R_s T_o \Delta p = \\ B_o \Delta T_o \gamma_o \Delta D + B_w \Delta T_w \gamma_w \Delta D + B_g \Delta T_g \gamma_g \Delta D - R_s B_g \Delta T_o \Delta D \\ + B_g \Delta R_s T_o \Delta D + B_w \Delta T_w \Delta p_{c,ow} - B_g \Delta T_g \Delta p_{c,go} \\ + \Big(B_o C_{2p} + B_w C_{1p} + B_g C_{3p} + \Big(\frac{\phi S_o B_g}{B_o}\Big)\frac{\Delta R_s}{\Delta p}\Big)\Delta_t p \\ + B_o q_o + B_w q_w + B_g q_g. \tag{3.4}$$

Using the finite volume method (see, for example Patankar 1978) for the space differential operation and writing equation implicitly for pressure p at unknown level of time $n+1$, we can obtain:

$$\mathbf{T}\mathbf{P}^{n+1} = \mathbf{D}(\mathbf{P}^{n+1} - \mathbf{P}^n) + \mathbf{G} + \mathbf{Q} \tag{3.5}$$

where $\mathbf{T}$ is a six-diagonal matrix, while $\mathbf{D}$ is a diagonal matrix. In this case the vector $\mathbf{G}$ includes gravity and capillary terms, $\mathbf{P}$ is a vector for pressures p of the blocks at time step n or $n+1$, $\mathbf{Q}$ - vector for source term.

For the problem of approximating the $i+1/2$ level in the space co-ordinate of the calculated parameters in the matrix $\mathbf{T}$ and $\mathbf{G}$, the solution method is following (Dake 1978; Aziz and Settari 1979; Dung Ngoc Hai and Dang The Ba 1997): The properties of the fluids are evaluated as the average of their values at adjacent grid block nodes; The absolute permeability is determined as the harmonic mean of its value at adjacent nodes; The relative permeabilities are calculated by one point upstream weighting.

Equation (3.5) which has many of coefficients determined at the unknown $(n+1)$-time level is solved by the iterative methods.

After the pressures at the new time step are obtained, the saturations are explicitly updated by substituting the received results into (2.1) and (2.3). When S_i^{n+1} are known, the new capillary pressures $p_{c,ow}^{n+1}$ and $p_{c,go}^{n+1}$ which are explicitly used at the next time step, are calculated.

Based on the system of basic equations with appropriate initial, boundary conditions and solution algorithm presented above, the numerical model (simulator) is designed and constructed for simulation and investigation of different situation occurred in experiment and real oil reservoir.

4. Case Study

4.1. One phase radial flow

This radial flow case is considered under certain assumption, namely: the formation is homogeneous and isotropic; the drainage is implemented by a fully penetrating well to ensure radial flow; the fluid itself must have a constant viscosity and a small and constant compressibility. In this case, the change of pressure in time and space can be obtained analytically (see, for example, Dake 1978).

$$p(r,t) = p^0 - \frac{q\mu}{4\pi kh} ei\left(\frac{m\mu cr^2}{4kt}\right) \tag{4.1}$$

To verify the computational model and working capability of the numerical code, this problem is solved numerically with the following data and constrains (see Table 1). For this problem, to compute the flow, the reservoir is divided into $101 \times 101 \times 1$ blocks. The production well is located at the center of central block $(51, 51, 1)$. The comparison of the numerical results with analytical result (4.1) is shown in Fig. 1.

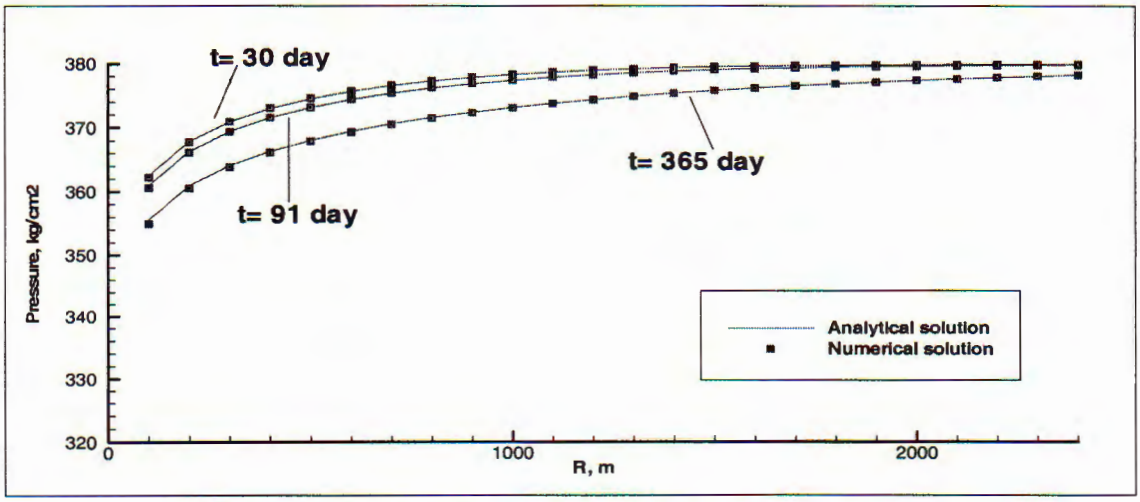


Fig. 1. Comparison of pressure distribution between numerical and analytic solutions for radial flow problem

In this figure, the calculated pressures in each block are regarded as the pressures at the center of blocks and they are compared with the pressures from (4.1), in which

the radius r are the distances from the well to the block centers. The pressure of the well block is not compared. The comparisons show a good agreement between numerical and analytical solutions.

Table 1. **Data and constraints for the radial flow problem**

reservoir dimension	10100×10100×20m	oil viscosity	0.01283 cp
porosity	0.3	oil compressibility	3.0E-04 $(kg/cm^2)^{-1}$
permeability	50 md	oil FVF	1.20 at 1.20 kg/cm^2
initial pressure	380 kg/cm^2	oil rate	507 $STDm^3/day$

4.2. 1D Water Injection (Buckley-Leverett) Problem

For the water displacing oil in linear one dimensional sample of reservoir, from the analysis of Buckley-Leverett (B-L), the position of different water saturation planes can be plotted at any given time moment from start of water injection by following expression (Buckley and Leverett 1942).

$$\chi s_w = \frac{W}{A\phi}\frac{df_w}{dS_w}\Big|_{s_w}; \quad f_w = \frac{1}{1+\frac{\mu_w}{k_{rw}}\frac{k_{ro}}{\mu_o}} \tag{4.2}$$

and the oil recovery can be calculated by the Welge technique (Welge 1952). For this problem, the results of comparison of numerical solution and analytical solution (4.2) are shown in Fig. 2 - 3. The statement of this problem is following : Reservoir is homogeneous, horizontal, incompressible, zero-capillary pressure; the fluids - water and oil are incompressible; the reservoir is initially fully saturated by oil; the water is injected from one end of the sample and the fluids are produced from the other end of the sample with the same rate: 1.0E-$3m^3/s$. The data and constraints of the problem are shown in Table 2.

Table 2. **Data and constraints for 1D-Water Injection problem**

cross section (cm^2)	1	oil viscosity (cp)	0.5
length (cm)	500	water viscosity (cp)	0.1
porosity	0.3	oil relative permeability	$(1-S_w)^2$
permeability (darcy)	1.0	water relative permeability	${S_w}^2$

In Fig. 2 - the comparison between numerical and analytical solutions of saturation distribution of water in different time moment is presented. From Fig. 2, it can be seen that the numerical solutions are in good agreement with B-L solution (4.2), except the region around the strong interrupted surface of the saturation. In the region around the saturation wave front - the strong interrupted front - the smoothness by numerical diffusion is clearly observed. From the figure it can be seen that in this case the saturation wave front is diffused and spread on several

grids. To reduce this effect several authors propose to use the self-adaptive finite difference method (see, for example, Douglas 1984; Todd 1971). It should be noted that, this kind of improvement can be used well in one dimensional cases but it will be difficult to apply for the multi-dimensional cases.

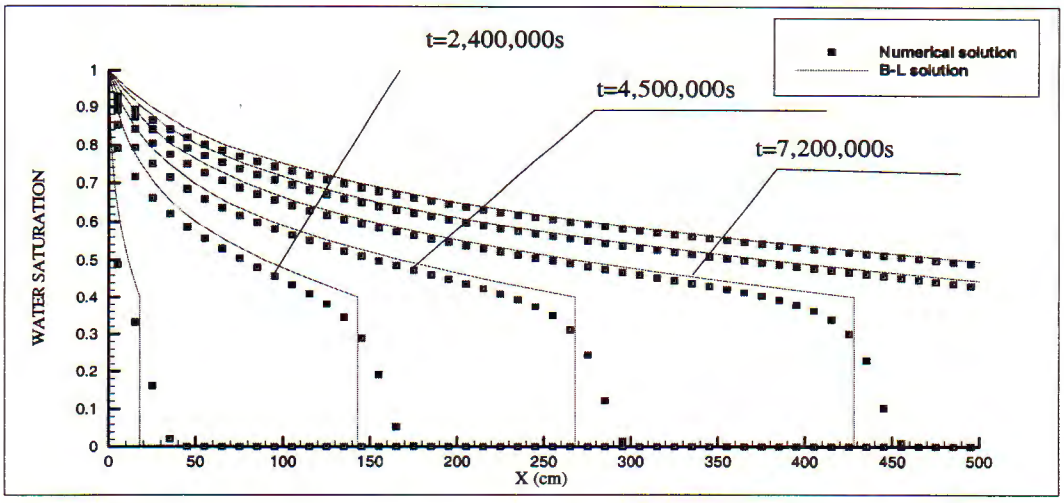


Fig. 2. The comparison of water saturation distribution between numerical solution and the B-L solution for 1D-water injection problem

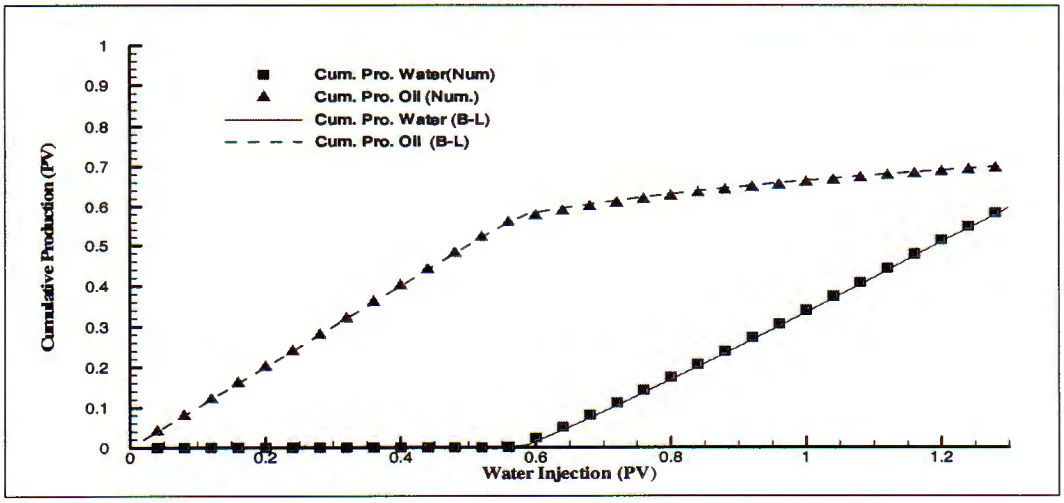


Fig. 3. The comparison of water and oil cumulative production (in PV unit) between numerical solution and the B-L solution for 1D-water injection problem

4.3. 3D Gas Injection - SPE1 Test Case

To verify and test the capability of the constructed simulator, the comparison of numerical solution of a three-dimensional black-oil reservoir problem with seven other simulators is made for the SPE1 Test Case.

The statements of the problem are described in the SPE1 Test Case (Aziz 1981) which can be resumed as follows:

Areal and cross-section views of the reservoir are presented in Fig. 4a and 4b. Pertinent data and constraints are given in Table 3 (In according to original paper of Aziz 1981, in this case the field - american measurement units (psi, ft, and ect.) are used to compare with the results of the other authors). The PVT properties and relative permeabilities are given in a table form.

The following results are compared:

Plot of: the pressures vs. time of the cells where the injector and producer are located (Fig. 5a and 5b); GOR vs. time (Fig. 6); Oil rate vs. time (Fig. 7);

Report at the end of 8^{th} year of: Gas saturation distribution (Fig. 8); Cell pressures (Fig. 9). In these figures, the results from presented in this paper model are marked by VCH.

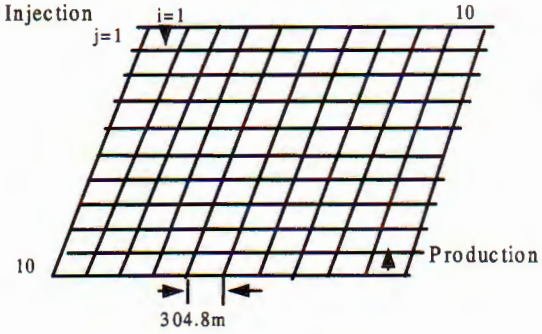


Fig. 4a. Reservoir and grid system

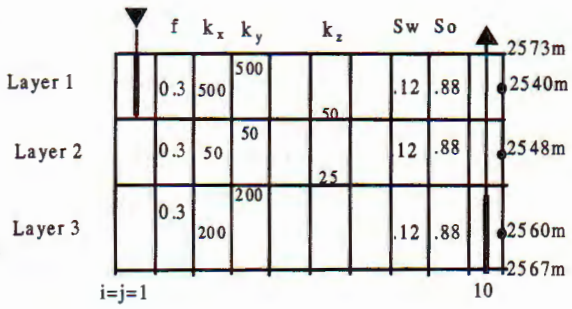


Fig. 4b. Diagonal cross section

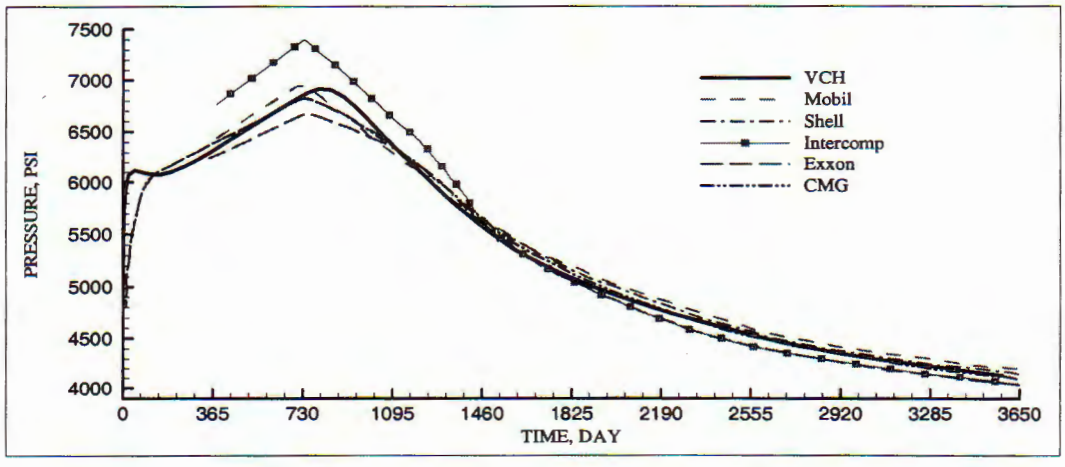


Fig. 5a. Pressure vs. time for injection well cell

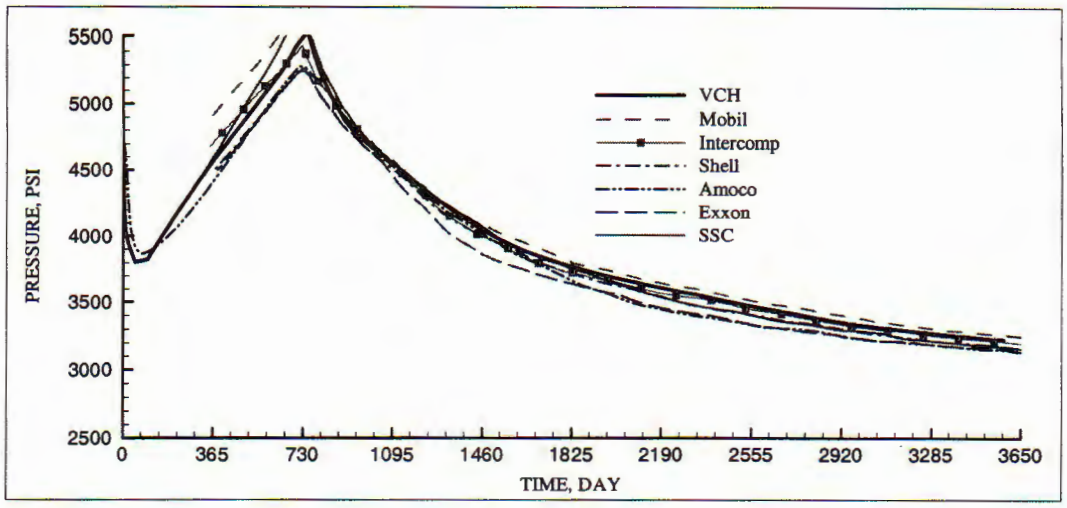


Fig. 5b. Pressure vs. time for producing well cell

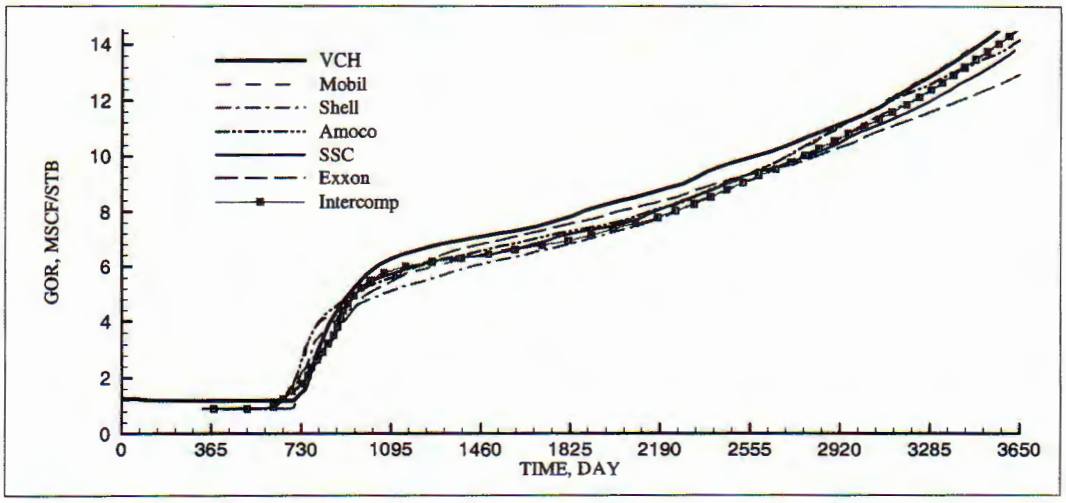


Fig. 6. GOR vs. time

Table 3. **Data and constraints for SPE1 Test Case**

Initial pressure	4800 psi	Rock compressibility	3E-6 1/psi
Gas injection rate	100 MMscf/D	Porosity at 14.7psi	0.3
Maximum oil production rate	20 MSTB/D	Well bore radius	0.25 ft
Minimum oil production rate	1 MSTB/D	Capillary pressure	0 psi
Minimum FBHP	1000 psi	Reservoir temperature	200 °F

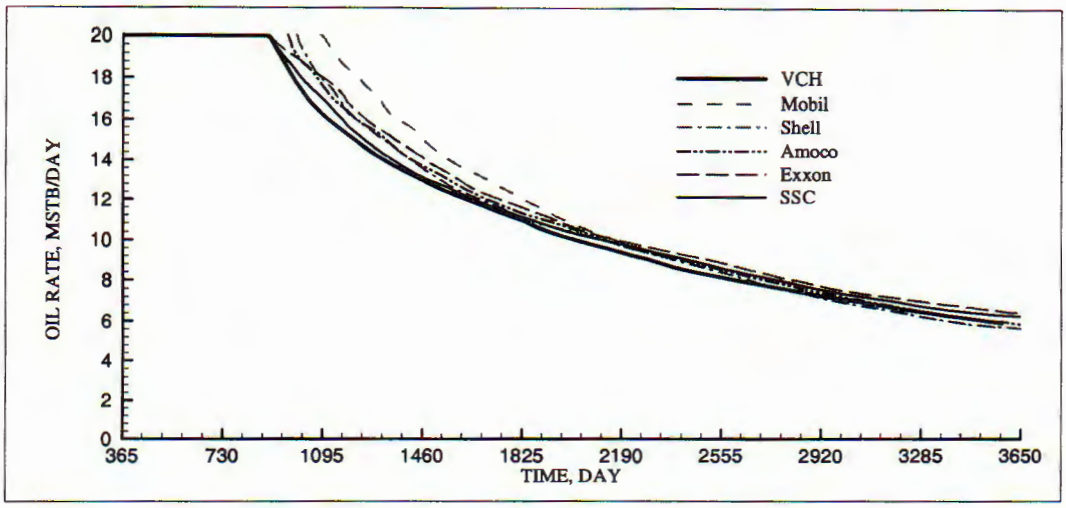


Fig. 7. Oil Rate vs. time

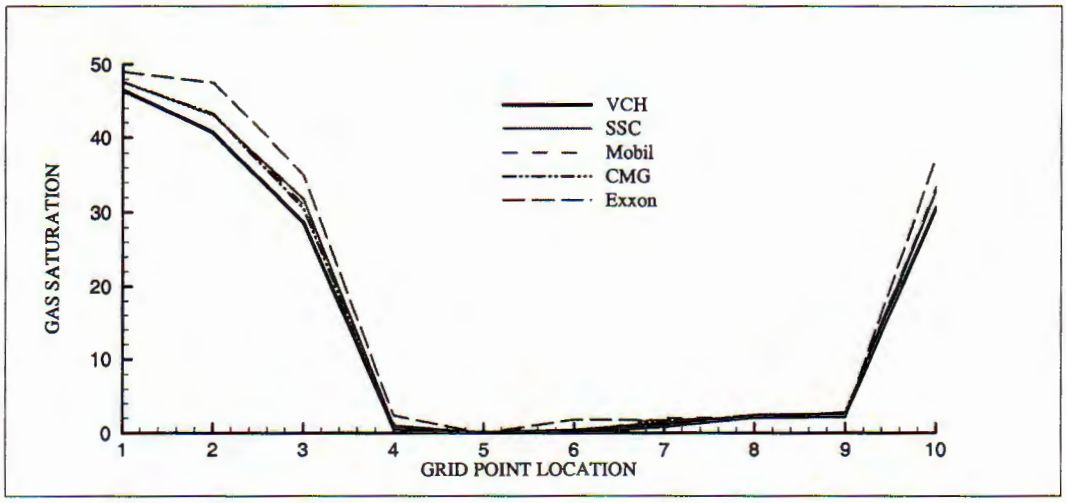


Fig. 8. Gas saturation vs. grid point location, $t = 8$ years, middle layer

For this problem, gas has been used to inject into a reservoir contained oil and connate water. The gas phase has the compressibility and mobility greater than the similar characteristics of oil and water phases and therefore, in this case, the non-linearity of the problem is stronger than the case without the presence of gas phase in the reservoir. In consequent, the problem is very sensible and essentially depends on the method of solution, technique for treatment of non-linearities, convergence and etc. For this case, the solutions obtained from the simulators are slightly different (see, Fig. 5-9).

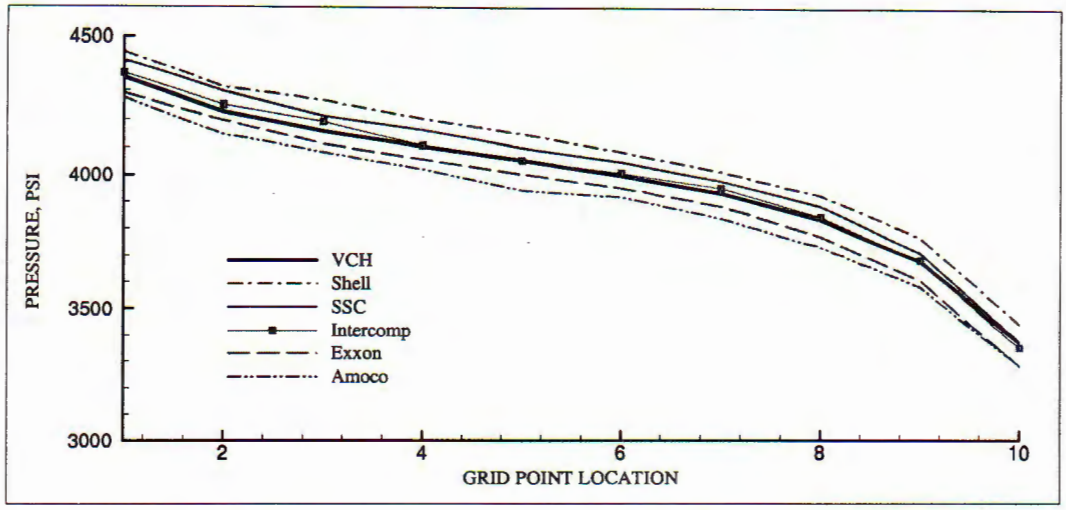


Fig. 9. Pressure vs. grid point location, $t = 8$ years, top layer

4.4. Simulation for the experiment of Grader et al.

Dynamic Displacement Experiments

The fluids used in Grader and O'Meara (1988) experiments were water, benzyl alcohol (BA) and decane. The porous medium was composed of glass beads of size 50-70μm. Decane ($\mu = 0.932$cp) was used in place of gas to reduce viscous fingering effects, and benzyl alcohol plays the role of oil phase. The viscosities of water and BA were 1.140 cp and 4.984 cp, respectively. The preparation of the sample was initiated by saturating the porous medium totally with water and then flooding it with oil until the irreducible water is reached. Uniform two phase saturation in the core was obtained by injecting a given fractional flow of oil and water to reach steady state. After that, the injection of gas (decane) was started.

Virnovsky (1984) and Grader and O'Meara (1988) extended the Welge/JBN method of measurement of relative permeability of two phase cases for the three phases flow cases. The experiments were performed with different initial water and oil saturation.

Simulation of three-phase displacement experiment

For verifying the numerical simulation, the numerical model of three-phase filtration flow has been used to simulate the three-phase displacement experiment. All of the properties of reservoirs and fluid given in the experiment of Grader et al. are used and they are presented in Table 4. The measured three-phase relative permeabilities of the phases are smooth by a fit polynomial of it's own saturation. The fit results are presented on Fig. 10. The results of simulation and comparison with the measurement data of Run #5 of Grader et al. experiment are shown in Fig. 11. In this experiment, the initial phase saturations are $S_w^0 = 0.447$, $S_{BA}^0 = 0.553$, $S_d^0 = 0.0$; and decane is injected to displace water and BA with rate of 0.15708 cm^3/s. The Fig. 11a presents the comparison of the change of the phase saturations at the end

of the sample in phase diagram (saturation path), and the comparison of recovery is presented in Fig. 11b. The comparison shows a good agreement between numerical solution and experimental data.

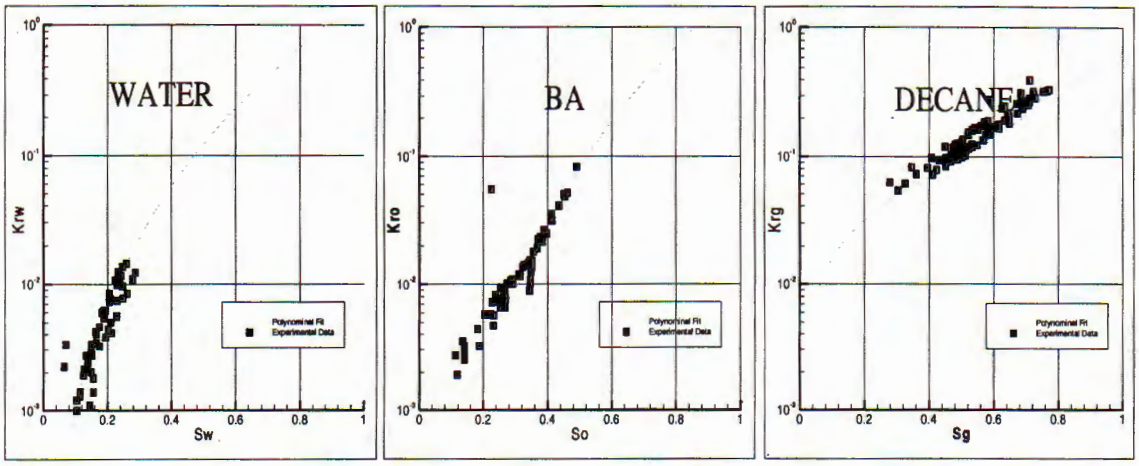
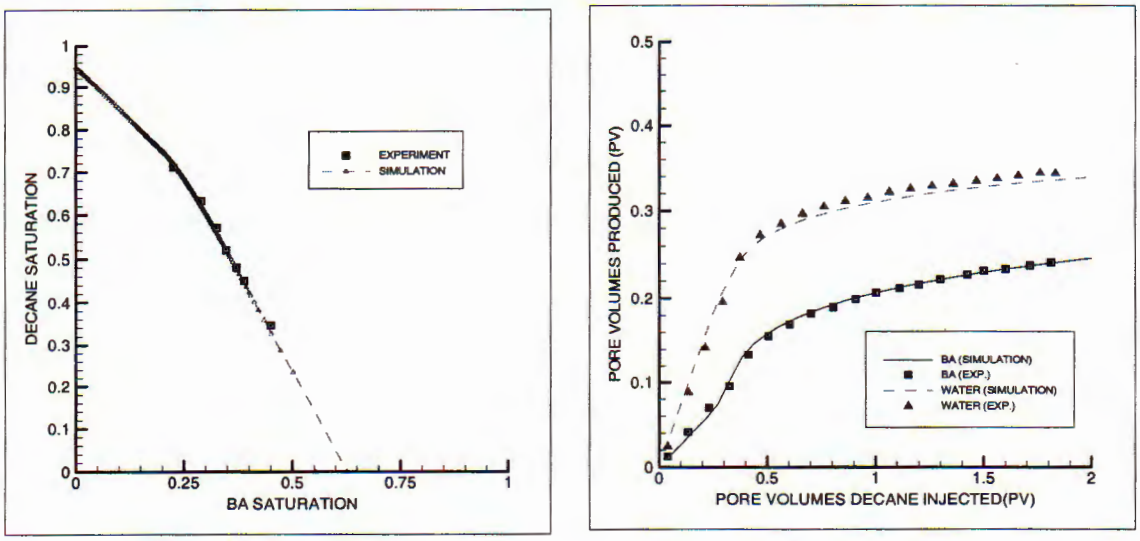


Fig. 10. Measured and fitted three-phase relative permeability



a) Saturation path b) Recoveries

Fig. 11. Comprison of simulative results and measured data for Grader's run # 5

The phase saturation distributions along reservoir at different time moment are presented in Fig. 12. From this figure, we can see the appearance and movement of two shock wave fronts of the saturation and it agrees with those obtained by characteristic analysis. This conduces to the creation of an oil bank in front of gas saturation front. This oil bank moves forward the production well and reaches it before the breakthrough of gas phase. And it leads to the sharp increase in oil recovery curve (see Fig. 11b).

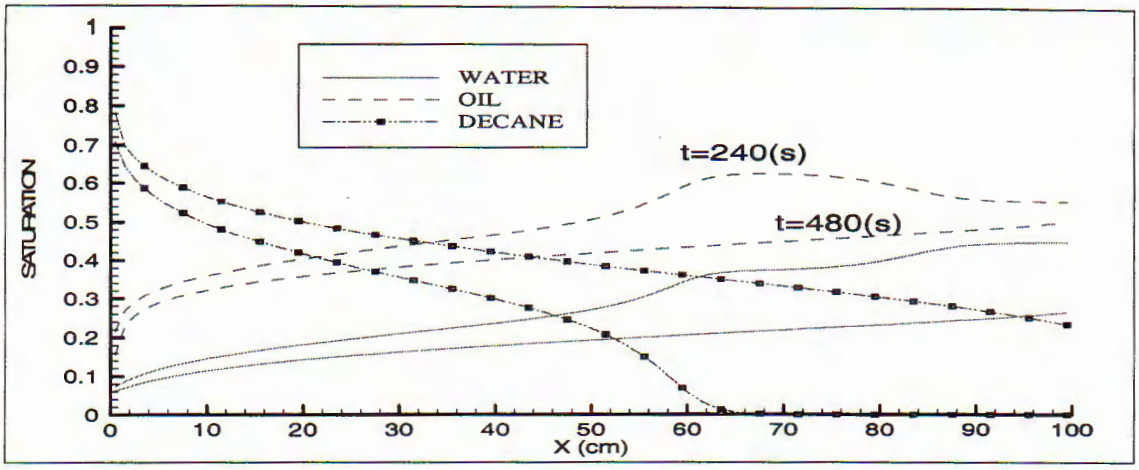


Fig. 12. Calculated distributrion of phase's saturation along reservoir

Table 4

a) Fluid properties

Fluid	Density (g/cm^3)	Viscosity (cp)	Interfacial tension (dynes/cm)
Water (w)	1.014	1.140	3.7(w-ba)
Benzyl Alcohol (BA)	1.026	4.984	2.5(BA-d)
Decane (d)	0.735	0.932	6.4(d-w)

b) Reservoir properties

Length	100 cm
Diameter	2.45 cm
Porosity	0.369
Absolute permeability	5.657 D

V. Conclusion

Based on the assumption of the extension of Darcy law for multiphase flow, the system of equations of three phase - water, oil and gas - flow in three dimension porous media is described. Using the IMPES method for solving numerically the mathematical equations, the algorithm is presented and the numerical program is realized.

The advantage of the IMPES method is simple in algorithm. The method is stable conditional, and has a limit in time step because of the explicit calculations of capillary pressure, phase saturation and of the non-linearities of obtained equations. Using the finite volume method for discretizing the differential equation guarantees the masse conservation in discrete form which is an important norm in solution of the fluid dynamic problems by numerical method.

For verifying the accuracy of the model and numerical method, the numerical code is used for solving the test cases and compared with analytical solution, numerical experimental measurement and results made by the other authors. The comparison shown a good agreement.

The program may be upgraded for research and applied purposes.

Acknowledgment

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Nomenclature

A: cross section of a reservoir;
AS: well skin factor;
B: formation volume factor;
c: compressibility;
D: depth;
f_w: fractional flow of water;
g: gravity;
k: permeability tensor;
k_r: Relative permeability;
p: pressure;
$p_{c,go}$: oil/gas capillary pressure;
$p_{c,ow}$: water/oil capillary pressure;
$p_{i,\ell}$: pressure of phase at the well block;
p_{wf}: well bottom hole pressure;
q: rate at STD;
r_e: outer diameter of well;
R_s: gas solution factor;
r_w: well bottom hole diameter;
S: phase saturation;
t: time;
W: cumulative water injected;
WI: well index;

Greek Letters

ρ: density;
μ: viscosity;
ϕ: porosity;

Subscripts

i = phase index;
i = w, o, g for water, oil and gas, respectively;
b: index for the variables on the boundary;
ℓ: index of well block;

Superscripts

0: for initial ($t = 0$) value of the variables

REFERENCES

1. Aziz K. and Settari A., 1979. Petroleum Reservoir Simulation. Applied Science Publishers LTD, London.

2. Aziz S. O., 1981. Comparison of Solution to a Three-Dimensional Black-Oil Reservoir Simulation Problem. JPT January, 13-25.

3. Bear J. and Corapcioglu Y. M., 1987. Advances in Transport Phenomena in Porous Media. Martinus Nijhoff Publishers.

4. Buckley S. E. and Leverett M. C., 1942. Mechanism of Fluid Displacement in Sands. Trans. AIME. 146, 107-116.

5. Calvin C. M. and Robert L. D., 1990. Reservoir Simulation. SPE monograph series, Richardson TX.

6. Dake L. P., 1978. Fundamentals of Reservoir Engineering. Elsevier Pub. Com., Amsterdam.

7. Ding Y., Renard G. and Weill L., 1995. Representation of Wells in Numerical Reservoir Simulation. SPE 13th Symposium on Reservoir Simulation, San Atonio TX, 12-15 Feb., 303-313 (SPE 29132)

8. Duong Ngoc Hai, 1994. On the Problem of Heat and Mass Transfer in Thermal Non-Isolated Reservoir. J. Mechanics, Vol. XVI, No.3, 11-16.

9. Duong Ngoc Hai, 1996. Modeling Multiphase Flow in Reservoir Engineering. J. Science and Technology, Vol. XXXV, No.1, 46-50.

10. Duong Ngoc Hai and Dang The Ba, 1997. Numerical Methods For Simulation of Multiphase Flow In Porous Media. In : Proc. of VI Nat. Conf. On Mechanics, Vol. 4, Hanoi 3-5/12, 168-177.

11. Duong Ngoc Hai, Kutushev A. G. and Nigmatulin R. I., 1988. On the Theory of the Filtration of a Liquid in Porous Medium under Bulk Heating by a High-Frequency Electro-Magnetic Fields. J. Applied Mathematics and Mechanics, Pergamon Press, Oxford, March, 20-28.

12. Duong Ngoc Hai, Musaev N. D. and Nigmatulin R. I., 1989. The Self-Similar Solution of the Problems of Heat and Mass Transfer in a Saturated Porous Medium with a Volume Heat Source. J. Applied Mathematics and Mechanics, Pergamon Press, Oxford, March, 749-757.

13. Duong Ngoc Hai and Nigmatulin R. I., 1992. The Unsteady One- Dimensional Flow in Saturated Porous Medium with a Volume Heat Source. New York, J.Fluid Dynamics, Jan.

14. Grader A. S. and O'Meara Jr., D.J., 1988. Dynamic Displacement Measurement of Three-Phase Relative Permeabilities Using Three Immisible Liquide. SPE paper 18293 presented at the 63rd Annual Technical Confrence and Exhibition of the SPE, Houston, TX, Oct. 2-5.

15. Nigmatulin R. I., Duong Ngoc Hai and Fedorov K.M., 1991. Mechanics, Heat and Mass Transfer in Saturated Porous Media. Application to Petroleum Technology. In: Convective Heat and Mass Transfer in Porous Media. NATO ASI Series E 196, Kluwer Pub. Co. Netherland, 939-961.

16. Patankar S. V., 1978. Numerical Heat Transfer and Fluid Flow. Mc Graw Hill Prees, London.

17. Todd M. R., O'Dell P. M. and Hirasaki G. J., 1971. Methods For Increase Accuracy In Numerical Reservoir Simulators. SPE J., December, Trans. AIME.

18. Virnovsky G. A., 1984. Determination of Relative Permeability in a Three-Phase Flow in a Porous Medium. Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza 5, 187-189.

19. Welge H. J., 1952. A Simplified Method for Computing Oil Recovery by Gas or Water Drive. Trans. AIME. 195, 91-98.

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MÔ HÌNH SỐ VỈA SỬ DỤNG MÔ HÌNH DÒNG CHẢY
3 PHA 3 CHIỀU TRONG MÔI TRƯỜNG RỖNG

Bài báo trình bày kết quả mô hình hóa và nghiên cứu dòng chảy hỗn hợp dầu, khí và nước trong môi trường rỗng sử dụng mô hình 3 pha, 3 chiều "dầu đen". Để rời rạc hóa hệ phương trình toán học đã sử dụng phương pháp sai phân thể tích hữu hạn. Phương pháp IMPES (ẩn cho áp suất và hiện cho độ bão hòa) và giải lặp được sử dụng để giải hệ phương trình rời rạc. Kết quả tính toán được so sánh với các lời giải giải tích, đo đạc thực nghiệm và giải số của một số tác giả khác, từ các so sánh cho thấy mô hình đã mô phỏng tốt các bài toán đặt ra trong các điều kiện khác nhau. Mô hình có thể được phát triển, làm phù hợp và sử dụng để nghiên cứu và mô phỏng dòng chảy hỗn hợp dầu, nước và khí trong những điều kiện thí nghiệm và hiện trường khác nhau.

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