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DYNAMICAL ANALYSIS OF MULTILAYERED REINFORCED COMPOSITE CYLINDRICAL SHELLS

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Abstract. In the present paper the governing dynamical equations for multilayered reinforced composite cylindrical shells based on Kirchhoff-Love's theory and Lekhnitsky's smeared stiffeners technique are derived. The shell is reinforced by longitudinal and ring stiffeners. The longitudinal stiffeners may be composite or sleeves with SMA wire. The linear problem of shell vibration is considered for illustrating the effects of the stiffeners, the shell geometry and altering the lamination scheme on fundamental frequencies of the shell.

1. INTRODUCTION

Unreinforced composite cylindrical shells has been analysed by analytical solution procedure or finite element technique [1-3, 7, 9, 10, 12-16]. Many structures in the form of cylindrical composite shells reinforced by stiffeners are used in the aircraft industry, flight objects construction, ship industry and civil engineering... Static and buckling analyses of these structures have been investigated in [4, 6, 11]. Dynamical analysis of them plays an important role in the anti-vibration practice.

In this paper the motion equations of reinforced composite cylindrical shells are developed based on the Kirchhoff-Love's theory and the Lekhnitsky's smeared stiffeners technique. The quantity analysis of the natural vibration problem of shells allows to discover the influence of stiffeners, the shell geometry and altering the lamination scheme on fundamental frequencies of the shells.

2. GOVERNING DYNAMICAL EQUATIONS

Consider a symmetrically laminated multilayered composite cylindrical shell. The shell is reinforced by longitudinal and ring composite stiffeners. The longitudinal stiffeners may be sleeves with SMA wire and the sleeves are bonded on the shell surface. The wire is not bonded to the sleeves, so it may slide freely along the stiffener. However the wire is embedded within the sleeves, so that it participates in bending of the stiffener and the shell.

According to the Kirchhoff-Love's theory nonlinear strain-displacement relationships for a shell with a middle surface radius R are of the form:

$$\varepsilon_1^0 = \frac{\partial u}{\partial x_1} + \frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2,$$
$$\varepsilon_2^0 = \frac{\partial v}{\partial x_2} + \frac{1}{2}\left(\frac{\partial w}{\partial x_2}\right)^2 + \frac{w}{R},$$
$$\varepsilon_6^0 = \frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1} + \frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2},$$

$$\begin{aligned}
\kappa_1 &= -\frac{\partial^2 w}{\partial x_1^2}, \\
\kappa_2 &= -\frac{\partial^2 w}{\partial x_2^2} + \frac{1}{R}\frac{\partial v}{\partial x_2}, \\
\kappa_6 &= -2\frac{\partial^2 w}{\partial x_1 \partial x_2} + \frac{1}{R}\frac{\partial v}{\partial x_1},
\end{aligned} \tag{2.1}$$

where u, v, w denote axial, circumferential and radial displacements; x_1 and $x_2 = R\theta$ are the axial and circumferential coordinates. The strains in the middle surface and the changes of curvature and twist are denoted by ε_i^0 and κ_i $(i = 1, 2, 6)$, respectively.

For simplicity the constitutive stress-strain equations are written as

$$\{\overline{\sigma}\} = \boxed{D}\,\{\varepsilon\}, \tag{2.2}$$

where

$$\begin{aligned}
\{\overline{\sigma}\} &= \{N_1,\ N_2, N_6,\ M_1,\ M_2, M_6\}^T, \\
\{\varepsilon\} &= \left\{\varepsilon_1^0,\ \varepsilon_2^0, \varepsilon_6^0,\ \kappa_1,\ \kappa_2, \kappa_6\right\}^T, \\
\boxed{D} &= \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix},
\end{aligned}$$

in which

$$(A_{ij},\ B_{ij},\ D_{ij}) = \sum_{k=1}^{n_1} \int_{h_{k-1}}^{h_k} Q_{ij}^{(k)} \left(1,\ z,\ z^2\right) dz \quad (i,\ j = 1,\ 2,\ 6).$$

n_1 is the number of composite layers of the shell, A_{ij}, B_{ij}, D_{ij} are extensional, coupling and bending stiffnesses of the shell without stiffeners. Note that in a multilayered symmetrically laminated material the coupling stiffnesses B_{ij} are equal to zero and the extensional A_{16}, A_{26} and the bending stiffnesses D_{16}, D_{26} are negligible compared to the other stiffnesses.

According to the Lekhnitsky's smeared stiffeners technique when expanding internal forces-deformations (2.2) combined with (2.1) we obtain the expression for stress resultants and internal moments of multilayered reinforced composite cylindrical shells

$$\begin{aligned}
N_1 &= \left(A_{11} + \frac{EA_1}{s_1}\right)\varepsilon_1^0 + A_{12}\varepsilon_2^0 + \left(\frac{EA_1}{s_1}\right) z_1\kappa_1 + \frac{N^r}{s_1}, \\
N_2 &= A_{12}\varepsilon_1^0 + \left(A_{22} + \frac{EA_2}{s_2}\right)\varepsilon_2^0 + \left(\frac{EA_2}{s_2}\right) z_2\kappa_2, \\
N_6 &= A_{66}\varepsilon_6^0, \\
M_1 &= \left(D_{11} + \frac{EI_1}{s_1}\right)\kappa_1 + D_{12}\kappa_2 + \left(\frac{EA_1}{s_1}\right) z_1\varepsilon_1^0,
\end{aligned}$$

$$M_2 = D_{12}\kappa_1 + \left(D_{22} + \frac{EI_2}{s_2}\right)\kappa_2 + \left(\frac{EA_2}{s_2}\right) z_2\varepsilon_2^0,$$
$$M_6 = D_{66}\kappa_6, \tag{2.3}$$

where E is the effective modulus in the axial direction of the corresponding stiffener, s_1, s_2 are the spacings of the longitudinal and ring stiffeners respectively, I_1, I_2 are the inertia moments of stiffener cross section and z_1, z_2 are eccentricities of the stiffener with respect to the middle surface of the shell, the torsional stiffness of the stiffener is disregarded. The recovery tensile force in SMA wire is denoted by N^r, this force does not generate a bending moment, because the wire can move freely along the sleeves.

The motion equations of a cylindrical shell are

$$\frac{\partial N_1}{\partial x_1} + \frac{\partial N_6}{\partial x_2} = J_0\frac{\partial^2 u}{\partial t^2} - J_1\frac{\partial^3 w}{\partial x_1 \partial t^2},$$
$$\frac{\partial N_6}{\partial x_1} + \frac{\partial N_2}{\partial x_2} + \frac{1}{R}\left(\frac{\partial M_6}{\partial x_1} + \frac{\partial M_2}{\partial x_2}\right) = J_0\frac{\partial^2 v}{\partial t^2} - J_1\frac{\partial^3 w}{\partial x_2 \partial t^2},$$
$$\frac{\partial^2 M_1}{\partial x_1^2} + 2\frac{\partial^2 M_6}{\partial x_1 \partial x_2} + \frac{\partial^2 M_2}{\partial x_2^2} - \frac{N_2}{R} + \frac{\partial}{\partial x_1}\left(N_1\frac{\partial w}{\partial x_1} + N_6\frac{\partial w}{\partial x_2}\right) + \frac{\partial}{\partial x_2}\left(N_6\frac{\partial w}{\partial x_1} + N_2\frac{\partial w}{\partial x_2}\right) =$$
$$= J_0\frac{\partial^2 w}{\partial t^2} + J_1\left(\frac{\partial^3 u}{\partial x_1 \partial t^2} + \frac{\partial^3 v}{\partial x_2 \partial t^2}\right) - J_2\left(\frac{\partial^4 w}{\partial x_1^2 \partial t^2} + \frac{\partial^4 w}{\partial x_2^2 \partial t^2}\right) + q, \tag{2.4}$$

where

$$J_i = \sum_{k=1}^{n_1}\int_{h_{k-1}}^{h_k} \rho^{(k)} z^i dz + \Big(\sum_{k=1}^{n_2}\int_{h_{k-1}}^{h_k} \rho^{(k)} z^i dz\Big)\frac{d_1}{s_1} + \Big(\sum_{k=1}^{n_3}\int_{h_{k-1}}^{h_k} \rho^{(k)} z^i dz\Big)\frac{d_2}{s_2} \quad (i = 0,\ 1,\ 2),$$

n_2, n_3 are the number of composite layers of the longitudinal and ring stiffeners respectively, $\rho^{(k)}$ is the mass density of k-th composite layer.

Combining (2.1) and (2.3) then substituting to (2.4) we receive the system of three nonlinear partial differential equations with respect to displacements

$$L_{11}\big(u\big) + L_{12}\big(v\big) + L_{13}\big(w\big) + P_1\big(w\big) = J_0\frac{\partial^2 u}{\partial t^2} - J_1\frac{\partial^3 w}{\partial x_1 \partial t^2},$$
$$L_{12}\big(u\big) + L_{22}\big(v\big) + L_{23}\big(w\big) + P_2\big(w\big) = J_0\frac{\partial^2 v}{\partial t^2} - J_1\frac{\partial^3 w}{\partial x_2 \partial t^2},$$
$$L_{13}\big(u\big) + L_{23}\big(v\big) + L_{33}\big(w\big) + P_3\big(w\big) + Q_3\big(u, w\big) + R_3\big(v, w\big)$$
$$= -q - J_0\frac{\partial^2 w}{\partial t^2} - -J_1\Big(\frac{\partial^3 u}{\partial x_1 \partial t^2} + \frac{\partial^3 v}{\partial x_2 \partial t^2}\Big) + J_2\Big(\frac{\partial^4 w}{\partial x_1^2 \partial t^2} + \frac{\partial^4 w}{\partial x_2^2 \partial t^2}\Big), \tag{2.5}$$

where linear operators L_{ij} $(i, j = 1, 2, 3)$ have the form

$$L_{11} = \Big(A_{11} + \frac{EA_1}{s_1}\Big)\frac{\partial^2}{\partial x_1^2} + A_{66}\frac{\partial^2}{\partial x_2^2},$$
$$L_{12} = L_{21} = \Big(A_{12} + A_{66}\Big)\frac{\partial^2}{\partial x_1 \partial x_2},$$

$$L_{13} = L_{31} = \left(\frac{A_{12}}{R}\right)\frac{\partial}{\partial x_1} - \left(\frac{EA_1}{s_1}\right)z_1\frac{\partial^3}{\partial x_1^3},$$

$$L_{22} = \left(A_{66} + \frac{D_{66}}{R^2}\right)\frac{\partial^2}{\partial x_1^2} + \left[A_{22} + \frac{D_{22}}{R^2} + \frac{EA_2}{s_2}\left(1 + 2\frac{z_2}{R}\right) + \frac{EI_2}{s_2R^2}\right]\frac{\partial^2}{\partial x_2^2},$$

$$L_{23} = L_{32} = \left(\frac{1}{R}\right)\left[A_{22} + \frac{EA_2}{s_2}\left(1 + \frac{z_2}{R}\right)\right]\frac{\partial}{\partial x_2} - \left(\frac{1}{R}\right)\left(D_{22} + 2D_{66}\right)\frac{\partial^3}{\partial x_1^2\partial x_2} -$$
$$- \left(\frac{D_{22}}{R} + \frac{EA_2z_2}{s_2} + \frac{EI_2}{s_2R}\right)\frac{\partial^3}{\partial x_2^3},$$

$$L_{33} = \left(D_{11} + \frac{EI_1}{s_1}\right)\frac{\partial^4}{\partial x_1^4} + 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4}{\partial x_1^2\partial x_2^2} + \left(D_{22} + \frac{EI_2}{s_2}\right)\frac{\partial^4}{\partial x_2^4} -$$
$$- 2\frac{EA_2z_2}{s_2R}\frac{\partial^2}{\partial x_2^2} + \left(\frac{1}{R^2}\right)\left(A_{22} + \frac{EA_2}{s_2}\right) - N^r\frac{\partial^2}{\partial x_1^2} \tag{2.6}$$

and nonlinear functions $P_i (i = 1,\, 2,\, 3)$, Q_3, R_3

$$P_1(w) = \left(A_{11} + \frac{EA_1}{s_1}\right)\frac{\partial w}{\partial x_1}\frac{\partial^2 w}{\partial x_1^2} + (A_{12} + A_{66})\frac{\partial w}{\partial x_2}\frac{\partial^2 w}{\partial x_1\partial x_2} + A_{66}\frac{\partial w}{\partial x_1}\frac{\partial^2 w}{\partial x_2^2},$$

$$P_2(w) = (A_{12} + A_{66})\frac{\partial w}{\partial x_1}\frac{\partial^2 w}{\partial x_1\partial x_2} + A_{66}\frac{\partial w}{\partial x_2}\frac{\partial^2 w}{\partial x_1^2} + \left[A_{22} + \frac{EA_2}{s_2}\left(1 + \frac{z_2}{R}\right)\right]\frac{\partial w}{\partial x_2}\frac{\partial^2 w}{\partial x_2^2},$$

$$P_3(w) = -\frac{A_{12}}{R}\left[\frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2 + w\frac{\partial^2 w}{\partial x_1^2}\right] - \left(A_{22} + \frac{EA_2}{s_2}\right)\left[\frac{1}{2R}\left(\frac{\partial w}{\partial x_2}\right)^2 + \frac{w}{R}\frac{\partial^2 w}{\partial x_2^2} +\right.$$
$$\left.\frac{3}{2}\left(\frac{\partial w}{\partial x_2}\right)^2\frac{\partial^2 w}{\partial x_2^2}\right] - 2\left(A_{11} + \frac{EA_1}{s_1}\right)\left(\frac{\partial w}{\partial x_1}\right)^2\frac{\partial^2 w}{\partial x_1^2} - 2\left(A_{12} + A_{66}\right)$$
$$\times\frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}\frac{\partial^2 w}{\partial x_1\partial x_2} - \left(\frac{A_{12}}{2} + A_{66}\right)\left[\left(\frac{\partial w}{\partial x_1}\right)^2\frac{\partial^2 w}{\partial x_2^2} + \left(\frac{\partial w}{\partial x_2}\right)^2\frac{\partial^2 w}{\partial x_1^2}\right],$$

$$Q_3(u, w) = -\left(A_{11} + \frac{EA_1}{s_1}\right)\left(\frac{\partial^2 u}{\partial x_1^2}\frac{\partial w}{\partial x_1} + \frac{\partial u}{\partial x_1}\frac{\partial^2 w}{\partial x_1^2}\right) - (A_{12} + A_{66})\frac{\partial^2 u}{\partial x_1\partial x_2}\frac{\partial w}{\partial x_2} -$$
$$- A_{66}\frac{\partial^2 u}{\partial x_2^2}\frac{\partial w}{\partial x_1} - A_{12}\frac{\partial u}{\partial x_1}\frac{\partial^2 w}{\partial x_2^2} - 2A_{66}\frac{\partial^2 w}{\partial x_1\partial x_2}\frac{\partial u}{\partial x_2},$$

$$R_3(v, w) = -(A_{12} + A_{66})\frac{\partial^2 v}{\partial x_1\partial x_2}\frac{\partial w}{\partial x_1} - A_{66}\frac{\partial^2 v}{\partial x_1^2}\frac{\partial w}{\partial x_2} -$$
$$- \left[A_{22} + \frac{EA_2}{s_2}\left(1 + \frac{z_2}{R}\right)\right]\left(\frac{\partial^2 v}{\partial x_2^2}\frac{\partial w}{\partial x_2} + \frac{\partial v}{\partial x_2}\frac{\partial^2 w}{\partial x_2^2}\right). \tag{2.7}$$

Note that, the obtained system of equations (2.5) is a generalization from the equations of equilibrium [4] to the equations for dynamical problem of a reinforced composite cylindrical shell.

With $R \to \infty$, the system of equations (2.5), (2.6), and (2.7) become a system of motion equations of a reinforced composite plate.

3. LINEAR VIBRATION OF A REINFORCED COMPOSITE CYLINDRICAL SHELL

Omitting nonlinear terms and putting $q = 0$ in the motion equations (2.5) we receive the equations system of natural vibration of a reinforced composite cylindrical shell

$$\begin{aligned}
L_{11}\Big(u\Big) + L_{12}\Big(v\Big) + L_{13}\Big(w\Big) &= J_0\frac{\partial^2 u}{\partial t^2} - J_1\frac{\partial^3 w}{\partial x_1 \partial t^2},\\
L_{12}\Big(u\Big) + L_{22}\Big(v\Big) + L_{23}\Big(w\Big) &= J_0\frac{\partial^2 v}{\partial t^2} - J_1\frac{\partial^3 w}{\partial x_2 \partial t^2},\\
L_{13}\Big(u\Big) + L_{23}\Big(v\Big) + L_{33}\Big(w\Big) &= -J_0\frac{\partial^2 w}{\partial t^2} - J_1\Big(\frac{\partial^3 u}{\partial x_1 \partial t^2} + \frac{\partial^3 v}{\partial x_2 \partial t^2}\Big) +\\
&\quad + J_2\Big(\frac{\partial^4 w}{\partial x_1^2 \partial t^2} + \frac{\partial^4 w}{\partial x_2^2 \partial t^2}\Big),
\end{aligned} \tag{3.1}$$

where operators L_{ij} are taken by (2.6).

The shell considered in the following analysis is simply supported and axial displacements of its ends cross sections are not restrained. If the end cross sections of the shell are supported by circular stiffeners then the circumferential displacements are practically prevented. These boundary conditions can be satisfied if we take the mode shape as follows:

$$\begin{aligned}
u &= \Big(U_{mn}\cos\frac{m\pi x_1}{L}\sin\frac{n x_2}{R}\Big)e^{i\omega t}, \qquad \omega \equiv \omega_{mn},\\
v &= \Big(V_{mn}\sin\frac{m\pi x_1}{L}\cos\frac{n x_2}{R}\Big)e^{i\omega t},\\
w &= \Big(W_{mn}\sin\frac{m\pi x_1}{L}\sin\frac{n x_2}{R}\Big)e^{i\omega t},
\end{aligned} \tag{3.2}$$

where L is the length of the shell, m, n are natural numbers representing the number of half-waves in the axial and circumferential directions.

Substituting expressions (3.2) into the equations of motion (3.1) we obtain the set of three linear homogenous algebraic equations with respect to U_{mn}, V_{mn} and W_{mn} which can be written in the matrix form:

$$\begin{pmatrix} m_{11} & m_{12} & m_{13}\\ m_{21} & m_{22} & m_{23}\\ m_{31} & m_{32} & m_{33}\end{pmatrix}\begin{Bmatrix} U_{mn}\\ V_{mn}\\ W_{mn}\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}, \tag{3.3}$$

where

$$\begin{aligned}
m_{11} &= -\frac{m^2}{L^2}\Big(A_{11} + \frac{EA_1}{s_1}\Big)\pi^2 - \frac{n^2}{R^2}A_{66} + J_0\omega^2,\\
m_{12} &= m_{21} = -\frac{m\pi}{L}\frac{n}{R}\Big(A_{66} + A_{12}\Big),\\
m_{13} &= m_{31} = \frac{m\pi}{L}\Big(-J_1\omega^2 + \frac{EA_1}{s_1}z_1\Big(\frac{m\pi}{L}\Big)^2 + \frac{A_{12}}{R}\Big),\\
m_{22} &= -\Big(\frac{m\pi}{L}\Big)^2\Big(A_{66} + \frac{D_{66}}{R^2}\Big) - \Big(\frac{n}{R}\Big)^2\Big(A_{22} + \frac{EA_2}{s_2} + 2\frac{EA_2}{s_2}\frac{z_2}{R} + \frac{1}{R^2}\Big(D_{22} + \frac{EI_2}{s_2}\Big)\Big) + J_0\omega^2,
\end{aligned}$$

$$m_{23} = m_{32} = \Big(\frac{n}{R}\Big)^3\Big(\frac{EA_2}{s_2}z_2 + \frac{1}{R}\Big(D_{22} + \frac{EI_2}{s_2}\Big)\Big) + \Big(\frac{m\pi}{L}\Big)^2\frac{n}{R}\frac{1}{R}\Big(2D_{66} + D_{12}\Big) +$$
$$+\frac{n}{R}\frac{1}{R}\Big(A_{22} + \frac{EA_2}{s_2} + \frac{1}{R}\frac{EA_2}{s_2}z_2\Big) - \frac{n}{R}J_1\omega^2,$$
$$m_{33} = \Big[J_2\Big(\frac{n}{R}\Big)^2 + J_2\Big(\frac{m\pi}{L}\Big)^2 + J_0\Big]\omega^2 - \Big(\frac{n}{R}\Big)^4\Big(D_{22} + \frac{EI_2}{s_2}\Big) - \Big(\frac{m\pi}{L}\Big)^4\Big(D_{11} + \frac{EI_1}{s_1}\Big) -$$
$$-2\Big(\frac{m\pi}{L}\Big)^2\Big(\frac{n}{R}\Big)^2\Big(2D_{66} + D_{12}\Big) - 2\Big(\frac{n}{R}\Big)^2\frac{1}{R}\frac{EA_2}{s_2}z_2 - \frac{1}{R^2}\Big(A_{22} + \frac{EA_2}{s_2}\Big), \qquad (3.4)$$

Because U_{mn}, V_{mn} and W_{mn} do not vanish simultaneously, then the determinant of coefficients in the equations (3.3) must be equal to zero

$$\mathrm{Det}\,|m_{ij}| = 0. \qquad (3.5)$$

This is an algebraic equation of 3-degree with respect to ω^2 for determining fundamental frequencies of the natural vibration of the shell.

4. NUMERICAL EXAMPLE

The material of the shells considered in this example is **AS4/3501** graphite/epoxy. The properties of this material are: $E_1 = 144.8\,\mathrm{GPa}$, $E_2 = 9.67\,\mathrm{GPa}$, $G_{12} = G_{13} = 4.14\,\mathrm{GPa}$, $G_{23} = 3.45\,\mathrm{GPa}$, $\nu_{12} = 0.3, \rho = 1389.23\,\mathrm{kg/m^3}$.

The skin of the shell has 4 plies, $[45/-45/-45/45]$, each ply being 0.75 mm thick. The material of the composite stiffeners is the same as that of the skin. The height of the stiffeners is equal to 12 mm, while their width is 2 mm. The purpose of this example being a comparison of the effectiveness of composite and SMA stiffeners, the spacing of both types of stiffener is equal. In the case of axial stiffeners, this spacing is 0.1 m. The spacing of composite ring stiffeners is equal to 0.2 m.

SMA stiffeners considered here were manufactured from a circular cross section wire with the diameter equal to 5 mm. The recovery stress is equal to 220 Mpa [8], SMA stiffeners were embedded within a resin sleeve bonded to the surface of the shell. The contribution of the material of the sleeves to the stiffness of the shell is neglected. Suppose, one can choose appropriate profile parameters of composite stiffeners such that using the same weight as that of SMA wires.

On the Table 1 some values of the 3 first natural frequencies of the mentioned cylindrical shell of length $L = 2\,\mathrm{m}$ and radius $R = 0.15\,\mathrm{m}$ are presented.

Table 1. Three first natural frequencies

Mode	Unreinforced	Axial CPS reinforced	Ring CPS reinforced	Axial SMA reinforced	Combined CPS reinforced	Combined CPS-SMA reinforced
1	1.3156	4.0405	3.2651	14.2744	26.2250	60.1510
2	2.0973	4.0728	11.1442	14.3426	57.4795	95.9752
3	2.2640	4.0754	20.1537	14.3774	73.7284	103.1939

Consider the case, when $R = 0.15\,\mathrm{m}$ fixed and L changes from 1 m to 4 m with the step 0.5 m. The lowest natural frequencies of the shell with various types of stiffeners depending on the length are represented on the Table 2 and shown in the Fig. 1.

Table 2. The lowest frequencies of the shell in 6 cases of reinforcement

Length	Unreinforced		Axial CPS reinforced		Ring CPS reinforced		Axial SMA reinforced		Combined CPS reinforced		Combined CPS-SMA reinforced	
1.0	2.264	(3)	8.105	(22)	11.144	(1)	20.235	(18)	73.728	(2)	103.193	(2)
1.5	1.561	(2)	5.401	(15)	5.556	(1)	16.496	(15)	45.616	(1)	78.644	(1)
2.0	1.315	(2)	4.040	(11)	3.265	(1)	14.274	(13)	26.225	(1)	60.151	(1)
2.5	1.234	(2)	3.227	(9)	2.134	(1)	12.766	(12)	16.960	(1)	48.572	(1)
3.0	1.201	(2)	2.684	(7)	1.499	(1)	11.647	(11)	11.846	(1)	40.686	(1)
3.5	1.185	(2)	2.293	(6)	1.109	(1)	10.766	(10)	8.734	(1)	34.983	(1)
4.0	0.853	(1)	2.012	(5)	0.853	(1)	10.069	(9)	6.702	(1)	30.673	(1)

Remark. The number in parentheses denotes the circumferential mode number n, while the axial mode number $m = 1$ in all cases.

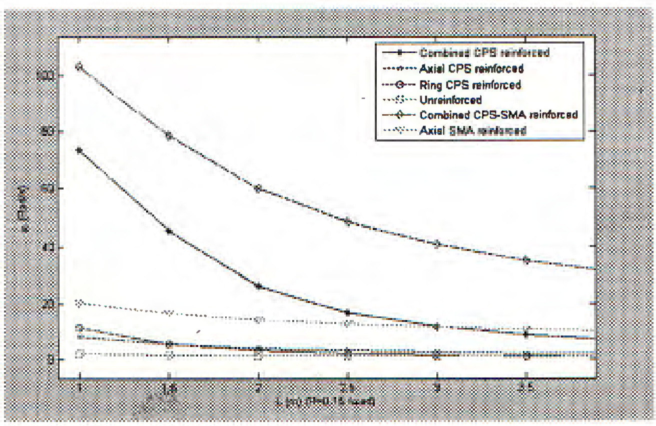

Fig. 1. Comparing the lowest frequencies of the shell in 6 cases of reinforcement

The effect of altering the lamination scheme on the fundamental frequency of a cross-ply shell of length $L = 2\,\mathrm{m}$ and radius $R = 0.15\,\mathrm{m}$ is shown in the Table 3.

Table 3. The effect of altering the lamination scheme

Length	Unreinforced		Axial CPS reinforced		Ring CPS reinforced		Axial SMA reinforced		Combined CPS reinforced		Combined CPS-SMA reinforced	
(0/0)	1.069	(3)	4.884	(28)	3.564	(1)	9.776	(19)	49.058	(2)	53.314	(2)
(30/-30)	1.238	(2)	4.306	(18)	4.376	(1)	11.523	(16)	42.910	(2)	60.224	(1)
(45/-45)	1.315	(2)	4.040	(11)	3.265	(1)	14.274	(13)	26.225	(1)	60.151	(1)
(60/-60)	1.691	(2)	4.288	(8)	2.142	(1)	16.847	(11)	21.205	(1)	60.099	(1)
(90/-90)	2.128	(2)	4.848	(7)	1.488	(1)	19.166	(10)	20.927	(1)	60.080	(1)
(90/0)	2.241	(2)	7.955	(13)	3.130	(1)	18.570	(10)	58.260	(1)	60.160	(1)

Note that in a (90/0) laminated shell the fibers of the outside layer are in the circumferential direction and those of the inside layer are along the longitudinal axis of the shell.

5. DISCUSSION

- From the results it is clear that stiffeners significantly increase the fundamental frequencies of the shell. Ring stiffeners appear to be less efficient than their axial counterparts for longer shells. Especially, SMA stiffeners strongly increase fundamental frequencies of the shell, combined axial SMA and the ring composite stiffeners affect stronger than combined axial and ring composite stiffeners. In all the cases considered the lowest frequencies decrease as the length of shell increase.

The previous discussion may result in the conclusion that the efficiency of reinforced shell is naturally higher than unreinforced one and SMA stiffeners compared to composite stiffeners affect stronger, because SMA stiffeners result in an identical increase of the lowest frequency for both axially reinforced and combined axially SMA-ring CPS reinforced shell.

- The numerical results indicate that for a laminated (90/0) unreinforced shell or a laminated (90/0) axially reinforced shell the fundamental frequencies is bigger than that of the others. Inversely for a laminated (90/-90) ring reinforced or combined CPS reinforced shell this frequency is smaller. Especially for a combined CPS-SMA reinforced shell the fundamental frequency slightly depends on the lamination scheme. According to the lamination scheme illustrated in the Table 3 the ring reinforcement reduces the value of the fundamental frequency, except laminae (90/0).

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PHÂN TÍCH ĐỘNG LỰC HỌC CỦA VỎ TRỤ COMPOSITE NHIỀU LỚP CÓ GÂN GIA CƯỜNG

Trên cơ sở lý thuyết vỏ của Kirchhoff-Love và cách phân tích gân gia cường theo Lekhnitsky đã thiết lập hệ các phương trình vi phân chuyển động của vỏ trụ composite lớp có gân gia cường theo phương dọc trục và theo phương vòng. Các gân có thể là gân composite hoặc gân dưới dạng sợi SMA lồng trong vỏ bọc cao su, vỏ cao su này gắn chặt vào vỏ trụ. Nghiên cứu ảnh hưởng của các yếu tố như các dạng gân, kích thước hình học và trật tự xếp lớp của vỏ đến tần số dao động riêng của vỏ.