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DYNAMICS OF A FIN DRIVEN BOAT - MODELING AND SIMULATION

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ABSTRACT. In this paper the motion of a small sport boat running on a lake is simulated using alaska tools when the external forces and torques from water and excitation force from a person are applied. Using the results from measurements we have done some validations.
Keywords. multibody system motion equations, fluid forces and torques to running boat, validations by measurement.

1. Introduction

The applications of hydrodynamics to naval architecture and ocean engineering have expanded dramatically in recent years. In Bird, Armstrong and Hassager 1987 and Wehausen and Laiton 1960 some theories are written about the influences of water force to the motion of Multibody Systems in a fluid. In this paper we would like to present some applications of these theories to calculate the motion of a small sport boat running on a lake. So in this problem we have to solve Lagrange's equation which describe the motion of Multibody Systems (see Maißer 1988 and 1997) under the influences of the external forces and torques from water. Using Alaska tool this motion of the boat is simulated. The Alaska is a general purpose tool to enable dynamic simulation of multibody system (MBS). The motion of a multibody system is described by nonlinear equations of motion. In some cases it is sufficient to solve the dynamic problem with the linearized equations of motion. Alaska provides both linear and nonlinear computation (see [12]). In [11] the external forces and torques acting on the finds are calculated by the help of added mass coefficients of Multibody Fluid Systems (MBFS). Besides this added forces and torques, in this paper the turbulence to the fins (see [6]), the hydrostatic force and torque (see [4]), the flow resistance to the boat (see [5]) are applied to simulate the external force to the boat.

2. Formulation of the problem

Let us consider a small boat running on a lake. This boat is regarded as one Multibody System (MBS) and consists of two kayaks used as hull. Between them, the articulated four bar driving system is placed. This mechanism is protected by patent which transforms the displacement of a driver standing on the lower platform into a vertical movement of the fin.

To investigate the behaviour of the boat, a multibody alaska model is used. The degree of freedom of the model and the number of bodies depend on the kind and number of fins. Considering only the boat driving system, there are 6 bodies in the Multibody System having a degree of freedom (DOF) of 10. All water forces are considered as applied forces calculated in a special subroutines.

Regarding to the aim to set-up an optimal fin driven boat, two kinds of fins are available: the elastic fin and the rigid profile fin. The hull, the rear band, the lower platform and the front band compose the four bar mechanism representing a constrained Multibody System (CMS). The driver modelled as a simple rigid body is connected by a rotational joint to the lower platform. By a constrained motion in this joint, an excitation can be applied to the MBS. The other person is fixed in the front of the boat as a not moving passenger. The figure described the boat is presenter in [11].

The fins generate the driving forces in this system. In this paper the profiled fin is modelled as one body connected by a rotational joint with spring. Its stiffness is based on measurements. The motion of an MBS (Lagrange equation) are written in [1] and [11].

3. The water forces acting on the boat running in a lake

In this part we will see the force and torque of fluid acting on the moving fins and boat. The fluid is assumed to be ideal and irrotational.

3.1. The water forces acting on the driving fins of the boat

By the same way in [11] we have the formulas of the forces and torques with respect to the k-body fixed reference frame (BFR):

$$\begin{aligned} \underset{k}{F}_j &= -\underset{k}{\dot{V}}_i \underset{k}{m}_{ji} - \varepsilon_{jql} \underset{k}{V}_i \underset{k}{\omega}_q \underset{k}{m}_{li}\,, \\ \underset{k}{M}_j &= -\underset{k}{\dot{V}}_i \underset{k}{m}_{j+3,i} - \varepsilon_{jql} \underset{k}{V}_i \underset{k}{\omega}_q \underset{k}{m}_{l+3,i} - \varepsilon_{jql} \underset{k}{V}_i \underset{k}{V}_q \underset{k}{m}_{li}\,. \end{aligned} \tag{3.1}$$

Here m_{i_1,i_2} $(i_1, i_2 = 1, \ldots, 6)$ - the added mass coefficients are written in [11], j, q, l take the values 1, 2, 3 and as the index i is used to denoted the six components of the velocity potential $i = 1, \ldots, 6$, ε_{jql} denotes the Levi-Civita symbol. In (3.1) the added mass coefficients are depending on the geometry of body and on velocity U. Substituting the force and torque (3.1) to the generalized force formula (2.4) in [11] we have the generalized force from water to each element of the fin. But if we put all this exterior generalized force to the right side of Langrange's equation (2.1) in [11], our program is shut-down because the acceleration of the fin at the first moment is too big then the following force also is too large. So in the calculation we put the force terms depending on acceleration into the left side of Langrange's equation then we have the generalized Lagrange's equations with modified metric and generalized Christoffel's symbol. Therefore, our calculation approach is stable. In figure 1 the velocity of the boat in one of our example models is shown. The initial velocity of the boat is zero and after 60 seconds the boat has the mean velocity 0.66 m/s. In some figures we take the interval of time only from 40 seconds to 50 seconds because after about 30 seconds our calculation comes to stationary state. Figure 2 and figure 3 show the water force and torque acting on one point of the fin in inertial frame (y is vertical direction).

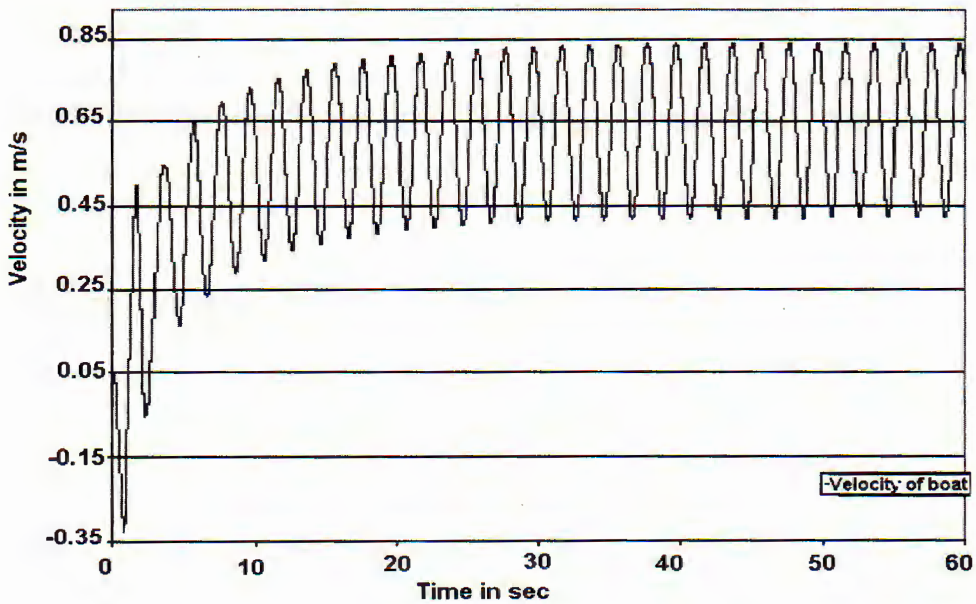

Fig. 1. The velocity way of driving boat

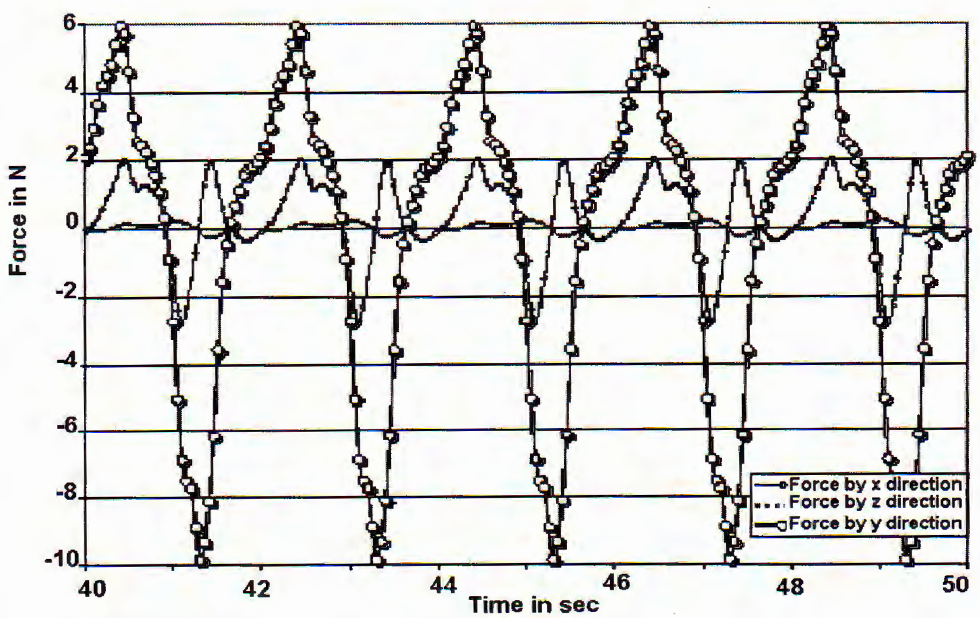

Fig. 2. Water forces to one point in the fin

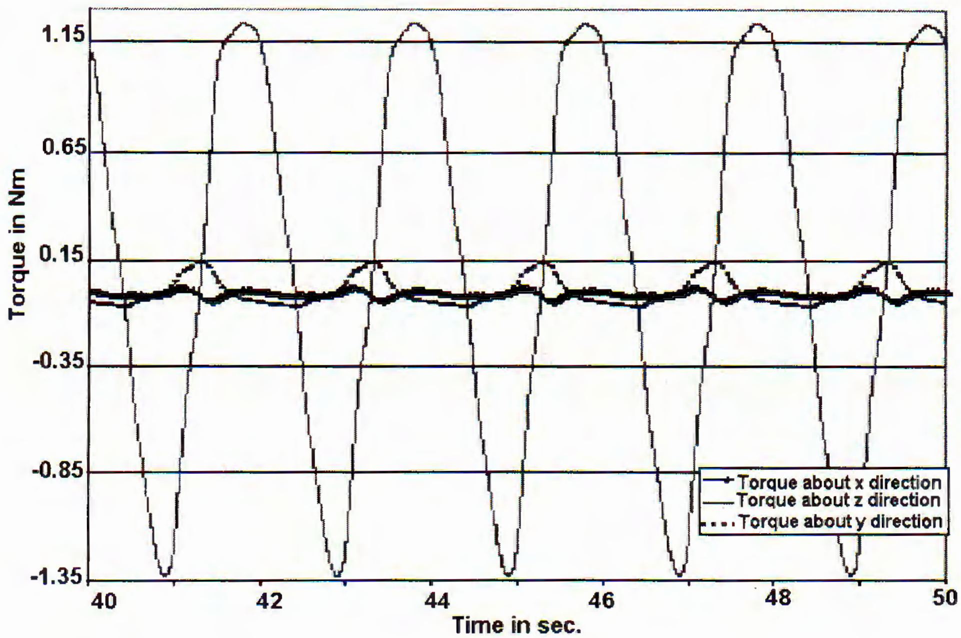

Fig. 3. Water torque to one point in the fin

In several papers (see Newman 1985, Chakrabarti 1987,...) the Morison's equation applying to a moving body in a fluid has been used. If the velocity of water is $\mathfrak{u}^f = (u_x^f, u_y^f, u_z^f)$ with respect to inertial frame (IFR) and the velocity of body is $\mathfrak{u}_b = (u_x^b, u_y^b, u_z^b)$ then the relative velocity in IFR is

$$\mathfrak{u} = \mathfrak{u}_f - \mathfrak{u}_b. \tag{3.2}$$

Denote by $\mathfrak{v} = (v_x, v_y, v_z)$ the relative velocity of a body in body frame (BFR). As shown in Newman 1985 and Chakrabarti 1987, the Morison's equation describing the water force to any cross section of an element is written in BFR as follows:

$$F_y = C_M \dot{v}_y + C_D |v_y| v_y, \tag{3.3}$$

here $C_M = c_m \dfrac{\rho \pi D^2}{4}$; $C_D = c_d \dfrac{\rho D}{2}$, where D is the linear characteristic dimension of the element cross section, ρ is water density, c_m and c_d respectively are added mass and drag coefficients. As in the previous section putting the force terms depending on acceleration to the left side of Lagrange's equation and substituting other terms from formula (3.3) to generalized force formula (2.4) in [11] we have the applied force of water to the fin. In calculation the Morison's formula (3.3) is more simple than formula (3.1) but we have to choose the correct coefficients C_M and C_D. Those coefficients can be obtained by several experiments and optimization problems with those measurement data. Using formula (3.1) in calculation we don't have to choose those coefficients but the time to calculate added mass coefficients $\underset{k}{m_{ij}}$ $(i,j = 1,\ldots,6)$ is long.

During the time of profile fin's movement in water there is a turbulence on the boundary layer of this fin (see Newman 1985). For the convenience, we will place the origin of the body frame so that the leading and trailing edges are situated at $x = \pm\frac{l_t}{2}$, here $l_t = l\cos\alpha(t)$, $\alpha(t)$ is the angle between the x-direction in IFR and the light from the leading and trailing edges (see figure 4).

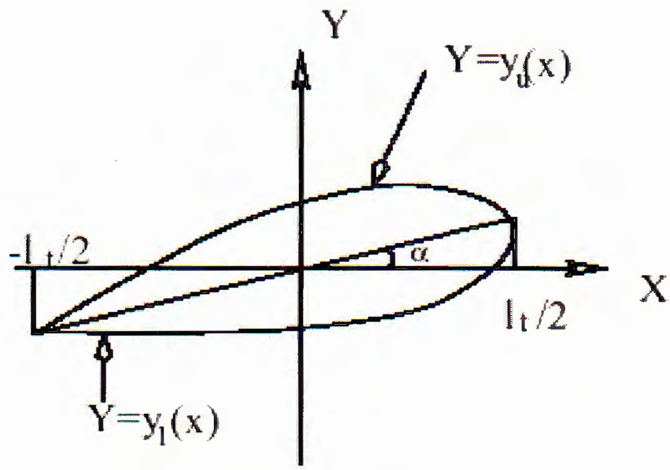

Fig. 4. Notation for two-dimensional profile fin

For easy to calculate we only study two dimensions in the turbulence problem of a moving fin. We assume that the vertical co-ordinates of the upper and lower fin surfaces $y = y_u(x,t)$ and $y = y_l(x,t)$ respectively are both much smaller than the chord length l of fin. The assumption of smooth tangential flow at the trailing edge is imposed mathematically by the Kutta condition requiring the velocity at the trailing edge to be finite. The conditions on the fluid flow are that the velocity vector should be equal to the free stream velocity $-U\vec{\imath}$ at infinity (in our model U is the velocity of the boat, $\vec{\imath}$ is the unit vector by the x-direction of IFR), tangential to the surface fin, and finite at the trailing edge. If the perturbation velocity potential $\phi(x,y)$ is defined such that $(u,v) = \nabla\phi$ then, as in Newman 1985, we have the following boundary value problem:

$$\begin{aligned}
&\nabla^2\phi = 0 \quad \text{throughout the fluid,}\\
&\frac{\partial\phi}{\partial \mathfrak{n}} = Un_x \quad \text{on the fin ,}\\
&\nabla\phi < 0 \quad \text{at the trailing edge,}\\
&\nabla\phi \to 0 \quad \text{at infinity.}
\end{aligned}$$

Let us denote the vertical co-ordinate of the mean camber line by $\eta(x,t) = \frac{1}{2}(y_u(x,t) + y_l(x,t))$. Using the results of Newman 1985, we have:

The corresponding boundary condition on the cut is:

$$u_y^f(x,0) = \frac{\partial \phi}{\partial y} = -U\eta_t'(x), \quad -\frac{1}{2}l_t < x < \frac{1}{2}l_t.$$

The vortex strength function $\gamma(x)$ is defined as follows:

$$\gamma(x) = \frac{2}{\pi\sqrt{(l_t^2/4 - x^2)}}\Big[\int\limits_{-l_t/2}^{l_t/2} \frac{u_y^f(\xi,0)\sqrt{(l_t^2/4-\xi^2)}}{\xi - x}\mathrm{d}\xi + \frac{1}{2}\Gamma\Big]$$

where Γ is the total circulation around the fin. In Newman 1985 the function Γ, the lift force L and the torque M about the z-axis acting on the fin are written as follows:

$$\Gamma = -2\int\limits_{-l_t/2}^{l_t/2} u_y^f(\xi,0)\Big[\frac{l_t/2-\xi}{l_t/2+\xi}\Big]^{1/2}\mathrm{d}\xi,$$

$$L = 2\rho U^2\int\limits_{-l_t/2}^{l_t/2} \frac{\mathrm{d}\eta(\xi)}{\mathrm{d}\xi}\Big[\frac{l_t/2-\xi}{l_t/2+\xi}\Big]^{1/2}\mathrm{d}\xi,$$

$$M = 2\rho U^2\int\limits_{-l_t/2}^{l_t/2} \frac{\mathrm{d}\eta(\xi)}{\mathrm{d}\xi}[{l_t}^2/4-\xi^2]^{1/2}\mathrm{d}\xi.$$

If the vortexes are distributed along the x-axis (by the direction of IFR) between the leading and trailing edges of the fin with local circulation density $\gamma(x)$ so the velocity components of water are given by the following integrals (see Newman 1985):

$$u_x^f(x,y) = -\frac{1}{2\pi}\int\limits_{-l_t/2}^{l_t/2} \frac{\gamma(\xi)y}{(x-\xi)^2+y^2}\mathrm{d}\xi, \tag{3.4}$$

$$u_y^f(x,y) = \frac{1}{2\pi}\int\limits_{-l_t/2}^{l_t/2} \frac{\gamma(\xi)(x-\xi)}{(x-\xi)^2+y^2}\mathrm{d}\xi. \tag{3.5}$$

Therefore, if the example of boat model has one profile fin in the front and one elastic fin in the rear, substituting those perturbation velocities u_x^f and u_y^f from formulas (3.4) and (3.5) into formula (3.2) we have the relative velocity $\mathfrak{u}$. Then, changing this relative velocity in IFR to the relative velocity $\mathfrak{v}$ in body frame (BFR) and putting the obtained components of both velocity v_y and acceleration $\dot{v}_y$ to the Morison's formula (3.3) we have the water force to elastic fin under the influence of the turbulence of the front profile fin.

Figure 5, figure 6 show the lift force and torque to profile fin about z direction in IFR arising from the turbulence of this fin in water.

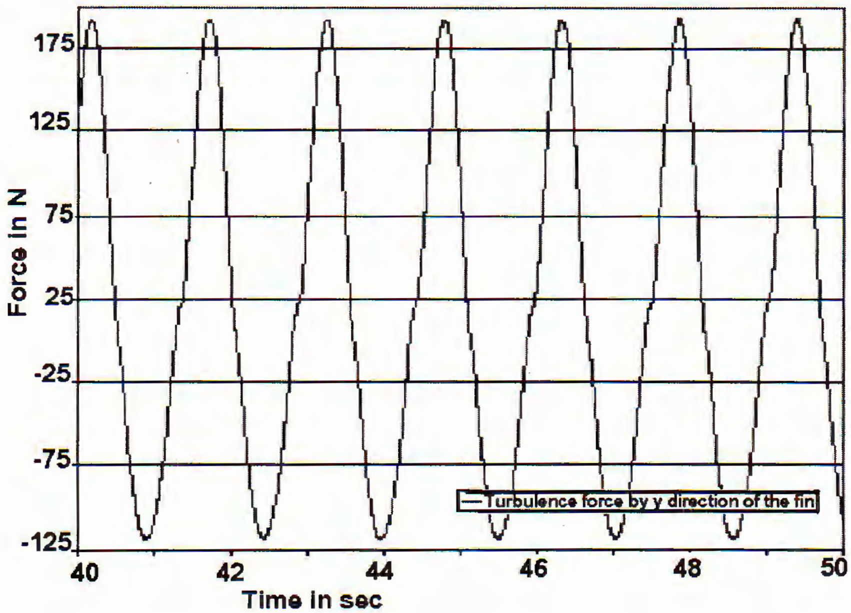

Fig. 5. Lift force arising from turbulence of profile fin

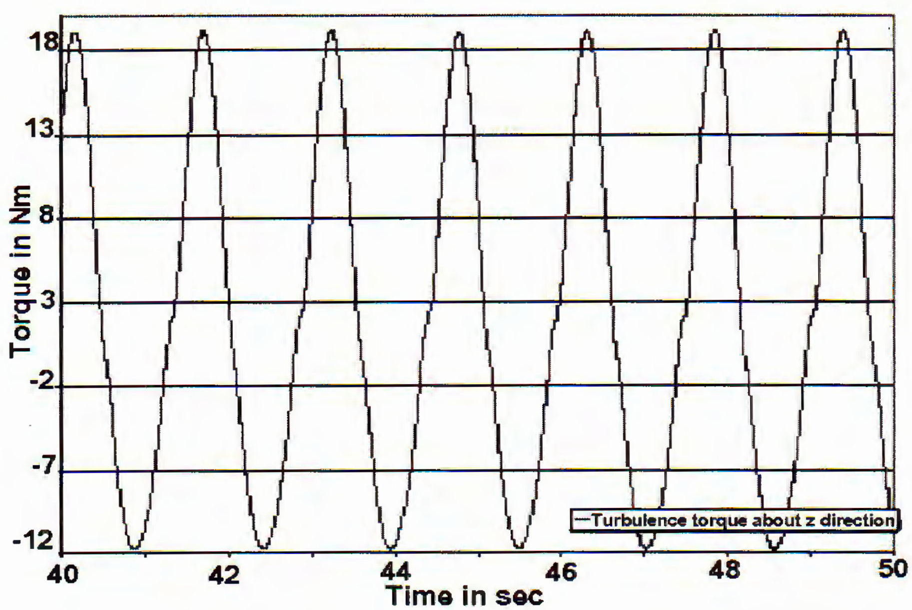

Fig. 6. Torque about z direction arising from turbulence of profile fin

3.2. The water forces acting on the running boat

1. Flow resistance to the boat.

In this part we will calculate the flow resistance by the Michell's formula (Stoker 1992, Timman, Hermans, Hsiao 1985, Wehausen and Laiton 1960). As in the previous point we assume that our boat is running with speed U by the direction of the x-axis in IFR. For the convenience we shall fix the co-ordinate system on the boat, or in other words we consider the flow past the boat with speed U in the negative x direction. We assume that the equation of the hull to be given by $z = \beta f(x, y)$. Where $f(x, y)$ is a smooth function defined on the projection of the boat hull on the x, y plane (in our model this function is obtained from measurement), $\beta = \frac{B}{L}$, here B is the width and L is the length of hull. The perturbation caused by the boat is described by velocity potential ϕ which satisfied Laplace equation together with the linearized free surface condition as follows (Stoker 1992, Timman, Hermans and Hsiao 1985):

$$\begin{aligned} \Delta\phi_j &= 0 \quad \text{except at } (\xi, \eta, \zeta)\,, \\ \frac{\partial^2 \phi_j}{\partial x^2} + \frac{g}{U^2}\frac{\partial \phi_j}{\partial y} &= 0 \quad \text{on the free surface} \quad y = 0,\ j = 1, \ldots, 6\,, \\ \frac{\partial \phi_j}{\partial y} &= 0 \quad \text{on the bottom} \quad y = -h\,, \end{aligned} \tag{3.6}$$

and the boundary condition on the boat hull. This boundary condition states that the total normal velocity relative to the boat vanishes and is written as follows (see Stoker 1992, Timman, Hermans and Hsiao 1985, Wehausen and Laiton 1960):

$\phi_z = \mp f_x$ inside the projection S_0 of the boat hull on the plane $z = 0$ and

$\phi_z = 0$ outside this projection.

Using the results of Stoker 1992 and Timman, Hermans and Hsiao 1985, the flow resistance R in the case of infinite deep water is written as follows:

$$R = \frac{4g^2\rho}{\pi U^2}\int_0^{\pi}(P^2(\mu) + Q^2(\mu))\sec^3\mu \mathrm{d}\mu\,, \tag{3.7}$$

$$P(\mu) = \iint_{S_0} f_x(x, y)\exp(\nu y \sec^2\mu)\cos(\nu x \sec\mu)\mathrm{dxdy}\,,$$

$$Q(\mu) = \iint_{S_0} f_x(x, y)\exp(\nu y \sec^2\mu)\sin(\nu x \sec\mu)\mathrm{dxdy}\,,$$

$$\text{and} \quad \nu = \frac{g}{U^2}\cdot$$

Formula (3.7) is Michell's integral. For the case of finite deep water h the flow resistance R is written as (Wehausen and Laiton 1960):

$$R = \frac{2\rho g U}{\pi} \int_{\mu_k}^{\infty} (P^2(\mu) + Q^2(\mu)) \sqrt{\frac{\mu}{\mu - \nu \tanh \mu \mathrm{h}}} \mathrm{d}\mu \,,$$

where

$$P(\mu) = \iint_{S_0} f_x(x,y) \frac{\cosh \mu(y+h)}{\cosh \mu h} \cos(x\sqrt{\nu\mu \tanh \mu h}) \mathrm{dxdy} \,,$$

$$Q(\mu) = \iint_{S_0} f_x(x,y) \frac{\cosh \mu(y+h)}{\cosh \mu h} \sin(x\sqrt{\nu\mu \tanh \mu h}) \mathrm{dxdy} \,.$$

Here μ_k is the nonzero solution of the equation: $\mu = \nu \tanh \mu h$ if such exists, i.e. if $U^2/gh > 1$; otherwise $\mu_k = 0$. In figure 7 the time history of the flow resistance to the boat while running is shown. It is easy to see that after approximately 50 seconds both functions nearly coincide.

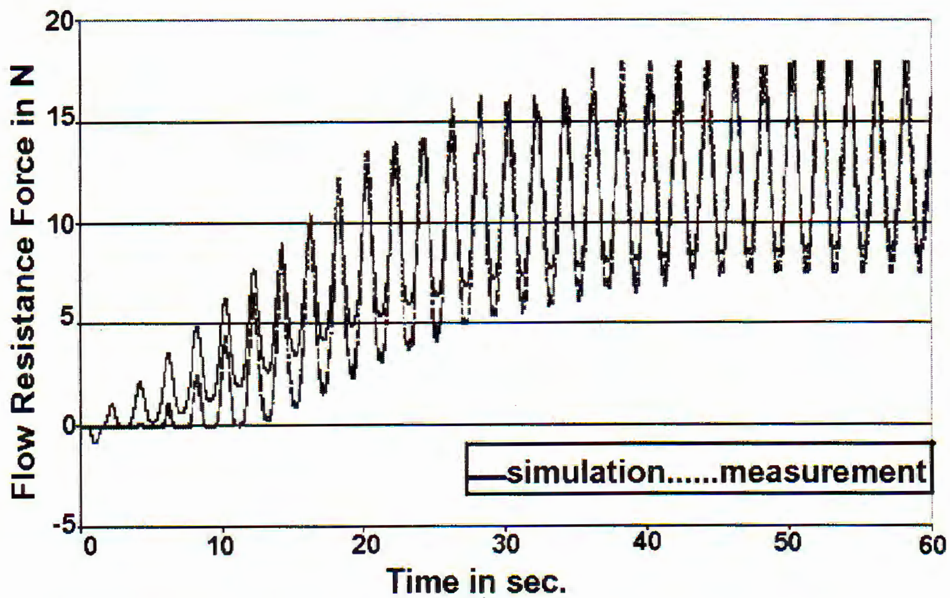

Fig. 7. Flow resistance

2. Hydrostatic force to the boat

Hydrostatics is the oldest and most elementary topic of naval architecture and fluid mechanics. The formulas of hydrostatic force and torque are written as follows (see Newman 1985)

$$\mathfrak{F} = -\rho g \iint_{S_B} \mathfrak{n} y \mathrm{d}S_B, \tag{3.8}$$

$$\mathfrak{M} = -\rho g \iint_{S_B} (\mathfrak{R} \times \mathfrak{n}) \mathrm{d}S_B. \tag{3.9}$$

Let us introduce two coordinate systems. The coordinate system ($\mathfrak{r}$) has the origin point in the center of mass of the boat hull in the equilibrium state and the directions of its are coincided with the direction of IFR. The coordinate system ($\mathfrak{r}'$) is the coordinate of BFR of the boat hull and its origin is lying in the center of mass of this. Then we can express the transformation between these coordinate systems as:

$$\mathfrak{r} = \mathfrak{r}' + \xi_T + \xi_R \times \mathfrak{r}'.$$

Here, $\xi_T = (\xi_1, \xi_2, \xi_3)$ and $\xi_R = (\xi_4, \xi_5, \xi_6)$ are the vectors formed by the translation and rotation of the boat.

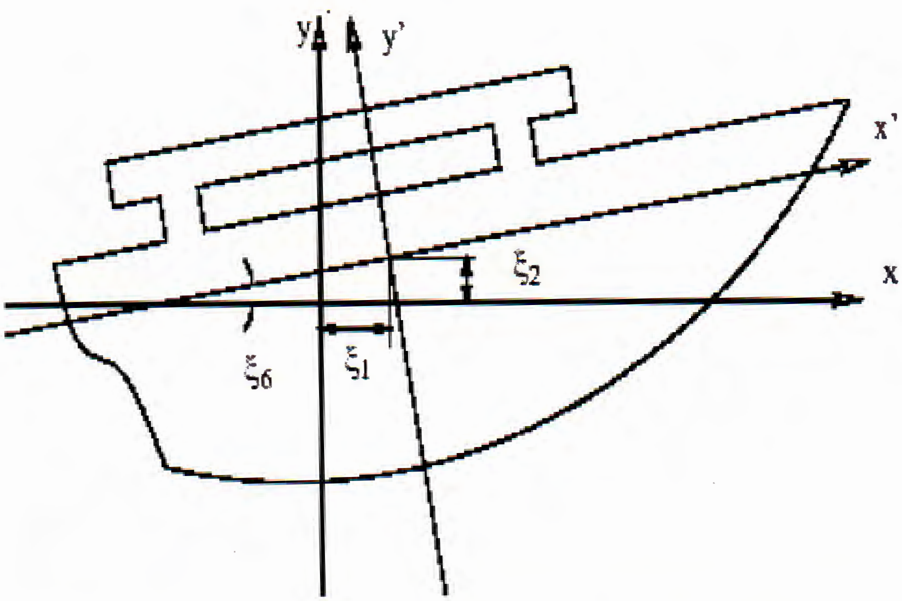

Fig. 8. Two-dimensional sketch showing 2 coordinate systems

By the direction of IFR as shown in figure 8 the hydrostatic force and torque (3.8), (3.9) can be expressed in the forms (see Newman 1985):

$$F_y = \rho g[V - \xi_2 S + \xi_4 S_2 - \xi_6 S_1],$$
$$M_x = -\rho g[V(z_B + \xi_4 y_B - \xi_5 x_B) - \xi_2 S_3 + \xi_4 S_{33} - \xi_6 S_{13}],$$
$$M_z = \rho g[V(x_B + \xi_5 z_B - \xi_6 y_B) - \xi_2 S_1 + \xi_4 S_{13} - \xi_6 S_{11}].$$

Here V is the displaced volume, $\mathfrak{r}_B$ - the center of buoyancy

$$\mathfrak{r}_B = \frac{1}{V} \iiint\limits_{\mathfrak{U}_0} \mathfrak{r} \mathrm{d}\mathfrak{U}, \tag{3.10}$$

S_j - the water-plane area and S_{ij} - the water-plane torques are defined as

$$S_j = \iint\limits_{S_0} x_j \mathrm{d}S \quad j = 1, 3,$$

$$S_{ij} = \iint\limits_{S_0} x_i x_j \mathrm{d}S \quad i, j = 1, 3,$$

where $\mathfrak{U}_0$ and S_0 denote the displaced volume and water-plane in the static condition. Although our hull boat does not have a simple structure in geometry as well as in material, using the result of C. D. Wolf and A. Keil 1989 we can easily calculate the center of buoyancy (3.10). Figure 9, figure 10 describe the hydrostatic force and torque to boat.

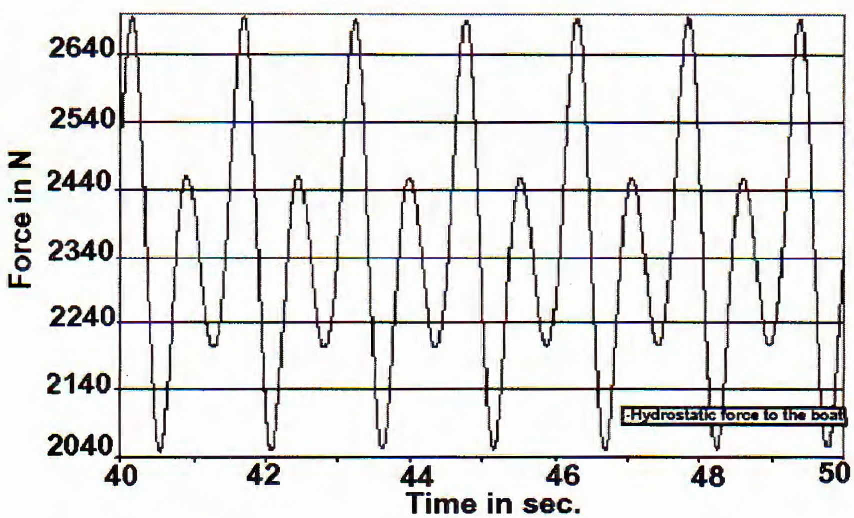

Fig. 9. Hydrostatic force to the boat by y direction

4. Some results in validation of the velocities of the boat

By experiments we have done several measurements with respect to the velocities of the boat with different models of fins under some excitation frequencies of the person. In this part we introduce one of our results in comparison with measurement. Figure 11 shows the velocity of the boat. From this figure we can see that after 50 seconds the behavior and the mean value of velocities in calculations and measurements are similar.

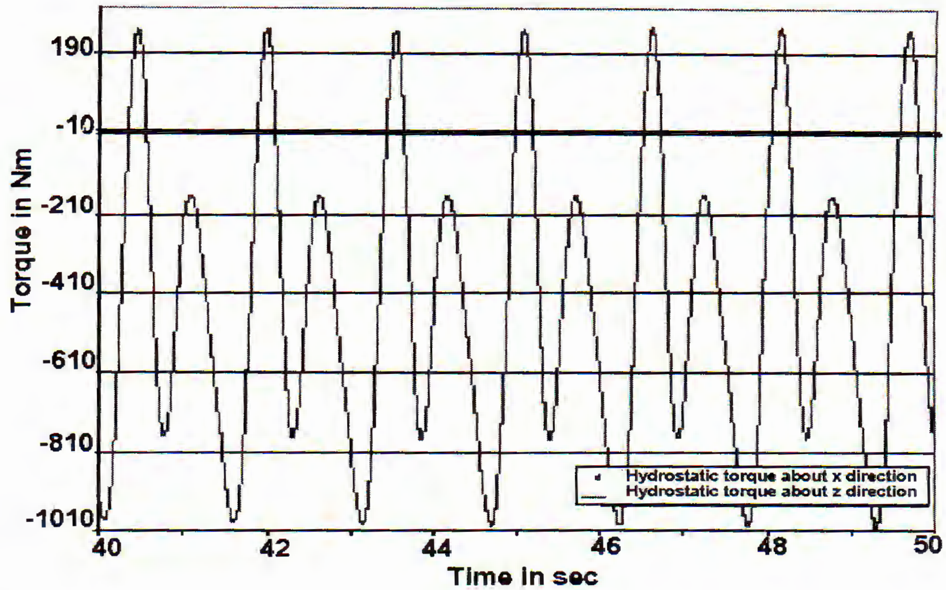

Fig. 10. Hydrostatic torque to the boat about x, z direction

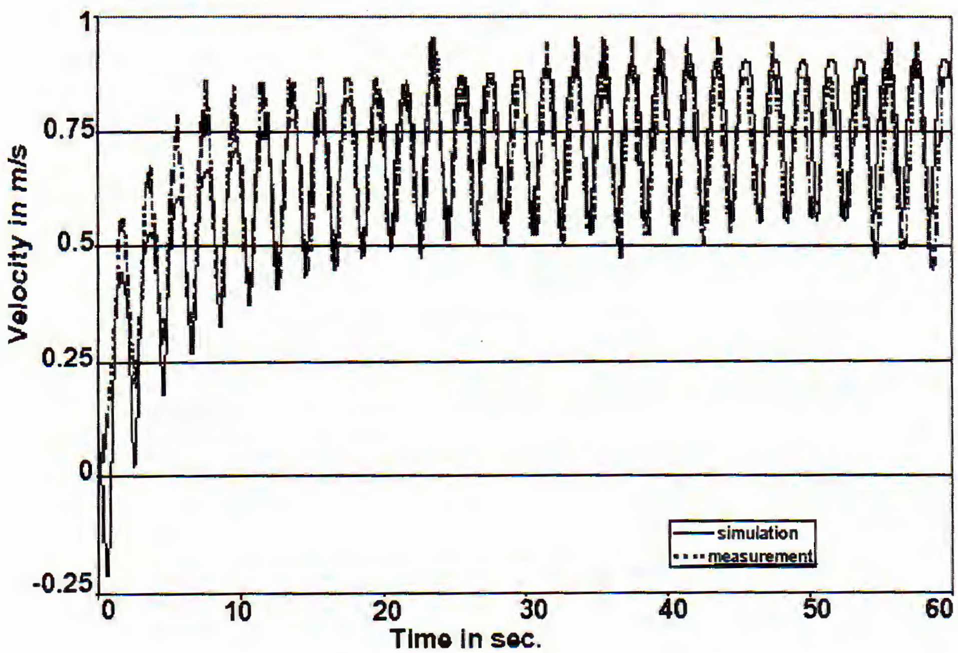

Fig. 11. Boat velocity

5. Conclusion

The problem to describe the motions of the driven fin boat running on a lake is difficult because of many influences acting on the boat. We have done some applications of hydrodynamic theories to get the water forces to the boat, the fins and we have some good comparison results between measurements and calculations. For getting a movement of the boat like in reality we need some damping parameters to the forces putting to the center of mass of this boat. In the next paper using the theories of Newman 1985 and Timman, Hermans and Hsiao 1985 we shall calculate the damping and added forces to the boat during the running time.

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ĐỘNG LỰC HỌC CỦA TÀU ĐẨY BẰNG TẤM THĂNG BẰNG- MÔ HÌNH VÀ MÔ PHỎNG

Trong bài báo này sự chuyển động của tàu thể thao nhỏ chạy trên hồ được mô phỏng bằng cách sử dụng công cụ Alaska khi lực và mô men ngoài từ nước và lực kích động trong suốt thời gian chuyển động được tính đến. Các kết quả tính được so sánh với kết quả đo.