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APPROXIMATED ANALYTICAL FREQUENCY RESPONSE OF A PENDULUM STRUCTURE ATTACHED WITH TWO ORTHOGONAL DYNAMIC VIBRATION ABSORBERS

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Abstract. Vibration of a pendulum can be reduced by single dynamic vibration absorbers (DVAs) moving in tangential or in normal direction of pendulum's orbit. The first DVA works in the linear zone while the second one works in the nonlinear zone. This paper uses the equivalent linearization technique to obtain an analytical form of the frequency response of a nonlinear pendulum structure, which is attached with both types of DVA. The numerical calculations are carried out to verify the analytical results. Some useful conclusions on the optimal parameters of the DVA can be found from the analytical solution.

Keywords: Dynamic vibration absorber, pendulum structures, Coriolis force, equivalent linearization, frequency response.

1. INTRODUCTION

Dynamic vibration absorber (DVA), which consists of a moving mass attached to the main structure through springs and dampers, is a well-known device to suppress vibration. The theory of linear DVA is well-developed in literature [1]. Beside the traditional sliding mass type, the pendulum types of DVA configuration are also investigated [2, 3]. Though there are some studies on the pendulum type of DVA, the primary structure is often modeled as a spring-mass system. In practice, the pendulum type structures also have high interest in research and engineering application. For example, some types of structures such as ropeway gondola, crane loads or floating structures (ships, tension leg platform) should be described by pendulum models. Using DVA was a mean for reducing the swing of pendulum structures [4, 5]. However, the effect of DVA on a pendulum structure can be quite different from that on a spring-mass structure. Especially, the effects of DVA's locations and pendulum's nonlinearity are important. Two types of installation of DVA in a pendulum structure are shown in Fig. 1a, b.

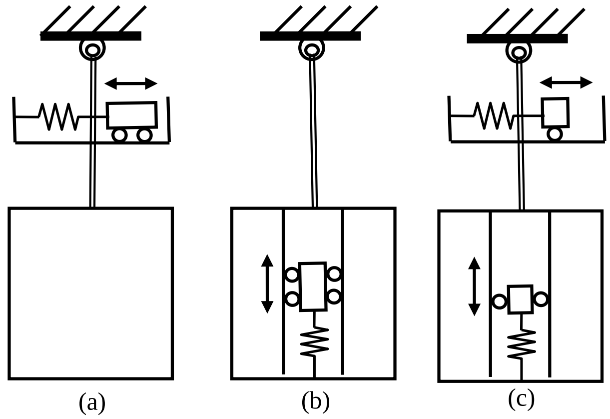


Fig. 1. Some types of DVA attachments in a pendulum structure
(a) Single DVA, tangential direction, (b) Single DVA, normal direction,
(c) Two DVAs, two directions

The first one moves in the tangential direction of the pendulum's orbit as shown in Fig. 1a [5]. The more general study on this type of DVA installed in the inverted pendulum type systems were also presented in [6]. This type of DVA works in the linear zone but it still has a disadvantage. If the DVA in Fig. 1a is located at the center of pendulum oscillation (the location of large motion), it has no effect at all. This phenomenon is very surprising but is proved in some previous works [5, 7]. Therefore, the DVA in Fig. 1a should be located as high as possible, which is not easy to satisfy in some practical situations. In the opposite way, the DVA in Fig. 1b moving in the radial direction of the pendulum's orbit still has good effect when it locates at the center of oscillation but it only works in the nonlinear zone [8, 9]. In this paper, we study the combination of two DVAs in Fig. 1c. While the system is nonlinear, the novelty of this paper comes from the analytical analysis of the pendulum attached with two DVAs as shown in Fig. 1c.

2. EQUATIONS OF MOTION

In Fig. 2, let's consider a pendulum structure having a concentrated mass m and a pendulum length l. Denote c as the structural damping coefficient, θ is the rotational angle of the pendulum and g is the acceleration of gravity. The notations u and v respectively are the displacements in normal and tangential directions of two DVAs, l_u and l_v respectively are the distances between the fulcrum and two DVAs in the static condition, m_u and m_v respectively are the masses of two DVAs, k_u and k_v respectively are the spring constants of two DVAs, c_u and c_v respectively are the damping coefficients of two DVAs.

Consider the coordinate system as shown in Fig. 2, the positions of the structure (x, y), the DVA moving in tangential direction (x_v, y_v) and the DVA moving in normal direction (x_u, y_u) are obtained as follows

$$\begin{gathered} x = l\sin\theta, y = l\cos\theta, \\ x_v = l_v\sin\theta + v\cos\theta, y_v = l_v\cos\theta - v\sin\theta, \\ x_u = (l_u - u)\sin\theta, y_u = (l_u - u)\cos\theta. \end{gathered} \tag{1}$$

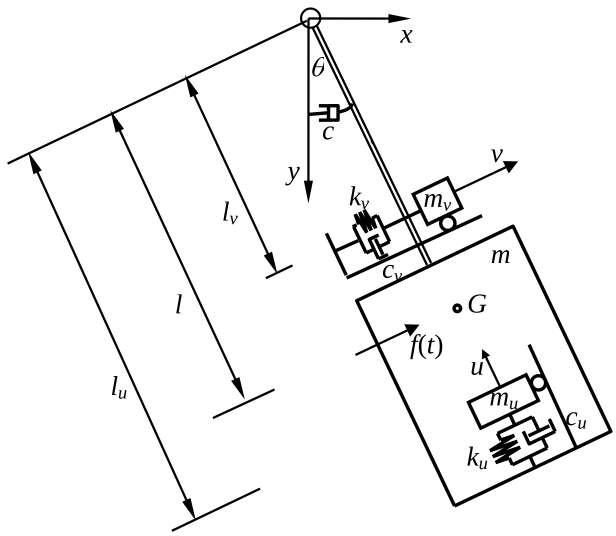


Fig. 2. Pendulum attached with two orthogonal DVAs

The kinetic energy T, the potential energy V and the energy dissipation function F are

$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}m_v\left(\dot{x}_v^2 + \dot{y}_v^2\right) + \frac{1}{2}m_u\left(\dot{x}_u^2 + \dot{y}_u^2\right), \tag{2}$$

$$V = mg\left(l - y\right) + m_v g\left(l_v - y_v\right) + m_u g\left(l_u - u - y_u\right) + \frac{1}{2}k_u u^2 + \frac{1}{2}k_u v^2, \tag{3}$$

$$F = \frac{1}{2}c\dot{\theta}^2 + \frac{1}{2}c_u\dot{u}^2 + \frac{1}{2}c_v\dot{v}^2. \tag{4}$$

Assume that the pendulum is subjected to a single harmonic external force $P\cos(\omega t)$, in which ω is the excitation frequency and P is the excitation magnitude. The Lagrange equations are given by

$$\left\{\begin{array}{l} \dfrac{d}{dt}\left(\dfrac{\partial\left(T-V\right)}{\partial\dot{\theta}}\right) - \dfrac{\partial\left(T-V\right)}{\partial\theta} + \dfrac{\partial F}{\partial\dot{\theta}} = lP\cos\left(\omega t\right) \\ \dfrac{d}{dt}\left(\dfrac{\partial\left(T-V\right)}{\partial\dot{v}}\right) - \dfrac{\partial\left(T-V\right)}{\partial v} + \dfrac{\partial F}{\partial\dot{v}} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial\left(T-V\right)}{\partial\dot{u}}\right) - \dfrac{\partial\left(T-V\right)}{\partial u} + \dfrac{\partial F}{\partial\dot{u}} = 0 \end{array}\right. \tag{5}$$

Using Eqs. (1), (2), (3), (4) and (5) gives

$$\begin{array}{l} \left(ml^2 + m_u\left(l_u - u\right)^2 + m_v\left(l_v^2 + v^2\right)\right)\ddot{\theta} + c\dot{\theta} + g\left(ml + m_u\left(l_u - u\right) + m_v l_v\right)\sin\theta + \\ \qquad + m_v l_v\ddot{v} + m_v g v\cos\theta + 2m_v v\dot{v}\dot{\theta} - 2lm_u\dot{\theta}\left(l_u - u\right)\dot{u} = lP\cos\left(\omega t\right), \\ m_v\ddot{v} + m_v l_v\ddot{\theta} - m_v\dot{\theta}^2 v + m_v g\sin\theta + c_v\dot{v} + k_v v = 0, \\ m_u\ddot{u} + m_u\dot{\theta}^2\left(l_u - u\right) - m_u g\left(1 - \cos\theta\right) + c_u\dot{u} + k_u u = 0. \end{array} \tag{6}$$

The following parameters are introduced

$$\begin{gathered}\mu_v=\frac{m_v}{m},\mu_u=\frac{m_u}{m},\omega_s=\sqrt{\frac{g}{l}},\zeta_s=\frac{c}{2l^2m\omega_s},\\ \alpha_u=\frac{k_u/m_u}{\omega_s^2},\zeta_u=\frac{c_u}{2m_u\omega_s},\alpha_v=\frac{k_v/m_v}{\omega_s^2},\zeta_v=\frac{c_v}{2m_v\omega_s},\\ \gamma_v=\frac{l_v}{l},\gamma_u=\frac{l_u}{l},z_u=\frac{u}{l},z_v=\frac{v}{l},\tau=\omega_s t,\beta=\frac{\omega^2}{\omega_s^2},\theta_m=\frac{P}{2\zeta_s\omega_s^2 ml}.\end{gathered}\tag{7}$$

In which, μ is the mass ratio, ω_s and ζ_s respectively are the natural frequency and the structural damping ratio of the pendulum, α_u and α_v respectively are the square of natural frequency ratios of two DVAs in normal and tangential directions, ζ_u and ζ_v respectively are the damping ratios of two DVAs in normal and tangential directions, γ_u and γ_v respectively are the location parameters specifying the positions of two DVAs, z_u and z_v respectively are the non-dimensional forms of the displacements of two DVAs in two directions, τ is the non-dimensional time with time scale ω_s^{-1}, β is the square of the ratio between excitation frequency and main structure frequency, θ_m is the maximum vibration angle at the resonance frequency. The motion equations (6) are simplified and rearranged as following non-dimensional form

$$\begin{aligned}&\left(1+\mu_u\left(\gamma_u-z_u\right)^2+\mu_v\left(\gamma_v^2+z_v^2\right)\right)\ddot{\theta}+2\zeta_s\dot{\theta}+\left(1+\mu_u\left(\gamma_u-z_u\right)+\mu_v\gamma_v\right)\sin\theta+\\ &\qquad+\mu_v\gamma_v\ddot{z}_v+\mu_v z_v\cos\theta+2\mu_v z_v\dot{z}_v\dot{\theta}-2\mu_u\dot{\theta}\left(\gamma_u-z_u\right)\dot{z}_u=2\zeta_s\theta_m\cos\left(\sqrt{\beta}\tau\right),\\ &\ddot{z}_v+2\zeta_v\dot{z}_v+\alpha_v z_v+\gamma_v\ddot{\theta}-\dot{\theta}^2 z_v+\sin\theta=0,\\ &\ddot{z}_u+2\zeta_u\dot{z}_u+\alpha_u z_u+\dot{\theta}^2\left(\gamma_u-z_u\right)-\left(1-\cos\theta\right)=0,\end{aligned}\tag{8}$$

in which the dot operator from now denotes the differentiation with respect to normalized time τ. The full nonlinear equations (8) are used in the numerical calculations in section 3.

3. ANALYTICAL FORM OF FREQUENCY RESPONSE CURVE

3.1. Equations in polynomial form

Because the system is nonlinear, some appropriate simplifications should be done to obtain the analytical form of the frequency response. Let us assume that the DVA normalized displacement z_u is small enough in comparison with the DVA location parameter γ_u and the DVA mass ratio μ_u is small enough in comparison with 1. The following approximations are obtained

$$1+\mu_u\left(\gamma_u-z_u\right)^2\approx 1+\mu_u\gamma_u^2,\quad 1+\mu_u\left(\gamma_u-z_u\right)\approx 1+\mu_u\gamma_u\ .\tag{9}$$

After that, the trigonometric terms are expanded by Taylor series, and then the Eq. (8) is rewritten as

$$\begin{aligned}&\left(1+\mu_u\gamma_u^2+\mu_v\gamma_v^2\right)\ddot{\theta}+2\zeta_s\dot{\theta}+\left(1+\mu_u\gamma_u+\mu_v\gamma_v\right)\theta\\ &\qquad+\mu_v\left(\gamma_v\ddot{z}_v+z_v\right)-2\gamma_u\mu_u\dot{\theta}\dot{z}_u+\text{h.o.t}=2\zeta_s\theta_m\cos\left(\sqrt{\beta}\tau\right),\\ &\ddot{z}_v+2\zeta_v\dot{z}_v+\alpha_v z_v+\gamma_v\ddot{\theta}+\theta+\text{h.o.t}=0,\\ &\ddot{z}_u+2\zeta_u\dot{z}_u+\alpha_u z_u+\gamma_u\dot{\theta}^2-\theta^2/2+\text{h.o.t}=0,\end{aligned}\tag{10}$$

in which "h.o.t" denotes the higher order terms, which have order larger than 2. If the higher order terms "h.o.t" are assumed to be ignored, the equations in second order approximation become

$$\begin{aligned}
&\left(1+\mu_u\gamma_u^2+\mu_v\gamma_v^2\right)\ddot{\theta}+2\zeta_s\dot{\theta}+\left(1+\mu_u\gamma_u+\mu_v\gamma_v\right)\theta+\\
&\quad+\mu_v\left(\gamma_v\ddot{z}_v+z_v\right)-2\gamma_u\mu_u\dot{\theta}\dot{z}_u=2\zeta_s\theta_m\cos\left(\sqrt{\beta}\tau\right),\\
&\ddot{z}_v+2\zeta_v\dot{z}_v+\alpha_v z_v+\gamma_v\ddot{\theta}+\theta=0,\\
&\ddot{z}_u+2\zeta_u\dot{z}_u+\alpha_u z_u+\gamma_u\dot{\theta}^2-\theta^2/2=0.
\end{aligned} \tag{11}$$

3.2. Frequency response

To obtain the analytical form of the frequency response, the equivalent linearization approach is used to approximate the Coriolis term $-2\gamma_u\mu_u\dot{\theta}\dot{z}_u$ in the first equation of (11). The Coriolis term contains both velocities of the pendulum and the DVA in normal direction. The Coriolis term can produce the damping effects (direct and cross terms) as well as stiffness effect (direct and cross terms), i.e. the general linearization should be

$$-2\gamma_u\mu_u\dot{\theta}\dot{z}_u\approx a_1\theta+a_2\dot{\theta}+a_3z_u+a_4\dot{z}_u+a_5z_v+a_6\dot{z}_v\ ,$$

where $a_i(i=1,...,6)$ are the linearization coefficients. However, this complete linearization can not result in the approximated analytical solution. Instead of that, we only consider the most important effect of the Coriolis term, namely the effect of direct damping to the pendulum. With this assumption, the Coriolis term is replaced by the linear effective damping as

$$-2\gamma_u\mu_u\dot{\theta}\dot{z}_u\approx 2\zeta_e\dot{\theta}, \tag{12}$$

in which ζ_e is the effective damping ratio, which is chosen to minimize the following error

$$E=\left\langle\left(2\gamma_u\mu_u\dot{\theta}\dot{z}_u+\zeta_e\dot{\theta}\right)^2\right\rangle, \tag{13}$$

in which $<>$ is the time average in one vibrational cycle. Setting the derivative of E with respect to ζ_e equal to zero gives

$$\zeta_e=\frac{-\mu_u\gamma_u\left\langle\dot{\theta}^2\dot{z}_u\right\rangle}{\left\langle\dot{\theta}^2\right\rangle}=\frac{-\mu_u\gamma_u\int\limits_0^{2\pi}\dot{\theta}^2\dot{z}_u d\tau}{\int\limits_0^{2\pi}\dot{\theta}^2 d\tau}. \tag{14}$$

Substituting (12) into (11) gives

$$\begin{aligned}
&\left(1+\mu_u\gamma_u^2+\mu_v\gamma_v^2\right)\ddot{\theta}+2\left(\zeta_s+\zeta_e\right)\dot{\theta}+\left(1+\mu_u\gamma_u+\mu_v\gamma_v\right)\theta+\\
&\quad+\mu_v\left(\gamma_v\ddot{z}_v+z_v\right)=2\zeta_s\theta_m\cos\left(\sqrt{\beta}\tau\right),\\
&\ddot{z}_v+2\zeta_v\dot{z}_v+\alpha_v z_v+\gamma_v\ddot{\theta}+\theta=0,\\
&\ddot{z}_u+2\zeta_u\dot{z}_u+\alpha_u z_u+\gamma_u\dot{\theta}^2-\theta^2/2=0.
\end{aligned} \tag{15}$$

We will show that the system of Eqs. (15) has stationary solutions in analytical forms. Indeed, let the stationary solutions express as

$$
\begin{aligned}
\theta &= \frac{(\theta_r + \mathrm{i}\theta_i)\,\mathrm{e}^{\mathrm{i}\sqrt{\beta}\tau} + (\theta_r - \mathrm{i}\theta_i)\,\mathrm{e}^{-\mathrm{i}\sqrt{\beta}\tau}}{2},\\
z_v &= \frac{(z_{vr} + \mathrm{i}z_{vi})\,\mathrm{e}^{\mathrm{i}\sqrt{\beta}\tau} + (z_{vr} - \mathrm{i}z_{vi})\,\mathrm{e}^{-\mathrm{i}\sqrt{\beta}\tau}}{2},\\
z_u &= \frac{(z_{ur} + \mathrm{i}z_{ui})\,\mathrm{e}^{2\mathrm{i}\sqrt{\beta}\tau} + (z_{ur} - \mathrm{i}z_{ui})\,\mathrm{e}^{-2\mathrm{i}\sqrt{\beta}\tau}}{2} + z_{st}\ ,
\end{aligned}
\tag{16}
$$

in which "i" denotes the imaginary unit and "e" denotes the exponential function, θ_r, z_{vr}, z_{ur} are the real parts and θ_i, z_{vi}, z_{ui} are the imaginary parts of the complex amplitudes, z_{st} is the constant displacement of the DVA moving in normal direction. Substituting (16) into (15), equating the coefficients of $\mathrm{e}^{\mathrm{i}\sqrt{\beta}\tau}$ and $\mathrm{e}^{-\mathrm{i}\sqrt{\beta}\tau}$ yields

$$
\begin{aligned}
&\left(1+\mu_v\gamma_v+\mu_u\gamma_u-\left(\mu_u\gamma_u^2+\mu_v\gamma_v^2+1\right)\beta\right)\theta_r-2\left(\zeta_s+\zeta_e\right)\sqrt{\beta}\theta_i+\mu_v\left(1-\gamma_v\beta\right)z_{vr}=2\zeta_s\theta_m\,,\\
&\left(1+\mu_v\gamma_v+\mu_u\gamma_u-\left(\mu_u\gamma_u^2+\mu_v\gamma_v^2+1\right)\beta\right)\theta_i+2\left(\zeta_s+\zeta_e\right)\sqrt{\beta}\theta_r+\mu_v\left(1-\gamma_v\beta\right)z_{vi}=0\,,\\
&(1-\gamma_v\beta)\,\theta_r + (\alpha_v-\beta)\,z_{vr} - 2\zeta_v\sqrt{\beta}z_{vi} = 0,\\
&(1-\gamma_v\beta)\,\theta_i + (\alpha_v-\beta)\,z_{vi} + 2\zeta_v\sqrt{\beta}z_{vr} = 0,\\
&(\alpha_u-4\beta)\,z_{ur} - 4\zeta_u\sqrt{\beta}z_{ui} = -\frac{1}{2}\left(\gamma_u\beta+\frac{1}{2}\right)\left(\theta_i^2-\theta_r^2\right),\\
&(\alpha_u-4\beta)\,z_{ui} + 4\zeta_u\sqrt{\beta}z_{ur} = \left(\gamma_u\beta+\frac{1}{2}\right)\theta_r\theta_i\,.
\end{aligned}
\tag{17}
$$

By using the identities

$$
\int_0^{2\pi}\mathrm{e}^{2\mathrm{i}\sqrt{\beta}\tau}d\tau = \int_0^{2\pi}\mathrm{e}^{-2\mathrm{i}\sqrt{\beta}\tau}d\tau = \int_0^{2\pi}\mathrm{e}^{4\mathrm{i}\sqrt{\beta}\tau}d\tau = \int_0^{2\pi}\mathrm{e}^{-4\mathrm{i}\sqrt{\beta}\tau}d\tau = 0,
$$

the effective damping ζ_e in (14) reduces to

$$
\zeta_e = \sqrt{\beta}\mu_u\gamma_u\frac{2\theta_r\theta_i z_{ur} + \theta_i^2 z_{ui} - \theta_r^2 z_{ui}}{\theta_r^2+\theta_i^2}. \tag{18}
$$

Eliminating z_{vr}, z_{ur}, z_{vi}, z_{ui} from (17) and using (18) give

$$
\begin{aligned}
&\left(1+\mu_v\gamma_v+\mu_u\gamma_u-\left(\mu_u\gamma_u^2+\mu_v\gamma_v^2+1\right)\beta-\frac{\mu_v\left(1-\gamma_v\beta\right)^2\left(\alpha_v-\beta\right)}{\left(\alpha_v-\beta\right)^2+4\zeta_v^2\beta}\right)^2 q+\\
&\quad+4\beta\left(\frac{\beta\mu_u\gamma_u\zeta_u\left(1+2\gamma_u\beta\right)}{\left(\alpha_u-4\beta\right)^2+16\zeta_u^2\beta}q+\zeta_s+\frac{\mu_v\left(1-\gamma_v\beta\right)^2\zeta_v}{\left(\alpha_v-\beta\right)^2+4\zeta_v^2\beta}\right)^2 q = 4\zeta_s^2\theta_m^2\,,
\end{aligned}
\tag{19}
$$

in which q is the square of vibration angle amplitude defined by

$$
q = \theta_r^2 + \theta_i^2. \tag{20}
$$

In brief, the analytical form of the frequency response is represented by the cubic equation (19) with respect to the square of vibration angle amplitude q. In the numerical simulations below, the cubic equation has only a unique real root. In general the equation can have multiple real roots. In this case, the stability analysis should be undertaken. However, the stability analysis is much complex, beyond the scope of this paper and needs to be done in a separate technical note or short communication.

The analytical form of the frequency response (19) under single harmonic excitation is quite simple, and can be used to speed up the process to find the optimal DVA's parameters. From (19), we can draw some preliminary observations as:

- To reduce q, the denominators in (19) should be small. Because the large vibration occurs near the resonance frequency, we have $\beta \approx 1$. Therefore, the optimal values of α_v and α_u should be near 1 and 4 respectively.

- Because the damping ratios ζ_v and ζ_u appear in both numerator and denominator, their values should not be too large or small.

- When $\gamma_v \approx 1$, the term $1 - \gamma_v \beta$ is very small and the DVA moving in the tangential direction (Fig. 1a) has very little effect. This is location problem discussed in the introduction section.

- The term containing parameters α_u and ζ_u is multiplied with q, which means the DVA moving in normal direction (Fig. 1b) only works in the nonlinear zone

3.3. Numerical verifications

In the numerical calculation, the results are obtained by solving the original differential equations (8). The total time of each simulation is 500 s. In the analytical approximation, the results are obtained by solving the cubic equation (19). Some cases of system parameters used in simulation are summarized in Tab. 1. In all cases, the structural damping ratio ζ_s is taken as 1%, the mass ratios $\mu_u = \mu_v = 0.05$ and the location parameters $\gamma_u = 1.3, \gamma_v = 0.8$.

The pendulum vibration amplitudes are plotted versus the frequency of single harmonic excitation. The comparisons are showed in Figs. 3-7.

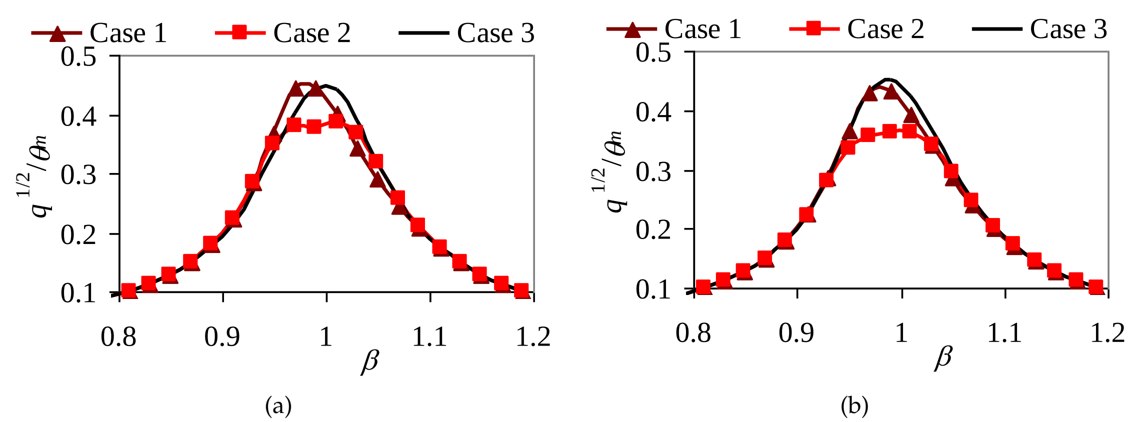
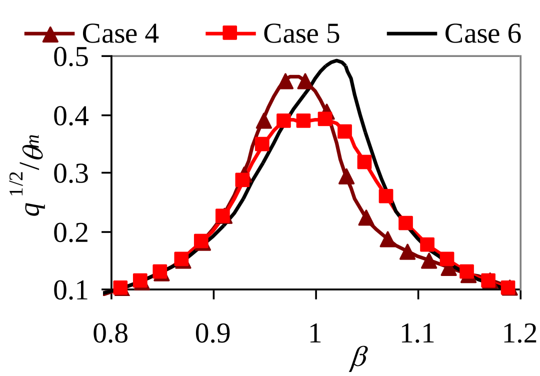


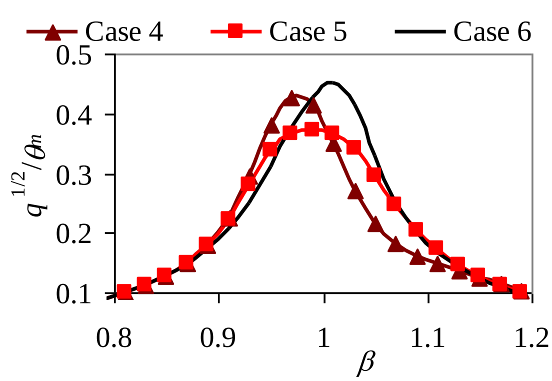
Fig. 3. Frequency response when α_u changes (cases are mentioned in Tab. 1)
(a) Simulated response, (b) Analytical response

Table 1. Cases used in numerical simulations

Parameter changed	Case	α_u	α_v	ζ_u	ζ_v	θ_m	$\max\limits_{\beta} \sqrt{q} \Big/ \theta_m$ (Simulated)	$\max\limits_{\beta} \sqrt{q} \Big/ \theta_m$ (Analytical)
α_u	1	3.6	1	0.05	0.05	25°	0.4445	0.4317
	2	4	1	0.05	0.05	25°	0.3848	0.3617
	3	4.5	1	0.05	0.05	25°	0.4425	0.4525
α_v	4	4	0.9	0.06	0.05	25°	0.4571	0.4258
	5	4	1	0.06	0.05	25°	0.3885	0.3717
	6	4	1.1	0.06	0.05	25°	0.4819	0.4499
ζ_u	7	4	1	0.005	0.04	25°	0.4328	0.3791
	8	4	1	0.05	0.04	25°	0.3616	0.3451
	9	4	1	0.5	0.04	25°	0.4072	0.4109
ζ_v	10	4	1	0.04	0.02	25°	0.4139	0.3761
	11	4	1	0.04	0.04	25°	0.3642	0.3424
	12	4	1	0.04	0.1	25°	0.4481	0.4
θ_m	13	4	1	0.04	0.04	15°	0.3905	0.3767
	14	4	1	0.04	0.04	25°	0.3642	0.3424
	15	4	1	0.04	0.04	35°	0.3459	0.3218



(a)

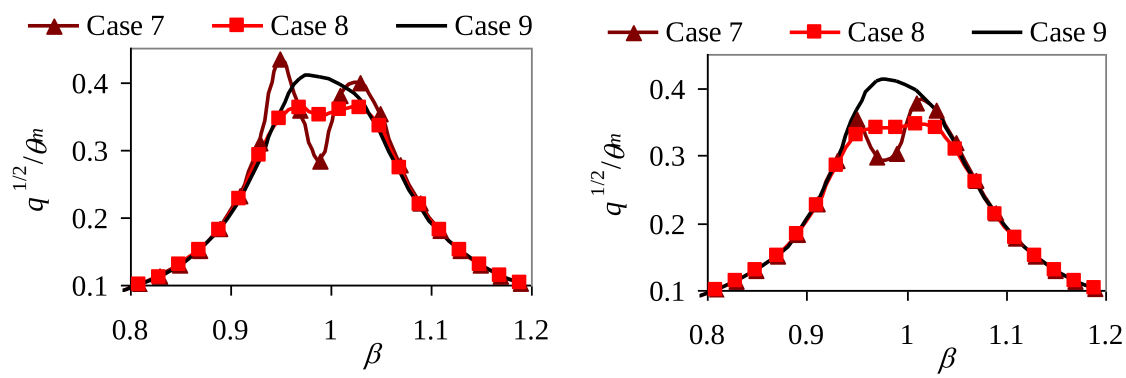


(b)

Fig. 4. Frequency response when α_v changes (cases are mentioned in Tab. 1), labels as Fig. 3

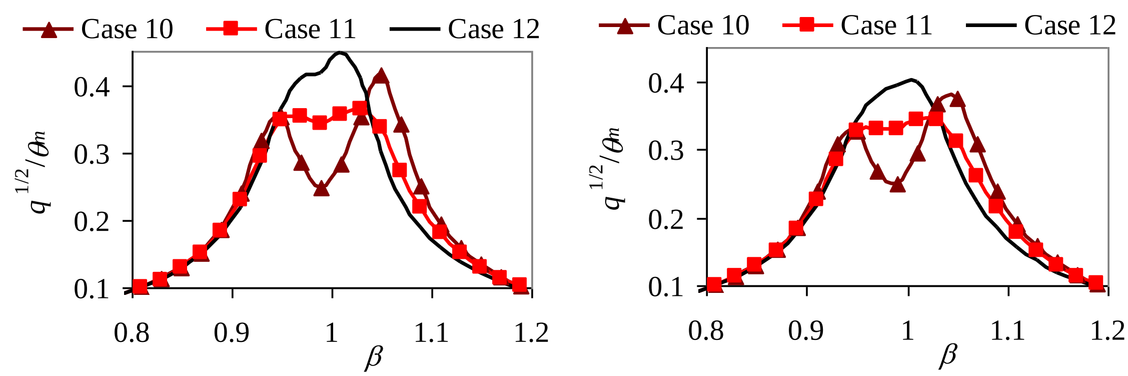
Some remarks are drawn from the comparisons:

- The errors between the numerical and analytical calculations are generally acceptable. In some cases (cases 7, 10, 12), the errors can be larger because of the effect of nonlinearity. It can be explained as follows. In cases 7, 10 or 12, too small values of the damping ratios ζ_u, ζ_v or too large value of the damping ratio ζ_v make the larger value



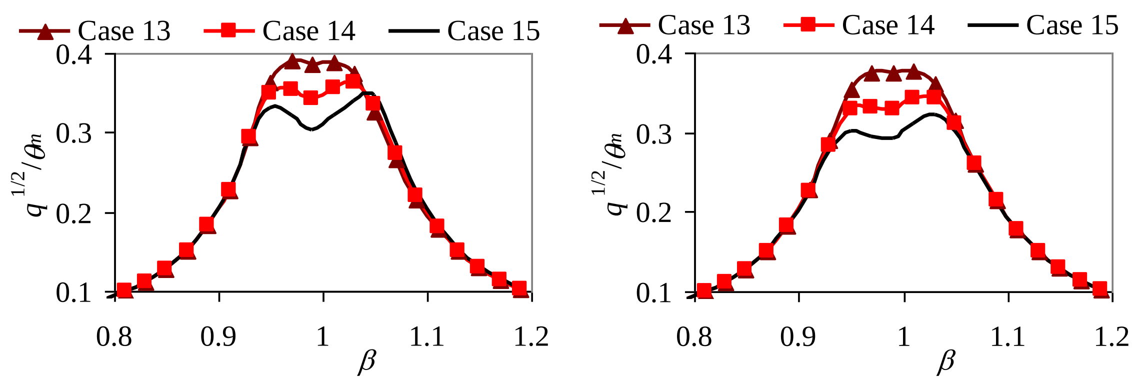
(a) (b)

Fig. 5. Frequency response when ζ_u changes (cases are mentioned in Tab. 1), labels as Fig. 3



(a) (b)

Fig. 6. Frequency response when ζ_v changes (cases are mentioned in Tab. 1), labels as Fig. 3



(a) (b)

Fig. 7. Frequency response when θ_m changes (cases are mentioned in Tab. 1), labels as Fig. 3

of $\dot{z}_u$ to absorb the vibrational energy. The larger value of velocity $\dot{z}_u$ can decrease the accuracy of the effective damping (12).

- The DVA's damping ratios ζ_u and ζ_v should be not too large or too small (Figs. 5 and 6).

- The optimal value of α_v is around 1 (Fig. 3) while the optimal value of α_u is around 4 (Fig. 4)

- The DVA effectiveness increases when the excitation increases (Fig. 7). This effect is the main difference between the linear and nonlinear systems.

4. CONCLUSION

This paper considers the vibration control problem of a pendulum attached with two orthogonal dynamic vibration absorbers (DVAs). The equivalent linearization technique is used to obtain the analytical form of the frequency response. Some fundamental natures of two DVAs are seen from the analytical solution. The optimal frequency ratios in tangential and normal directions should be near 1 and 4 respectively. The damping ratios should not be too large or small. The DVA in tangential direction meets a location problem while the DVA in normal direction only works in the nonlinear zone. All the aforementioned natures are verified by numerical simulations of harmonic vibration of a pendulum.

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