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COMPLETE SHAKING FORCE AND SHAKING MOMENT BALANCING OF SPATIAL MULTIBODY SYSTEMS WITH OPEN KINEMATIC CHAINS

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Abstract. This paper deals with a solution for the problem of full shaking force and shaking moment balancing of spatial multibody systems with open kinematic chains. Firstly, the general conditions for complete dynamic balancing of spatial multibody systems are formulated. These include formulae for complete elimination of the resultant inertia force and inertia couple caused by all moving bodies. In the following example, the equations of complete shaking force and shaking moment balancing of a nine-bar direct drive manipulator are given.

1. INTRODUCTION

The balancing of machinery has became an important research topic because a balanced machine leads to better dynamic characteristics and less vibrations. A considerable amount of research on the dynamic balancing of planar mechanisms has been carried out for several decades. It is well known that the resultant inertia force (called as shaking force) and the resultant inertia couple (or shaking moment) caused by all moving links can be reduced by either internal mass redistribution, or using counterweights [5, 7] or adding supplementary links such as cams, dyads to the initial mechanism [6].

In contrast to the rapid progress in balancing theory of planar mechanisms, the development on the balancing theory of spatial multibody systems is still limited. Balancing methods of planar mechanisms can not be directly applicable to spatial multibody systems since kinematic and dynamic properties of spatial multibody systems are much more complicated. The literature on this respect therefore is very little (see [1]-[4]).

Based on theory of multibody dynamics, the general conditions for complete shaking force and shaking moment balancing of spatial multibody systems have been derived theoretically in this paper. These include formulae for complete elimination of the resultant inertia force and inertia couple caused by all moving links. A first attempt is made here to present a solution for the problem of complete balancing of spatial multibody systems with open kinematic chains. In the following example, the equations of shaking force and shaking moment balancing of a nine-link direct drive manipulator are given. A specialized code has been developed on the MAPLE environment for this study.

2. GENERAL CONDITIONS FOR COMPLETE SHAKING FORCE AND SHAKING MOMENT BALANCING OF SPATIAL MULTIBODY SYSTEMS

We consider a spatial multibody system with holonomic and rheonomic constraints as a set of linked rigid bodies in an open or a closed loop structure shown in Fig. 1. The shaking force $\boldsymbol{F}^*$ and the shaking moment $\boldsymbol{M}_O^*$ that caused by all moving bodies can be expressed in the compact matrix form [1], [8]:

$$\boldsymbol{F}^* = -\frac{d}{dt}\boldsymbol{p} = -\frac{d}{dt}\sum_{i=1}^{n} m_i \boldsymbol{v}_i, \tag{2.1}$$

$$\boldsymbol{M}_O^* = -\frac{d}{dt}\boldsymbol{L}_O = -\frac{d}{dt}\sum_{i=1}^{n} (\boldsymbol{I}_i\boldsymbol{\omega}_i + \tilde{\boldsymbol{r}}_i m_i \boldsymbol{v}_i), \tag{2.2}$$

where m_i is the mass of the i-th body, n - the number of bodies, $\mathbf{r}_i$ - the position vector of the center of mass C_i, $\boldsymbol{v}_i$ - the velocity of the center of mass C_i, $\boldsymbol{\omega}_i$ - the angular velocity of the i-th body, $\mathbf{I}_i$ - the matrix of the inertia tensor of the i-th body referred to C_i, $\mathbf{p}$ - the linear momentum of the system, $\mathbf{L}_O$ - the angular momentum of the system taken about fixed point.

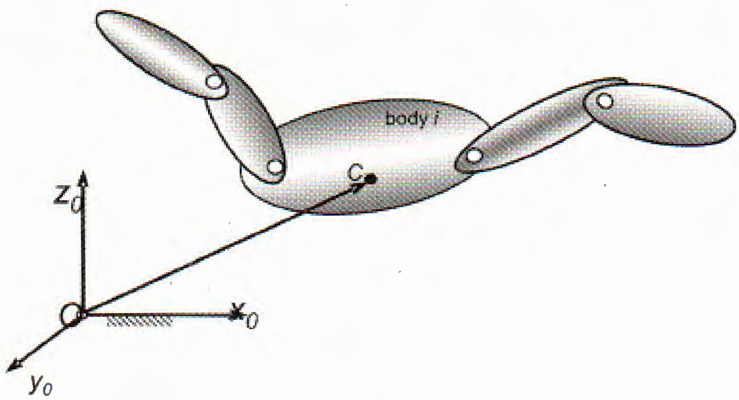

Fig. 1. A spatial multibody system

Now we choose f independent generalized coordinates $\boldsymbol{q} = [q_1, q_2, ..., q_f]^T$ corresponding to the f degrees of freedom of the system. Assumed that the system has holonomic constraints, in this case the velocity $\boldsymbol{v}_i$ and the angular velocity $\boldsymbol{\omega}_i$ are given by

$$\boldsymbol{v}_i = \boldsymbol{J}_{Ti}(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2.3}$$

$$\boldsymbol{\omega}_i = \boldsymbol{J}_{Ri}(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2.4}$$

where $\boldsymbol{J}_{Ti}(\omega q)$ and $\boldsymbol{J}_{Ri}(\omega q)$ denote the $3 \times f$ Jacobian matrices that relate velocity $\boldsymbol{v}_i$ and angular velocity $\boldsymbol{\omega}_i$ to the generalized velocities $\dot{\boldsymbol{q}} = [\dot{q}_1, \dot{q}_2, ..., \dot{q}_f]^T$.

$$\boldsymbol{J}_{Ti}(q) = \frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}}, \quad \boldsymbol{J}_{Ri}(q) = \frac{\partial \boldsymbol{\omega}_i}{\partial \dot{\boldsymbol{q}}}. \tag{2.5}$$

Substituting (2.3) and (2.4) into equations (2.1) and (2.2), the shaking force $\boldsymbol{F}^*$ and the shaking moment $\boldsymbol{M}_O^*$ can be written in the form

$$\boldsymbol{F}^* = -\frac{d}{dt}\left\{\left[\sum_{i=1}^{n} m_i \boldsymbol{J}_{Ti}(\boldsymbol{q})\right]\dot{\boldsymbol{q}}\right\}, \tag{2.6}$$

$$\boldsymbol{M}_O^* = -\frac{d}{dt}\left\{\left[\sum_{i=1}^{n} \boldsymbol{I}_i \boldsymbol{J}_{Ri}(\boldsymbol{q}) + m_i \tilde{r}_i \boldsymbol{J}_{Ti}(\boldsymbol{q})\right]\dot{\boldsymbol{q}}\right\}. \tag{2.7}$$

So, the spatial multibody system can be completely balanced if the shaking force and the shaking moment vanish, this yields

$$\sum_{i=1}^{n} m_i \boldsymbol{J}_{Ti}(\boldsymbol{q}) = 0, \tag{2.8}$$

$$\sum_{i=1}^{n} [\boldsymbol{I}_i \boldsymbol{J}_{Ri}(\boldsymbol{q}) + m_i \tilde{\boldsymbol{r}}_i \boldsymbol{J}_{Ti}(\boldsymbol{q})] = 0. \tag{2.9}$$

Equations (2.8) and (2.9) are the general conditions for complete dynamic balancing of spatial multibody systems. If the mass distribution of the bodies that satisfies equation (2.8), the shaking force will be fully balanced (static balancing). However, in general, the shaking moment can not be fully balanced by internal mass redistribution [4]. One of current trend in dynamic balancing is minimizing the shaking moment of the full force balanced system [5].

Note that equations (2.8) and (2.9) are expressed in the inertial reference system. The elements of inertia matrix $\mathbf{I}_i$ are time dependent. The term $\boldsymbol{I}_i \boldsymbol{J}_{Ri}(\boldsymbol{q})$ of equation (2.9) can be developed most conveniently in the body-fixed coordinate frame because the inertia components of the body are constant in this coordinate frame.

The inertia matrix $\mathbf{I}_i$ can be expressed as [8]

$$\boldsymbol{I}_i = \boldsymbol{A}_i \boldsymbol{I}_i^{(i)} \boldsymbol{A}_i^T, \tag{2.10}$$

where $\boldsymbol{A}_i$ denotes the rotation matrix of the $i-$th body referred to the fixed coordinate frame, $\boldsymbol{I}_i^{(i)}$ is the inertia matrix determined in the body-fixed coordinate frame. Let $\boldsymbol{\omega}_i^{(i)}$ be the angular velocity of the i-th body with respect to the body-fixed coordinate frame, it can be shown that

$$\boldsymbol{J}_{Ri}(\boldsymbol{q}) = \frac{\partial \boldsymbol{\omega}_i}{\partial \dot{\boldsymbol{q}}} = \frac{\partial (\boldsymbol{A}_i \boldsymbol{\omega}_i^{(i)})}{\partial \dot{\boldsymbol{q}}} = \boldsymbol{A}_i \frac{\partial \boldsymbol{\omega}_i^{(i)}}{\partial \dot{\boldsymbol{q}}}. \tag{2.11}$$

Substituting equations (2.10) and (2.11) into expression $\boldsymbol{I}_i \boldsymbol{J}_{Ri}(\boldsymbol{q})$, we obtain

$$\boldsymbol{I}_i \boldsymbol{J}_{Ri}(\boldsymbol{q}) = \boldsymbol{A}_i \boldsymbol{I}_i^{(i)} \boldsymbol{A}_i^T \boldsymbol{A}_i \frac{\partial \boldsymbol{\omega}_i^{(i)}}{\partial \dot{\boldsymbol{q}}} = \boldsymbol{A}_i \boldsymbol{I}_i^{(i)} \boldsymbol{J}_{Ri}^{(i)}, \tag{2.12}$$

where
$$\boldsymbol{J}_i^{(i)} = \frac{\partial \boldsymbol{\omega}_i^{(i)}}{\partial \dot{\boldsymbol{q}}} \tag{2.13}$$

Substituting (2.12) into equation (2.9), the conditions for shaking moment balancing take the form

$$\sum_{i=1}^{n} \left[\boldsymbol{A}_i \boldsymbol{I}_i^{(i)} \boldsymbol{J}_{Ri}^{(i)}(\boldsymbol{q}) + m_i \tilde{\boldsymbol{r}}_i \boldsymbol{J}_{Ti}(\boldsymbol{q}) \right] = \mathbf{0}. \tag{2.14}$$

3. EXAMPLE

In the following example we introduce the application of the general balancing conditions described above to a spatial multibody system with open kinematic chains. Fig. 2 shows a nine-link direct drive manipulator designed by Abdel-Rahman and Elbestawi [2].

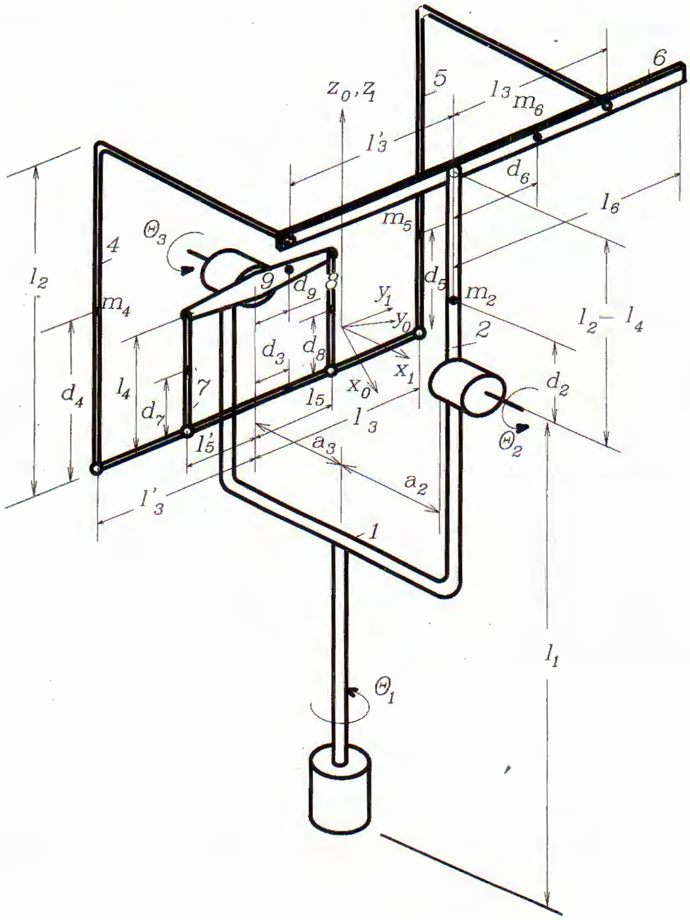

Fig. 2. A nine-link direct drive manipulator

This manipulator is connected directly to three high torque actuators. The first actuator drives link 1, the second actuator drives link 2, which drives links 4, 5, 7 and 8 consecutively. The third actuator drives link 9 and consequently, links 3 and the end-effector link 6. Link 1 carries the other links. Links 2 and 6 are assumed to be in a plane while the rest of the links move in a parallel plane.

First, we introduce the fixed coordinate frame $\{x_0, y_0, z_0\}$ which is located at point O as shown in Fig. 2. In addition, the z_0-axis is chosen to be in line with the first actuator axis. For convenience, coordinate frame $\{x_i, y_i, z_i\}$ is attached to link i ($i = 2...9$) at the revolute joint O_i in accordance with the following rule: The y_i - axis coincides with the largest of the link, the x_i - axis is perpendicular to the moving plane of the link. For the link 1, the x_1 - axis coincides with the second and third actuator axes and the z_1 - axis coincides with the z_0 - axis of the fixed frame. The configuration of the manipulator is also prescribed by three rotation angle θ_1, θ_2, θ_3, in which θ_1 represents the rotation of the link 1 around the z_0 - axis, θ_2 and θ_3 denote the rotation of the links 2 and 9 around the x_1 - axis respectively. The independent generalized coordinates $\boldsymbol{q} = [\theta_1, \theta_2, \theta_3]^T$ are chosen.

The position vector $\mathbf{r}_i$ of the center of mass C_i in the fixed coordinate frame $\{x_0, y_0,$

$z_0\}$ is given by

$$\boldsymbol{r}_i = \boldsymbol{r}_{Oi} + \boldsymbol{A}_i \boldsymbol{r}_i^{(i)}, \qquad (i = 1, ..., 9), \tag{3.1}$$

where $\boldsymbol{r}_i^{(i)}$ is position vector of C_i in the moving coordinate frame $\{x_i,\ y_i,\ z_i\}$ and $\boldsymbol{r}_{Oi}$ is position vector of origin O_i with respect to the fixed frame $\{x_0,\ y_0,\ z_0\}$. The rotation matrix $\boldsymbol{A}_i$ of the i-th link of the manipulator can be expressed as

$$\boldsymbol{A}_1 = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

$$\boldsymbol{A}_2 = \boldsymbol{A}_4 = \boldsymbol{A}_5 = \boldsymbol{A}_7 = \boldsymbol{A}_8 = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_2 & -S_2 \\ 0 & S_2 & C_2 \end{bmatrix} = \begin{bmatrix} C_1 & -C_2S_1 & S_1S_2 \\ S_1 & C_1C_2 & -C_1S_2 \\ 0 & S_2 & C_2 \end{bmatrix} \tag{3.3}$$

$$\boldsymbol{A}_9 = \boldsymbol{A}_6 = \boldsymbol{A}_3 = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_3 & -S_3 \\ 0 & S_3 & C_3 \end{bmatrix} = \begin{bmatrix} C_1 & -C_3S_1 & S_1S_3 \\ S_1 & C_1C_3 & -C_1S_3 \\ 0 & S_3 & C_3 \end{bmatrix} \tag{3.4}$$

where C_k is a shorthand notation for $\cos\theta_k$ and S_k for $\sin\theta_k$, $k = 1, 2, 3$.

Thus, position vectors of the mass centers can be derived from equations (3.1) - (3.4) and from Fig. 2 as

$$\boldsymbol{r}_1 = \begin{bmatrix} 0 & 0 & -d_1 \end{bmatrix}^T, \tag{3.5}$$

$$\boldsymbol{r}_2 = \begin{bmatrix} -d_2C_2S_1 + a_2C_1 \\ d_2C_2C_1 + a_2S_1 \\ d_2S_2 \end{bmatrix}, \tag{3.6}$$

$$\boldsymbol{r}_3 = \begin{bmatrix} -(-l_4C_2 + d_3C_3)S_1 - a_3C_1 \\ (-l_4C_2 + d_3C_3)C_1 - a_3S_1 \\ -l_4S_2 + d_3S_3 \end{bmatrix}, \tag{3.7}$$

$$\boldsymbol{r}_4 = \begin{bmatrix} -(d_{44}C_2 - l_3'C_3)S_1 - a_3C_1 \\ (d_{44}C_2 - l_3'C_3)C_1 - a_3S_1 \\ d_{44}S_2 - l_3'S_3 \end{bmatrix}, \tag{3.8}$$

$$\boldsymbol{r}_5 = \begin{bmatrix} -(d_{54}C_2 + l_3C_3)S_1 - a_3C_1 \\ (d_{54}C_2 + l_3C_3)C_1 - a_3S_1 \\ d_{54}S_2 + l_3S_3 \end{bmatrix}, \tag{3.9}$$

$$\boldsymbol{r}_6 = \begin{bmatrix} -(l_{24}C_2 + d_6C_3)S_1 + a_2C_1 \\ (l_{24}C_2 + d_6C_3)C_1 + a_2S_1 \\ l_{24}S_2 + d_6S_3 \end{bmatrix}, \tag{3.10}$$

$$\boldsymbol{r}_7 = \begin{bmatrix} -(d_{74}C_2 - l_5'C_3)S_1 - a_3C_1 \\ (d_{74}C_2 + l_5'C_3)C_1 - a_3S_1 \\ d_{74}S_2 - l_5'S_3 \end{bmatrix}, \tag{3.11}$$

$$\boldsymbol{r}_8 = \begin{bmatrix} -(d_{84}C_2 + l_5C_3)S_1 - a_3C_1 \\ (d_{84}C_2 + l_5C_3)C_1 - a_3S_1 \\ d_{84}S_2 + l_5S_3 \end{bmatrix}, \tag{3.12}$$

$$\boldsymbol{r}_9 = \begin{bmatrix} -d_9C_3S_1 - a_3C_1 \\ d_9C_3C_1 - a_3S_1 \\ d_9S_3 \end{bmatrix}, \tag{3.13}$$

where $l_{24} = l_2 - l_4$ and $d_{i4} = d_i - l_4,\ i = 4, 5, 7, 8$.

The angular velocities of the links with respect to the link-fixed coordinate frame are given by

$$\boldsymbol{\omega}_1^{(1)} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}, \tag{3.14}$$

$$\boldsymbol{\omega}_2^{(2)} = \boldsymbol{\omega}_4^{(4)} = \boldsymbol{\omega}_5^{(5)} = \boldsymbol{\omega}_7^{(7)} = \boldsymbol{\omega}_8^{(8)} = \begin{bmatrix} \dot{\theta}_2 \\ 0 \\ 0 \end{bmatrix} + \boldsymbol{A}_2^T \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ S_2 & 0 & 0 \\ C_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}, \tag{3.15}$$

$$\boldsymbol{\omega}_3^{(3)} = \boldsymbol{\omega}_6^{(6)} = \boldsymbol{\omega}_9^{(9)} = \begin{bmatrix} \dot{\theta}_3 \\ 0 \\ 0 \end{bmatrix} + \boldsymbol{A}_3^T \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ S_3 & 0 & 0 \\ C_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}. \tag{3.16}$$

Assumed that axes x_i, y_i, z_i of the link-fixed coordinate frame are principal axes. The inertia matrix $\boldsymbol{I}_i^{(i)}$ of i-th link about the center of mass C_i, referred to the principal axes, can be written in the form

$$\boldsymbol{I}_i^{(i)} = \begin{bmatrix} \boldsymbol{I}_{xi} & 0 & 0 \\ 0 & \boldsymbol{I}_{yi} & 0 \\ 0 & 0 & \boldsymbol{I}_{zi} \end{bmatrix}, \quad i = 1, 2, ..., 9. \tag{3.17}$$

By using the expressions from (3.5) to (3.13) we obtain

$$\sum_{i=1}^{9} m_i \boldsymbol{J}_{Ti}(\boldsymbol{q}) = \begin{bmatrix} f_1S_1 + f_2C_1C_2 + f_3C_1C_3 & -f_2S_1S_2 & -f_3S_1S_3 \\ -f_1C_1 + f_2S_1C_2 + f_3S_1C_3 & f_2S_2C_1 & f_3S_3C_1 \\ 0 & -f_2C_2 & -f_3C_3 \end{bmatrix}, \tag{3.18}$$

where

$$f_1 = -(m_2 + m_6)a_2 + (m_3 + m_4 + m_5 + m_7 + m_8 + m_9)a_3, \tag{3.19}$$

$$f_2 = -m_2d_2 - \sum_{i=4}^{8} m_id_i + l_4 \sum_{i=3}^{8} m_i + m_6(d_6 - l_2), \tag{3.20}$$

$$f_3 = -m_3d_3 + m_4l_3' - m_5l_3 - m_6d_6 + m_7l_5' - m_8l_5 - m_9d_9. \tag{3.21}$$

Expression (3.18) contains three equations for shaking force balancing of the manipulator. Finally, by using expressions from (3.2) to (3.17) we get

$$\sum_{i=1}^{9} \left[\boldsymbol{A}_i \boldsymbol{I}_i^{(i)} \boldsymbol{J}_{Ri}^{(i)}(\boldsymbol{q}) + m_i \tilde{\boldsymbol{r}}_i \boldsymbol{J}_{Ti}(\boldsymbol{q}) \right] = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}, \tag{3.22}$$

with the following elements

$$h_{11} = f_4S_2C_1 + f_5S_3C_1 + f_6(S_1S_3C_2 + S_1S_2C_3) + f_8S_1S_2C_2 + f_9S_1S_3C_3, \tag{3.23}$$

$$h_{12} = -f_4 S_1 C_2 + f_6(S_2 S_3 C_1 + C_1 C_2 C_3) + f_{10} C_1, \tag{3.24}$$
$$h_{13} = -f_5 S_1 C_3 + f_6(S_2 S_3 C_1 + C_1 C_2 C_3) + f_{11} C_1, \tag{3.25}$$
$$h_{21} = f_4 S_1 S_2 + f_5 S_1 S_3 - f_6(S_2 C_1 C_3 + S_3 C_1 C_2) - f_8 S_2 C_1 C_2 - f_9 S_3 C_1 C_3, \tag{3.26}$$
$$h_{22} = f_4 C_1 C_2 + f_6(S_1 S_2 S_3 + S_1 C_2 C_3) + f_{10} S_1, \tag{3.27}$$
$$h_{23} = f_5 C_1 C_3 + f_6(S_1 S_2 S_3 + S_1 C_2 C_3) + f_{11} S_1, \tag{3.28}$$
$$h_{31} = 2 f_6 C_2 C_3 + f_8 C_2^2 + f_9 C_3^2 + f_7, \tag{3.29}$$
$$h_{32} = f_4 S_2, \tag{3.30}$$
$$h_{33} = f_5 S_3, \tag{3.31}$$

where

$$f_4 = -m_2 a_2 d_2 - m_3 a_3 l_4 + m_4 a_3 d_{44} + m_5 a_3 d_{54} - m_6 a_2 l_{24} + m_7 a_3 d_{74} + m_8 a_3 d_{84}, \tag{3.32}$$
$$f_5 = (m_3 d_3 - m_4 l_3' + m_5 l_3 - m_7 l_5' + m_8 l_5 + m_9 d_9) a_3 - m_6 a_2 d_6, \tag{3.33}$$
$$f_6 = -m_3 l_4 d_3 - m_4 l_3' d_{44} + m_5 l_3 d_{54} + m_6 l_{24} d_6 - m_7 l_5' d_{74} + m_8 l_5 d_{84}, \tag{3.34}$$
$$f_7 = (m_2 + m_6) a_2^2 + (m_3 + m_4 + m_5 + m_7 + m_8 + m_9) a_3^2 + \sum_{i=2}^{9} I_{yi} + I_{z1}, \tag{3.35}$$
$$\begin{aligned} f_8 = {} & m_2 d_2^2 + m_3 l_4^2 + m_4 d_{44}^2 + m_5 d_{54}^2 + m_6 l_{24}^2 + m_7 d_{74}^2 + m_8 d_{84}^2 \\ & + I_{z2} + I_{z4} + I_{z5} + I_{z7} + I_{z8} - I_{y2} - I_{y4} - I_{y5} - I_{y7} - I_{y8} \end{aligned}, \tag{3.36}$$
$$\begin{aligned} f_9 = {} & m_3 d_3^2 + m_4 l_3'^2 + m_5 l_3^2 + m_6 d_6^2 + m_7 l_5'^2 + m_8 l_5^2 + m_9 d_9^2 \\ & + I_{z3} + I_{z6} + I_{z9} - I_{y3} - I_{y6} - I_{y9} \end{aligned}, \tag{3.37}$$
$$\begin{aligned} f_{10} = {} & m_2 d_2^2 + m_3 l_4^2 + m_4 d_{44}^2 + m_5 d_{54}^2 + m_6 l_{24}^2 + m_7 d_{74}^2 + m_8 d_{84}^2 \\ & + I_{x2} + I_{x4} + I_{x5} + I_{x7} + I_{x8} \end{aligned}, \tag{3.38}$$
$$f_{11} = m_3 d_3^2 + m_4 l_3'^2 + m_5 l_3^2 + m_6 d_6^2 + m_7 l_5'^2 + m_8 l_5^2 + m_9 d_9^2 + I_{x3} + I_{x6} + I_{x9}. \tag{3.39}$$

Expression (3.22) contains eight equations for shaking moment balancing of the manipulator.

4. DISCUSSION AND CONCLUSIONS

The manipulator is statically balanced if expressions of f_1, f_2, f_3 in (3.19), (3.20) and (3.21) vanish, this yields the following conditions

$$\begin{aligned} & -(m_2 + m_6) a_2 + (m_3 + m_4 + m_5 + m_7 + m_8 + m_9) a_3 = 0, \\ & -m_2 d_2 - \sum_{i=4}^{8} m_i d_i + l_4 \sum_{i=3}^{8} m_i + m_6 (d_6 - l_2) \qquad = 0, \\ & -m_3 d_3 + m_4 l_3' - m_5 l_3 - m_6 d_6 + m_7 l_5' - m_8 l_5 - m_9 d_9 = 0 \end{aligned} \tag{4.1}$$

Conditions (4.1) can be satisfied by internal mass redistribution or using counterweights mounted on the links. These conditions may give simple design guidelines for shaking force balancing of the manipulator.

Furthermore, the manipulator is completely dynamically balanced if the following expressions vanish

$$f_k = 0, \quad k = 4, 5, ..., 11. \tag{4.2}$$

Note that these conditions can not be completely satisfied in practice. For example, the values of f_7, f_{10} and f_{11} are not equal to zero in any case. However, the static balanced manipulator can be partially dynamically balanced if some conditions in (4.2) are satisfied, for example $f_4 = f_5 = f_6 = 0$. Another solution for this problem is the minimization of the shaking moment, this yields a set of optimizing values for geometrical and inertia parameters of the links

$$f_k \to \min, \quad k = 4, 5, ..., 11. \tag{4.3}$$

The research for shaking moment minimization of spatial multibody systems will be done in the future.

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CÂN BẰNG LỰC QUÁN TÍNH VÀ MÔ MEN LỰC QUÁN TÍNH CỦA HỆ NHIỀU VẬT KHÔNG GIAN CÓ CẤU TRÚC CHUỖI ĐỘNG HỞ

Bài báo này đề cập tới một giải pháp cho vấn đề cân bằng hoàn toàn lực quán tính và mô men lực quán tính của hệ nhiều vật không gian có cấu trúc chuỗi động hở. Trước hết, các điều kiện tổng quát để cân bằng lực quán tính và mô men lực quán tính của hệ nhiều vật không gian được thiết lập. Các điều kiện này bao gồm các công thức dẫn đến sự triệt tiêu hoàn toàn véc tơ chính và mô men chính của hệ lực quán tính gây ra bởi các vật thuộc hệ. Các hệ thức cân bằng lực quán tính và mô men lực quán tính của một tay máy chín khâu được trình bày trong một thí dụ áp dụng.