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A FORM OF EQUATIONS OF MOTION OF CONSTRAINED MECHANICAL SYSTEMS

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ABSTRACT. In the present paper a form of equations of motion of a constrained mechanical system is constructed. These equations only contain a minimum number of accelerations. In the other words, such equations are written in independent accelerations while the configuration of the system is described by dependent coordinates. It is important that the equations obtained are applied conveniently for the mechanisms in which the use of independent generalized coordinates is not suitable.

1. Introduction

As known [5, 6], the use of holonomic coordinates for writting equations of motion is very convenient due to simplicity. However, in the case of constrained mechanical systems including holonomic systems, for example, in the problem of dynamics of mechanisms, the choice of independent coordinates in many case is impossible (in the case of mechanisms of closed loops). Moreover, in the problem of determining dynamic reactions of kinematic joints it is necessary to introduce redundant coordinates. Such a situation is related also to multibody systems, for example, kinematics and dynamics of robotics.

2. Equations of motion of a mechanical system subjected to stationary constraints

Let us consider a constrained mechanical system (holonomic and nonholonomic) of n degrees of freedom.

Denote by q_i, Q_i $(i = \overline{1, m})$ the generalized coordinates and forces, respectively.

In general, the generalized forces are functions of coordinates, velocities and time.

Consider the system subjected to stationary constraints of the form

$$\sum_{i=1}^{m} b_{\alpha i}\dot{q}_i = 0; \quad \alpha = \overline{1, s}, \tag{2.1}$$

which can be written in the matrix form

$$\mathbf{b}\,\dot{\mathbf{q}} = \mathbf{0}, \tag{2.2}$$

where $\mathbf{b}(q)$ is an $(s \times m)$ matrix, the elements of which are functions of coordinates, that is

$$\mathbf{b} = \left\| b_{\alpha i} \right\|_{\alpha=\overline{1,s}\ i=\overline{1,m}}, \tag{2.3}$$

but $\dot{\mathbf{q}}$ - the $(m \times 1)$ matrix of generalized velocities:

$$\dot{\mathbf{q}}^T = \| \dot{q}_1\ \dot{q}_2\ \ldots\ \dot{q}_m \|. \tag{2.4}$$

The letter T at the high right corner designates the transposition.

The kinetic energy of the system under consideration has the form

$$T = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{A}\, \dot{\mathbf{q}}, \tag{2.5}$$

where $\mathbf{A}$ is a quadratic, symmetric and nonsingular matrix of m order, the elements of which depend only on generalized coordinates.

By the Principle of Compatibility [2] the equations of motion of the system under consideration can be written in the form

$$\mathbf{A}\, \ddot{\mathbf{q}} = \mathbf{Q} + \mathbf{G} + \mathbf{R}, \tag{2.6}$$

where $\mathbf{R}$ is the matrix of unknown reaction forces of the constraints (2.1), which is defined by means of the condition for equations (2.6) being compatible with the constraint equations (2.1).

Moreover, $\mathbf{Q}$ and $\mathbf{G}$ are the $(m \times 1)$ matrices: $\mathbf{Q}$ is the matrix of generalized forces, but $\mathbf{G}$ - the matrix of components including quadratic velocities. The latter matrix is defined by means of only the matrix of inertia $\mathbf{A}$. The $\ddot{\mathbf{q}}$ denotes the matrix of generalized accelerations, which is an $(m \times 1)$ matrix too.

Let us introduce the pseudovelocities $\dot{\pi}_\sigma$ $(\sigma = \overline{1,n} = m - s)$:

$$\dot{\pi}_\sigma = \sum_{i=1}^{m} f_{\sigma i} \dot{q}_i, \quad \sigma = \overline{1,n} = m - s, \tag{2.7}$$

where $f_{\alpha i}$ $(\alpha = \overline{1,s}.i = \overline{1,m})$ are functions of generalized coordinates q_i $(i = \overline{1,m})$.

Only one condition need to be imposed on the linear form (2.7)

$$\det \left\| \begin{matrix} b_{\alpha_i} \\ f_{\sigma_i} \end{matrix} \right\|_{\alpha=\overline{1,s};\ \sigma=\overline{1,n};\ i=\overline{1,m}} \neq 0. \tag{2.8}$$

In the other words, $(n + s)$ linear forms must constitute a complete system made up linear independent form. By such a way the quantities $\dot{\pi}_\sigma$ $(\sigma = \overline{1,n} = m - s)$ can take on arbitrary values. Solving the set of linear equations (2.1) and (2.7) in consideration of the condition (2.8) we get

$$\dot{q}_i = \sum_{\sigma=1}^{n} d_{i\sigma} \dot{\pi}_\sigma, \quad i = \overline{1,m}, \tag{2.9}$$

where $d_{i\sigma}$ $(i = \overline{1,m}, \sigma = \overline{1,n})$ are functions of generalized coordinates q_i $(i = \overline{1,m})$.
In the matrix form the relations (2.9) can be written as

$$\dot{\mathbf{q}} = \mathbf{D}\dot{\boldsymbol{\pi}}, \tag{2.10}$$

where $\mathbf{D}$ is the $(m \times n)$ matrix, but $\dot{\boldsymbol{\pi}}$ is the $(n \times 1)$ matrix of pseudovelocities.
Derivating (2.10) we obtain

$$\ddot{\mathbf{q}} = \mathbf{D}\ddot{\boldsymbol{\pi}} + \dot{\mathbf{D}}\dot{\boldsymbol{\pi}}, \tag{2.11}$$

where $\dot{\mathbf{D}}$ is the matrix, the elements of which consist of the derivatives of elements of the matrix $\mathbf{D}$. Hereafter the matrix $\dot{\mathbf{D}}$ is called the derivative matrix of the matrix $\mathbf{D}$.

As known [2], the condition of ideality of the constraints (2.1) gives us

$$\mathbf{D}^T\mathbf{R} = \mathbf{0}. \tag{2.12}$$

Using the condition (2.12) in consideration of (2.11), the equation (2.6) is written as

$$\mathbf{D}_0\ddot{\boldsymbol{\pi}} = \mathbf{D}^T(\mathbf{Q} + \mathbf{G} + \mathbf{D}_1\dot{\mathbf{D}}\dot{\boldsymbol{\pi}}), \tag{2.13}$$

where

$$\mathbf{D}_0 = \mathbf{D}^T\mathbf{A}\,\mathbf{D}, \quad \mathbf{D}_1 = \mathbf{D}^T\mathbf{A}. \tag{2.14}$$

The equation (2.13) together with the constraint equation (2.2) describes the motion of the system under consideration.

By such a way the motion of the system under consideration is described by a system of algebraic - differential equations.

Note. Let us consider the case of constraints of the form

$$f_\alpha(q_1, q_2, \ldots, q_m) = 0, \quad \alpha = \overline{1,s}. \tag{2.15}$$

In this case the coefficients in (2.1) must be calculated by the formula

$$b_{\alpha i} = \frac{\partial f_\alpha}{\partial q_i} \quad i = \overline{1,m}, \ \alpha = \overline{1,s}. \tag{2.16}$$

For illustration let us consider the following example.

Example 1. A planar slider crank mechanism consists of a rigid crank, a rigid sliding block and two revolving and one translational joint. The crank is balanced and has the moment of inertia J_1 about its rotation axis. The lengths of the crank and connecting rod are denoted by r and L respectively, but the centre of gravity C of the connecting rod locates a distance a from the joint A. The sliding block has the mass m_3. The driving moment acting on the crank is denoted by M_d and the efficient resistance force by F. The friction at the joints is neglected. Write the equation of motion of the mechanism.

Let us choose φ and θ as generalized coordinates. They are redundant coordinates because the mechanism has only one degree of freedom. It is easy to write the constraint equation

$$f \equiv r \sin \varphi - L \sin \theta = 0, \tag{2.17}$$

which can be written as follows

$$\dot{f} \equiv r \cos \varphi \dot{\varphi} - L \cos \theta \dot{\theta} = 0. \tag{2.18}$$

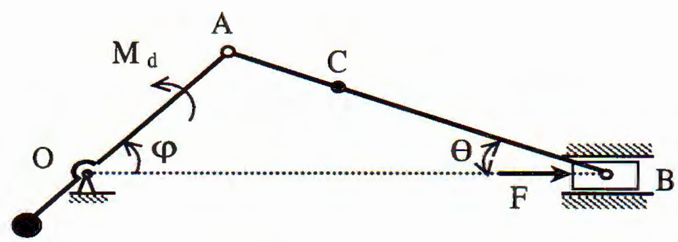

Fig. 1

The kinetic energy of the mechanism is calculated by the formula

$$T = \frac{1}{2}[J_1 + (m_2 + m_3 \sin^2 \varphi) r^2] \dot{\varphi}^2 + \frac{1}{2}(J_2 + M_2 a^2 + m_3 L^2 \sin^2 \theta) \dot{\theta}^2$$
$$+ [m_3 L \sin \varphi \sin \theta - m_2 a \cos(\varphi - \theta)] \dot{\varphi} \dot{\theta}. \tag{2.19}$$

The (2×2) matrix of inertia of the mechanism is of the form

$$\mathbf{A} = \begin{Vmatrix} J_1 + (m_2 + m_3 \sin^2 \varphi) r^2 & m_3 L \sin \varphi \sin \theta - m_2 a \cos(\varphi - \theta) \\ m_3 L \sin \varphi \sin \theta - m_1 a \cos(\varphi - \theta) & J_2 + m_2 a^2 + m_3 L^2 \sin^2 \theta \end{Vmatrix}. \tag{2.20}$$

Let us choose φ as an independent coordinate. The (2×1) matrix $\mathbf{D}$ in the formula (2.10) is of the form

$$\mathbf{D}^T = \begin{Vmatrix} 1 & \dfrac{r \cos \varphi}{L \cos \theta} \end{Vmatrix}, \tag{2.21}$$

and therefore

$$\dot{\mathbf{D}}^T = \begin{Vmatrix} 0 & -\dfrac{r}{L}\left(\dfrac{\sin \varphi}{\cos \theta} \dot{\varphi} - \dfrac{\cos \varphi}{\cos \theta} \mathrm{tg} \theta \, \dot{\theta} \right) \end{Vmatrix}. \tag{2.22}$$

The (2×1) matrix of generalized forces takes the form

$$\mathbf{Q} = \begin{Vmatrix} M_d - (F \sin \varphi - m_2 g \cos \varphi) r \\ -(m_2 g a \cos \theta + F L \sin \theta) \end{Vmatrix}. \tag{2.23}$$

It is easily to calculate the (2×1) matrix $\mathbf{G}$

$$\mathbf{G} = \begin{Vmatrix} m_3 r L \sin\varphi \cos\theta\, \dot{\theta}^2 - m_2 a \cos(\varphi - \theta)\dot{\varphi}^2 \\ -m_3 L^2 \sin\theta \cos\theta\, \dot{\theta}^2 - m_2 a \sin(\varphi - \theta)\dot{\varphi}^2 \end{Vmatrix}. \tag{2.24}$$

Equation (2.13) now will be

$$\begin{Vmatrix} 1 & \dfrac{r}{L}\dfrac{\cos\varphi}{\cos\theta} \end{Vmatrix} \times$$

$$\begin{Vmatrix} J_1 + r^2(m_2 + m_3 \sin^2\varphi) & [m_3 L \sin\varphi \sin\theta - m_2 a \cos(\varphi - \theta)]r \\ [m_3 L \sin\varphi \sin\theta - m_2 a \cos(\varphi - \theta)]r & J_2 + m_2 a^2 + m_3 L^2 \sin^2\theta \end{Vmatrix} \begin{Vmatrix} 1 \\ \dfrac{r}{L}\dfrac{\cos\varphi}{\cos\theta} \end{Vmatrix} \ddot{\varphi}$$

$$= \begin{Vmatrix} 1 & -\dfrac{r}{L}\dfrac{\cos\varphi}{\cos\theta} \end{Vmatrix} \times$$

$$\times \begin{Vmatrix} M_d - (F \sin\varphi - m_2 g \cos\varphi) r + m_3 r L \sin\varphi \cos\theta\, \dot{\theta}^2 - m_2 a r \cos(\varphi - \theta)\dot{\varphi}^2 \\ -m_2 g a \sin\theta \cos\theta - F L \sin\theta - m_3 L^2 \sin\theta \cos\theta \dot{\theta}^2 - m_2 a r \sin(\varphi - \theta)\dot{\varphi}^2 \end{Vmatrix}$$

$$+ \begin{Vmatrix} 1 & \dfrac{r}{L}\dfrac{\cos\varphi}{\cos\theta} \end{Vmatrix} \times$$

$$\times \begin{Vmatrix} J_1 + (m_2 + m_3 \sin^2\varphi) r^2 & [m_3 L \sin\varphi \sin\theta - m_2 a \cos(\varphi - \theta)]r \\ [m_3 L \sin\varphi \sin\theta - m_2 a \cos(\varphi - \theta)]r & J_2 + m_2 a^2 + m_3 L^2 \sin^2\theta \end{Vmatrix} \times$$

$$\times \begin{Vmatrix} 0 \\ -\dfrac{r}{L}\Big(\dfrac{\sin\varphi}{\cos\theta}\dot{\varphi} - \dfrac{\cos\varphi}{\cos\theta}\mathrm{tg}\theta\, \dot{\theta}\Big) \end{Vmatrix} \dot{\varphi}.$$

This equation is written as follows

$$\Big\{ J_1 + (m_2 + m_3 \sin^2\varphi) r^2 + \frac{r}{L}\frac{\cos\varphi}{\cos\theta}\big[m_3 L \sin\varphi - m_2 a \cos(\varphi - \theta)\big] r$$

$$+ \big[m_3 r L \sin\varphi \sin\theta - m_2 a r \cos(\varphi - \theta) + \frac{r}{L}\frac{\cos\varphi}{\cos\theta}\big(J_2 + m_2 a^2 + m_3 L^2 \sin^2\theta\big)\big]\frac{r}{L}\frac{\cos\varphi}{\cos\theta}\Big\}\ddot{\varphi}$$

$$= M_d - (F \sin\varphi - m_2 g \cos\varphi) r + m_3 r L \sin\varphi \cos\theta \dot{\theta}^2 - m_2 a r \cos(\varphi - \theta)\dot{\varphi}^2$$

$$+ \frac{r}{L}\frac{\cos\varphi}{\cos\theta}\big[m_2 g a \cos\theta \sin\theta + F L \sin\theta + m_3 L^2 \sin\theta \cos\theta\, \dot{\theta}^2 + m_2 a r \sin(\varphi - \theta)\dot{\varphi}^2\big]$$

$$- \frac{r}{L}\Big\{ \big[m_3 L \sin\varphi \cos\theta - m_2 a \cos(\varphi - \theta)\big] r$$

$$+ \frac{r}{L}\frac{\cos\varphi}{\cos\theta}\big(J_2 + m_2 a^2 + m_3 L^2 \sin^2\theta\big)\Big\}\Big(\frac{\sin\varphi}{\cos\theta}\dot{\varphi} - \frac{\cos\varphi}{\cos\theta}\mathrm{tg}\theta\, \dot{\theta}\Big)\dot{\varphi}.$$

This equation together with the algebraic equation (2.17) describes the motion of the crank slider mechanism.

3. Equations of motion of a mechanical system subjected to arbitrary constraints

Let us consider a mechanical system subjected to the unstationary constraints of the form

$$\sum_{i=1}^{m} b_{\alpha i}\dot{q}_i + b_\alpha = 0, \quad \alpha = \overline{1,s}. \tag{3.1}$$

Unlike above, the coefficients $b_{\alpha i}$ and b_α are the functions of generalized coordinates and time.

In the matrix form the equations (3.1) can be written as

$$\mathbf{b}\,\dot{\mathbf{q}} + \mathbf{b}_0 = \mathbf{0}, \tag{3.2}$$

where $\mathbf{b}(\mathbf{q},t)$, $\mathbf{b}_0(\mathbf{q},t)$ are the $(s \times m)$ and $(s \times 1)$ matrices respectively.

The kinetic energy of such a system can be calculated by the expression

$$T = T_2 + T_1 + T_0. \tag{3.3}$$

The functions T_i $(i = 0,1,2)$ are homogeneous functions of ith degree, which take the following forms

$$T_2 = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{A}_2\dot{\mathbf{q}}, \quad T_1 = \mathbf{A}_1\dot{\mathbf{q}}, \quad T_0 = A_0, \tag{3.4}$$

where $\mathbf{A}_1$, $\mathbf{A}_2$ respectively are the $(m \times m)$ and $(1 \times m)$ matrices, the elements of which depend on generalized coordinates and time, but A_0 is a function of generalized coordinates and time too.

By introducing the pseudovelocities in accordance with (2.7), we can express the generalized velocities in term of pseudovelocities, that is

$$\dot{q}_i = \sum_{\sigma=1}^{n} d_{i\sigma}\dot{\pi}_\sigma + d_i, \quad i = \overline{1,m}, \tag{3.5}$$

which can be written in the matrix form

$$\dot{\mathbf{q}} = \mathbf{D}\dot{\boldsymbol{\pi}} + \mathbf{d}, \tag{3.6}$$

where $\mathbf{D}$ and $\mathbf{d}$ are respectively the $(m \times n)$ and $(m \times 1)$ matrices, the elements of which are the functions of generalized coordinates and time.

As above, by applying the Principle of Compatibility the equations of motion of the system under consideration can be written as follows

$$\mathbf{A}_2\ddot{\mathbf{q}} = \mathbf{Q} + \mathbf{G} + \mathbf{Q}^g + \mathbf{Q}^0 + \mathbf{R}, \tag{3.7}$$

As above, $\mathbf{Q}$ is the matrix of generalized forces, $\mathbf{R}$ - the matrix of generalized reaction forces, satisfying the ideal conditions (2.2), $\mathbf{G}$ - the matrix of components of quadratic velocities defined only by means of the matrix $\mathbf{A}_2$.

The matrix $\mathbf{Q}^g$ and $\mathbf{Q}^0$ are the $(m \times 1)$ matrices, which consist of elements Q_i^g and Q_i^0 respectively, that are

$$Q_i^g = \sum_{j=1}^{m} \Big(\frac{\partial a_i}{\partial q_j} - \frac{\partial a_j}{\partial q_i}\Big)\dot{q}_j + \frac{\partial a_i}{\partial t}, \quad i = \overline{1,m}, \tag{3.8}$$

$$Q_i^0 = \frac{\partial A_0}{\partial q_i}, \quad i = \overline{1,m}. \tag{3.9}$$

In consideration of (2.12) and (3.6) the equation (3.7) will take the following form

$$\mathbf{D}_0^* \ddot{\boldsymbol{\pi}} = \mathbf{D}^T\big[\mathbf{Q} + \mathbf{G} + \mathbf{Q}^g + \mathbf{Q}^0 + \mathbf{A}_2(\dot{\mathbf{D}}\,\dot{\boldsymbol{\pi}} + \dot{\mathbf{d}})\big], \tag{3.10}$$

where $\mathbf{D}_0^* = \mathbf{D}^T\mathbf{A}_2\mathbf{D}$, $\dot{\mathbf{D}}$ and $\dot{\mathbf{d}}$ are the $(m \times 1)$ derivative matrices of $\mathbf{D}$ and $\mathbf{d}$ respectively.

Equation (3.10) together with (3.2) describes the motion of the system under consideration. We have obtained then a system of algebraic - differential equations.

In order to determine the reaction forces of the constraints (3.2) it is possible to apply the equations (3.7), that is

$$\mathbf{R} = \mathbf{A}_2\ddot{\mathbf{q}} - \mathbf{Q} - \mathbf{G} - \mathbf{Q}^g - \mathbf{Q}^0, \tag{3.11}$$

where $\ddot{\mathbf{q}}$ is calculated by (2.11) in consideration of (3.10), (2.10) and (2.11).

Of course, in the case of stationary constraints we have:

$$\mathbf{Q}^g \equiv 0; \quad \mathbf{Q}^0 \equiv 0, \quad \mathbf{d} \equiv 0. \tag{3.12}$$

By putting these quantities into (3.10) we immediately obtain (2.13).

Example 2. Consider a hammer grinder. The rotating drum is a homogeneous cylinder of radius R and the moment of inertia J_0 about the rotation axis. The hammer of mass m is connected to the drum by the revolution joint A located distance R from the axis O. The moment of inertia of the hammer about its centre of gravity C is denoted by J with $AC = a$. The drum rotates informly with angular velocity ω_0. Write the equation of motion of the hammer grinder and determine the reaction forces at the revolution joint A (Fig. 2).

For determining the reaction forces at the hinge A let us release the hammer from the drum and consider the released system shown in Fig. 3.

Denote the generalized coordinates by θ, u, v. The kinetic energy of the released system is calculated by means of the expression (3.3), in which

$$T_2 = \frac{1}{2}(J + ma^2)\dot{\theta}^2 + \frac{1}{2}m\dot{u}^2 + \frac{1}{2}m\dot{v}^2 - ma\sin\theta\dot{u}\dot{\theta} + ma\cos\theta\sin\theta\dot{v}\dot{\theta}, \tag{3.13}$$

$$T_1 = mRa\omega_0\cos(\theta - \omega_0 t)\dot{\theta} - mR\omega_0\sin\omega_0 t\cos\omega_0 t\dot{u} + mR\omega_0\cos\omega_0 t\dot{v}, \tag{3.14}$$

$$T_0 = \frac{1}{2}(J_0 + mR^2)\omega_0^2. \tag{3.15}$$

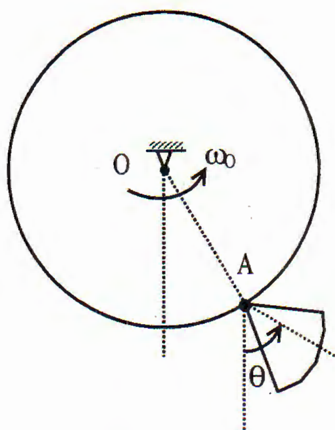

Fig. 2

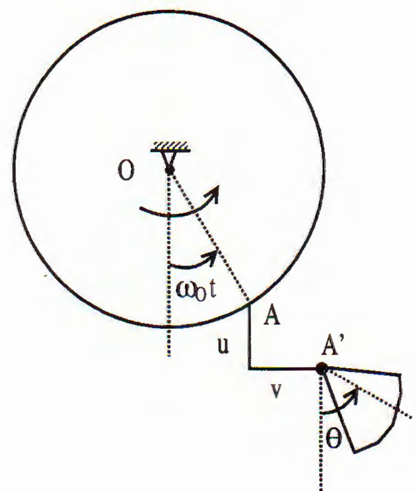

Fig. 3

We have then the (3×3) matrix $\mathbf{A}_2$

$$\mathbf{A}_2 = \begin{Vmatrix} J + ma^2 & -ma\sin\theta & ma\cos\theta \\ -ma\sin\theta & m & 0 \\ ma\cos\theta & 0 & m \end{Vmatrix}, \tag{3.16}$$

but the (1×3) matrix $\mathbf{A}_1$ and the function A_0 are

$$\mathbf{A}_1 = \begin{Vmatrix} mRa\omega_0\cos(\theta - \omega_0 t) & -mR\omega_0\sin\omega_0 t & mR\omega_0\cos\omega_0 t \end{Vmatrix}, \tag{3.17}$$

$$A_0 = \frac{1}{2}(J + mR^2)\omega_0^2. \tag{3.18}$$

The constraint equations are of the following form

$$u = 0, \tag{3.19}$$

$$v = 0. \tag{3.20}$$

Let us choose θ as a independent coordinate.

The (3×1) matrix $\mathbf{D}$ takes the form

$$\mathbf{D}^T = \|1 \quad 0 \quad 0\|. \tag{3.21}$$

For writting the equations (3.10) let us calculate the (3×1) matrices $\mathbf{Q}$, $\mathbf{G}$, $\mathbf{Q}^g$ and $\mathbf{Q}^0$

$$\mathbf{Q}^T = \| - mga \sin\theta \quad mg \quad 0 \|, \tag{3.22}$$

$$\mathbf{G}^T = \| 0 \quad ma\cos\theta\,\dot\theta^2 \quad ma\sin\theta\,\dot\theta^2 \|, \tag{3.23}$$

$$\mathbf{Q}^g = \| - mRa\sin(\theta - \omega_0 t)\omega_0^2 \quad mR\omega_0^2 \sin\omega_0 t \quad mR\omega_0^2\cos\omega_0 t \|^T, \ \mathbf{Q}^0 = \mathbf{0}, \tag{3.24}$$

$$\mathbf{D}_0^* = \mathbf{D}^T\mathbf{A}_2\mathbf{D} = \|1 \ 0 \ 0\| \begin{Vmatrix} J + ma^2 & -ma\sin\theta & ma\cos\theta \\ -ma\sin\theta & m & 0 \\ ma\cos\theta & 0 & m \end{Vmatrix} \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix} = J + ma^2. \tag{3.25}$$

Besides

$$\dot{\mathbf{D}} = \mathbf{0}, \quad \mathbf{d} = 0, \quad \dot{\mathbf{d}} = \mathbf{0}. \tag{3.26}$$

Therefore, the right member of the equation (3.10) will be

$$\begin{aligned} &\|1 \ 0 \ 0\| \begin{Vmatrix} -mga\cos\theta - mRa\sin(\theta - \omega_0 t)\omega_0^2 \\ mg + ma\sin\theta\,\dot\theta^2 + mR\omega_0^2\sin\omega_0 t \\ ma\sin\theta\,\dot\theta^2 + mR\omega_0^2\cos\omega_0 t \end{Vmatrix} = \\ &= -ma[g\cos\theta + R\sin(\theta - \omega_0 t)\omega_0^2]. \end{aligned} \tag{3.27}$$

The equation (3.10) will be now

$$(J + ma^2)\ddot\theta = -ma\big[g\cos\theta + R\sin(\theta - \omega_0 t)\omega_0^2\big], \tag{3.28}$$

which describes the motion of the hammer grinder under consideration.

The reaction force $\mathbf{R}^T = \|R_\theta \ R_u \ R_v\|$ is defined by the equations (2.12) and (3.11) in consideration of $\ddot u = 0$, $\ddot v = 0$, that is

$$\|1 \ 0 \ 0\| \begin{Vmatrix} R_\theta \\ R_u \\ R_v \end{Vmatrix} = \mathbf{0}, \tag{3.29}$$

$$\begin{Vmatrix} R_\theta \\ R_u \\ R_v \end{Vmatrix} = \begin{Vmatrix} J + ma^2 & -ma\sin\theta & ma\cos\theta \\ -ma\sin\theta & m & a \\ ma\cos\theta & 0 & m \end{Vmatrix} \begin{Vmatrix} \ddot\theta \\ 0 \\ 0 \end{Vmatrix} - \begin{Vmatrix} -ma[g\cos\theta + R\sin(\theta - \omega_0 t)\omega_0^2] \\ mg + ma\cos\theta\dot\theta^2 + mR\omega_0^2\sin\omega_o t \\ ma\sin\theta\,\dot\theta^2 + mR\omega_0^2\sin\omega_0 t \end{Vmatrix} \tag{3.30}$$

By these equations we get

$$R_\theta = 0, \tag{3.31}$$

$$R_u = -ma\sin\theta\,\ddot\theta - mg - ma\cos\theta\,\dot\theta^2 - mR\omega_0^2\sin\omega_0 t, \tag{3.32}$$

$$R_v = ma\cos\theta\,\ddot\theta - ma\sin\theta\,\dot\theta^2 - mR\omega_0^2\cos\omega_0 t, \tag{3.33}$$

where $\ddot{\theta}$ is calculated by (3.28).

It is easy to notice that R_u and R_v just are the vertical and horizontal components of reaction forces at the hinge A.

4. Conclusion

The represented method and the obtained equations are convenient to investigate mechanisms with closed loops and to determine the reaction forces at kinematic joints. Such equations are also applied usefully for constrained mechanical systems (holonomic and nonholonomic)

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MỘT DẠNG PHƯƠNG TRÌNH CHUYỂN ĐỘNG CỦA CÁC HỆ CHỊU LIÊN KẾT

Trong bài báo đề xuất một dạng phương trình chỉ chứa các gia tốc độc lập khi cấu hình của cơ hệ (hôlônôm và không hôlônôm) được mô tả nhờ các tọa độ suy rộng thừa. Như đã biết thậm chí trong trường hợp cơ hệ hôlônôm việc chọn tọa độ suy rộng độc lập không phải lúc nào cũng thuận tiện, ví dụ trong trường hợp các cơ cấu với các chuỗi đóng. Đối với trường hợp này việc áp dụng các phương trình được đưa ra trong bài báo tỏ ra rất tiện lợi. Các phương trình như vậy cũng giúp cho việc xác định các phản lực tại các khớp động của các cơ cấu được dễ dàng.

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