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ON A VARIANT OF THE ASYMPTOTIC PROCEDURE
(WEAKLY NONLINEAR SYSTEMS IN A SPECIAL CASES)

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ABSTRACT. The variant the asymptotic procedure presented in [2, 3] is applied to oscillating systems in a special case.

1 Introduction

The modified asymptotic procedure presented in the part I is now applied to study weakly nonlinear systems in a special case. As known, this case is characterized by the absence of a part of variables in the governing differential equations and, consequently, the determination of stationary oscillations needs higher approximations.

Malkin J. G. [3] has paid attention on special cases, using the Poincaré method [3]. The usual asymptotic procedure with full amplitude and full dephase angle as variables is not convenient: it cannot use the mentioned characteristic (the absence of a part of variables) and moreover, the equations for determining stationary oscillations in the end of the asymptotic procedure are often complicated enough.

Contrarily, with the variant of the asymptotic procedure given in the part I, regarding to the mentioned characteristic, we can determine stationary oscillation successively in each step of appoximation.

We restrict ourselves in examining a system of the form:

$$\ddot{x} + \omega^2 x = \varepsilon f(x, \dot{x}, \varphi), \quad \varphi = \omega t, \tag{1.1}$$

where x is an oscillatory variable, overdots denote derivation with respect to time t; ω is the exciting frequency; $\varepsilon > 0$ is a small parameter; $f(x, \dot{x}, \varphi)$ is a function of $(x, \dot{x}, \varphi)$, 2π-periodic with respect to φ. For simplicity, $f(x, \dot{x}, \varphi)$ is assumed to be a trigonometrical polynomial i.e. it can be expanded in finite Fourier series in ωt with polynomial in $(x, \dot{x})$ coefficients.

2 Some properties related to a special case

Let us briefly recall some properties of the function $f(x, \dot{x}, \varphi)$ in a special case.

By $f(a, \theta, \psi)$ we denote the function $f(x, \dot{x}, \varphi)$ after replacing x, $\dot{x}$, φ by $a\cos\psi$, $-\omega a\sin\psi$, $\psi - \theta$, respectively:

$$f(a, \theta, \psi) = f(a\cos\psi, -\omega a\sin\psi, \psi - \theta). \tag{2.1}$$

The Fourier series of $f(a, \theta, \psi)$ is of the form:

$$f(a,\theta,\psi) = f_0(a,\theta) + \sum_{n=1}^{N} \big[S_n(a,\theta)\sin n\psi + C_n(a,\theta)\cos n\psi\big], \tag{2.2}$$

here N is a positive integer.

Let us consider the case in which

$$S_1(a,\theta) = \frac{1}{\pi}\int_0^{2\pi} f(a\cos\psi, -\omega a\sin\psi, \psi-\theta)\sin\psi d\psi \equiv S_1(a), \tag{2.3}$$

$$C_1(a,\theta) = \frac{1}{\pi}\int_0^{2\pi} f(a\cos\psi, -\omega a\sin\psi, \psi-\theta)\cos\psi d\psi \equiv 0, \tag{2.4}$$

i.e. the first harmonic $\cos\psi$ is absent and the coefficient of the first harmonic $\sin\psi$ depends only on a.

Differentiating (2.3), (2.4) with respect to a and θ yields:

$$\frac{\partial S_1}{\partial a} = \frac{dS_1}{da} = S_1'(a) = \frac{1}{\pi}\int_0^{2\pi} \big(\cos\psi\cdot f_x - \omega\sin\psi\cdot f_{\dot{x}}\big)\sin\psi d\psi, \tag{2.5}$$

$$\frac{\partial C_1}{\partial a} = \frac{1}{\pi}\int_0^{2\pi} \big(\cos\psi\cdot f_x - \omega\sin\psi\cdot f_{\dot{x}}\big)\cos\psi d\psi \equiv 0, \tag{2.6}$$

$$\frac{\partial S_1}{\partial \theta} = \frac{-1}{\pi}\int_0^{2\pi} f_\varphi\cdot\sin\psi d\psi \equiv 0, \tag{2.7}$$

$$\frac{\partial C_1}{\partial \theta} = \frac{-1}{\pi}\int_0^{2\pi} f_\varphi\cdot\cos\psi d\psi \equiv 0, \tag{2.8}$$

where f_x, $f_{\dot{x}}$, f_φ are the partial derivatives of $f(x.\dot{x},\varphi)$ with respect to x, $\dot{x}$, φ, respectively; $S_1'(a)$ is the derivative of $S_1(a)$ with respect to a.

From (2.5), (2.6) it follows that the expression $(\cos\psi\cdot f_x - \omega\sin\psi f_{\dot{x}})$ does not contain the first harmonic $\cos\psi$ and the coefficient of its first harmonic $\sin\psi$ depends only on a i.e. we can write:

$$\cos\psi\cdot f_x - \omega\sin\psi\cdot f_{\dot{x}} = S_1'(a)\sin\psi + \langle\ldots\rangle, \tag{2.9}$$

here and below, $\langle\ldots\rangle$ represents "constant" and higher harmonic terms.

Analogously, from (2.7), (2.8), it follows:

$$f_\varphi(a\cos\psi, -\omega a\sin\psi, \psi-\theta) = \langle\ldots\rangle. \tag{2.10}$$

With regard to (2.10), by comparing the two expressions of the first partial derivative $\dfrac{\partial f}{\partial \psi}$ calculated from (2.1) and (2.2), that is

$$\frac{\partial f}{\partial \psi} = -a \sin\psi \cdot f_x - \omega a \cos\psi \cdot f_{\dot{x}} + f_\varphi = S_1(a)\cos\psi + \langle \dots \rangle, \tag{2.11}$$

we can write

$$f_x \cdot \sin\psi + \omega f_{\dot{x}} \cdot \cos\psi = -\frac{1}{a} S_1(a)\cos\psi + \langle \dots \rangle. \tag{2.12}$$

Differentiating (2.10) with respect to θ gives:

$$f_{\varphi^2}(a\cos\psi, -\omega a \sin\psi, \psi - \theta) = \langle \dots \rangle. \tag{2.13}$$

Analogously, with regard to (2.13), by differentiating (2.10) with respect to ψ, we get:

$$\sin\psi \cdot f_{\varphi x} + \omega \cos\psi \cdot f_{\varphi \dot{x}} = \langle \dots \rangle. \tag{2.14}$$

Finally, with regard to (2.9), (2.13), (2.14), by comparing the two expressions of the second partial derivative $\dfrac{\partial^2 f}{\partial \psi^2}$ calculated from (2.1) and (2.2), that is:

$$\begin{aligned}\frac{\partial^2 f}{\partial \psi^2} &= -a\big(\cos\psi \cdot f_x - \omega \sin\psi \cdot f_{\dot{x}}\big) \\ &\quad + a^2\big(\sin^2\psi \cdot f_{x^2} + 2\omega \sin\psi\cos\psi \cdot f_{x\dot{x}} + \omega^2\cos^2\psi \cdot f_{\dot{x}^2}\big) \\ &\quad - 2a\big(\sin\psi \cdot f_{x\varphi} + \omega\cos\varphi \cdot f_{\dot{x}\varphi}\big) + f_{\varphi^2} = -S_1(a)\sin\psi + \langle \dots \rangle,\end{aligned} \tag{2.15}$$

we can write:

$$\sin^2\psi \cdot f_{x^2} + 2\omega\sin\psi\cos\psi \cdot f_{x\dot{x}} + \omega^2\cos^2\psi \cdot f_{\dot{x}^2} = \widetilde{S}_1(a)\sin\psi + \langle \dots \rangle, \tag{2.16}$$

where

$$\widetilde{S}_1(a) = \frac{1}{a} S_1'(a) - \frac{1}{a^2} S_1(a). \tag{2.17}$$

3 Stationary oscillation from the usual asymptotic procedure

For comparison, we briefly recall the usual asymptotic procedure. Following asymptotic expansions will be used

$$x = a\cos\psi + \varepsilon u_1(a,\theta,\psi) + \varepsilon^2 u_2(a,\theta,\psi) + \dots, \quad \psi = \varphi - \theta = \omega t - \theta, \tag{3.1}$$

$$\dot{a} = \varepsilon A_1(a,\theta) + \varepsilon^2 A_2(a,\theta) + \dots, \tag{3.2}$$

$$\dot{\theta} = \varepsilon B_1(a,\theta) + \varepsilon^2 B_2(a,\theta) + \dots, \tag{3.3}$$

where a and θ are the full amplitude and the full dephase angle of the first harmonic; A_i, B_i $(i = 1, 2, \dots)$ are functions of (a, θ); u_i $(i = 1, 2, \dots)$ are functions of (a, θ, ψ), 2π-periodic with respect to ψ, do not containing the first harmonics.

Substituting (3.1) into (1.1), using (3.2), (3.3), expanding $f(x, \dot{x}, \varphi)$ in Taylor series of ε, then equating the terms of like powers of ε yield in the first approximation:

$$-2\omega A_1 \sin\psi - 2\omega a B_1 \cos\psi + \omega^2\Big(\frac{\partial^2 u_1}{\partial\psi^2} + u_1\Big) = f(a\cos\psi, -\omega a \sin\psi, \psi - \theta). \tag{3.4}$$

Using (2.2), (2.3), (2.4), equating the terms of like harmonics, from (3.4) it follows:

$$A_1(a, \theta) = A_1(a) = -\frac{1}{2\omega} S_1(a), \tag{3.5}$$

$$B_1(a, \theta) = -\frac{1}{2\omega a} C_1(a, \theta) = 0, \tag{3.6}$$

$$\omega^2\Big(\frac{\partial^2 u_1}{\partial\psi^2} + u_1\Big) = f_0(a, \theta) + \sum_{n=2}^{N} \big[S_n(a, \theta)\sin n\psi + C_n(a, \theta)\cos n\psi\big], \tag{3.7}$$

$$u_1 = \frac{1}{\omega^2}\Big\{f_0(a, \theta) - \sum_{n=2}^{N} \frac{1}{n^2 - 1}\big[S_n(a, \theta)\sin n\psi + C_n(a, \theta)\cos n\psi\big]\Big\}. \tag{3.8}$$

The same procedure gives successively $A_i(a, \theta)$, $B_i(a, \theta)$ $(i = 2, 3, \dots)$ in which both variables a and θ are present.

In the n-th approximation, to determine the full amplitude a_* and the full dephase angle θ_* of stationary oscillation, we use the stationarity conditions which are expressed as two equations

$$\begin{aligned} A(a, \theta) &= \varepsilon A_1(a) + \varepsilon^2 A_2(a, \theta) + \dots + \varepsilon^n(a, \theta) = 0, \\ B(a, \theta) &= \varepsilon^2 B_2(a, \theta) + \dots + \varepsilon^n B_n(a, \theta) = 0. \end{aligned} \tag{3.9}$$

Stability conditions are two inequalities

$$\Big(\frac{\partial A}{\partial a} + \frac{\partial B}{\partial \theta}\Big)_* < 0, \qquad \Big(\frac{\partial A}{\partial a}\frac{\partial B}{\partial \theta} - \frac{\partial A}{\partial \theta}\frac{\partial B}{\partial a}\Big)_* > 0. \tag{3.10}$$

Note that no information is given in intermediate step and (a_*, θ_*) can only be determined at the end of the asymptotic procedure.

4 Stationary oscillation from a variant of the asymptotic procedure

The variant of the asymptotic procedure presented in part I allows us to determine stationary oscillation successively in each step of approximation.

First, the same expansions (3.1), (3.2), (3.3) are used but a and θ are now the amplitude and the dephase angle of order ε^0 of the first harmonic; consequently, $u_i(a, \theta, \psi)$ contain the first harmonics $a_i \cos\psi + b_i \sin\psi$.

In the first approximation, from (3.5), using the stationarity condition

$$A_1(a,\theta) = A_1(a) = -\frac{1}{2\omega}S_1(a) = 0, \tag{4.1}$$

we obtain immediately a_0 - the stationary amplitude of order ε^0 of the first harmonic.

The stationary dephase angle θ_0 remains undeterminate; this means that in the first approximation, there exists a family of stationary oscillations with amplitude a_0 but with arbitrary dephase angle (although the system is non-autonomous).

The expression (3.8) of u_1 is replaced by

$$u_1 = \frac{1}{\omega^2}\Big\{f_0(a,\theta) - \sum_{n=2}^{N}\frac{1}{n^2-1}[S_n(a,\theta)\sin n\theta + C_n(a,\theta)\cos n\theta]\Big\}$$
$$+ a_1\cos\psi + b_1\sin\psi, \tag{4.2}$$

where a_1, b_1 are two constants to be chosen, N is a positive integer.

Let us move to the second and third approximation. We have

$$\begin{aligned}
&-2\omega A_2\sin\psi - 2\omega a B_2\cos\psi + \omega^2\Big(\frac{\partial^2 u_2}{\partial\psi^2} + u_2\Big)\\
&= -A_1\frac{\partial A_1}{\partial a}\cos\psi - 2\omega A_1\frac{\partial^2 u_1}{\partial\psi\partial a} + u_1 f_x(a\cos\psi, -\omega a\sin\psi, \psi-\theta)\\
&+ \Big(A_1\cos\psi + \omega\frac{\partial u_1}{\partial\psi}\Big)f_{\dot{x}}(a\cos\psi, -\omega a\sin\psi, \psi-\theta),
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
&-2\omega A_3\sin\psi - 2\omega a B_3\cos\psi + \omega^2\Big(\frac{\partial^2 u_3}{\partial\psi^2} + u_3\Big) = -A_2\frac{\partial A_1}{\partial a}\cos\psi + 2A_1B_2\sin\psi\\
&-2\omega A_2\frac{\partial^2 u_1}{\partial\psi\partial a} - 2\omega B_2\frac{\partial^2 u_1}{\partial\psi\partial\theta} - 2\omega B_2\frac{\partial^2 u_1}{\partial\psi^2} - A_1\frac{\partial A_2}{\partial a}\cos\psi + A_1\frac{\partial B_2}{\partial a}\sin\psi\\
&-A_1^2\frac{\partial^2 u_1}{\partial a^2} - A_1\frac{\partial A_1}{\partial a}\frac{\partial u_1}{\partial a} + u_2 f_x + \Big(A_1\frac{\partial u_1}{\partial a} + \omega\frac{\partial u_2}{\partial\psi}\Big)f_{\dot{x}} + \frac{1}{2}u_1^2 f_{x^2}\\
&+ u_1\Big(A_1\cos\psi + \omega\frac{\partial u_1}{\partial\psi}\Big)f_{x\dot{x}} + \frac{1}{2}\Big(A_1\cos\psi + \omega\frac{\partial u_1}{\partial\psi}\Big)^2 f_{\dot{x}^2}.
\end{aligned} \tag{4.4}$$

Denoting the right hand side of (4.3) by $f^{(2)}(a,\theta,\psi)$ we expand it in Fourier series

$$f^{(2)}(a,\theta,\psi) = f_0^{(2)}(a,\theta) + \sum_{n=1}^{M}[S_n^{(2)}(a,\theta)\sin n\psi + C_n^{(2)}(a,\theta)\cos n\psi], \tag{4.5}$$

where M is a positive integer.

Note that a_1, b_1 are present only in the sum $u_1 f_x + \omega\dfrac{\partial u_1}{\partial\psi}f_{\dot{x}}$ and the latter can be written as:

$$u_1 f_x + \omega\frac{\partial u_1}{\partial\psi}f_{\dot{x}} = a_1\big(\cos\psi\cdot f_x - \omega\sin\psi\cdot f_{\dot{x}}\big) + b_1\big(\sin\psi\cdot f_x + \omega\cos\psi\cdot f_{\dot{x}} + \ldots, \tag{4.6}$$

or with regard to (2.9), (2.12):

$$u_1 f_x + \omega \frac{\partial u_1}{\partial \psi} f_{\dot{x}} = a_1 S_1'(a) \sin\psi - \frac{1}{a} b_1 S_1(a) \cos\psi + \ldots, \tag{4.7}$$

where non-written terms do not contain a_1 and b_1.

Therefore, $S_1^{(2)}$ and $C_1^{(2)}$ are of the form:

$$S_1^{(2)}(a,\theta) = a_1 S_1'(a) + \overline{S}_1^{(2)}(a,\theta), \quad C_1^{(2)}(a,\theta) = -\frac{b_1}{a} S_1(a) + \overline{C}_1^{(2)}(a,\theta). \tag{4.8}$$

Equating the terms of like harmonics yields:

$$A_2(a,\theta) = -\frac{1}{2\omega}\{a_1 S_1'(a) + \overline{A}_1^{(2)}(a,\theta)\}, \tag{4.9}$$

$$B_2(a,\theta) = \frac{-1}{2\omega a}\Big\{-\frac{b_1}{a} S_1(a) + \overline{C}_1^{(2)}(a,\theta)\Big\}, \tag{4.10}$$

$$u_2 = \frac{1}{\omega^2}\Big\{f_0^{(2)}(a,\theta) - \sum_{n=2}^{M} \frac{1}{n^2-1}\big[S_n^{(2)}(a,\theta)\sin n\psi + C_n^{(2)}(a,\theta)\cos n\psi\big]\Big\}$$
$$+ a_2 \cos\psi + b_2 \sin\psi, \tag{4.11}$$

where a_2, b_2 are two constants which are still undeterminate.

We impose on $A_2(a,\theta)$ and $B_2(a,\theta)$ the stationarity conditions:

$$A_2(a_0,\theta_0) = 0 \quad \text{or} \quad a_1 S_1(a_0) + \overline{S}_1^{(2)}(a_0,\theta_0) = 0, \tag{4.12}$$

$$B_2(a_0,\theta_0) = 0 \quad \text{or} \quad \overline{C}_1^{(2)}(a_0,\theta_0) = 0. \tag{4.13}$$

The dephase angle θ_0 of order ε^0 is obtained by solving the equation

$$\overline{C}_1^{(2)}(a_0,\theta_0) = 0. \tag{4.14}$$

Then, with the assumption $S_1'(a_0) \neq 0$, the constant a_1 is chosen such that

$$a_1 S_1'(a_0) + \overline{S}_1^{(2)}(a_0,\theta_0) = 0 \quad \text{or} \quad a_1 = a_{10} = -\overline{A}_1^{(2)}(a_0,\theta_0)/S_1'(a_0). \tag{4.15}$$

To determine b_1 we have to examine the right hand side of the equation (4.4), whose Fourier expansion is of the form:

$$f^{(3)}(a,\theta,m\psi) = f_0^{(3)}(a,\theta) + \sum_{n=1}^{k}\big[S_n^{(3)}(a,\theta)\sin n\psi + C_n^{(3)}(a,\theta)\cos n\psi\big], \tag{4.16}$$

where K is a positive integer.

It is not difficult to show that $C_1^{(3)}$ is of the form:

$$C_1^{(3)}(a,\theta) = \frac{-1}{a} b_2 S_1(a) + b_1 \widetilde{C}_1^{(3)}(a,\theta) + \overline{C}_1^{(3)}(a,\theta). \tag{4.17}$$

Indeed:

- The absence of a_2 and the presence of b_2 in (4.17) result from the structure of the sum

$$u_2 f_x + \omega \frac{\partial u_2}{\partial \psi} = a_2\big(\cos\psi \cdot f_x - \omega \sin\psi \cdot f_{\dot{x}}\big) + b_2\big(\sin\psi \cdot f_x + \omega\cos\psi f_{\dot{x}}\big) + \ldots$$
$$= a_2 S_1'(a)\sin\psi - \frac{1}{a} b_2 S_1(a)\cos\psi + \ldots, \tag{4.18}$$

where non-written terms do not contain a_2 and b_2

- With regard to (2.6), the absence of b_1^2 in (4.17) results from the structure of the sum:

$$\frac{1}{2}u_1^2 f_{x^2} + u_1\Big(A_1\cos\psi + \omega\frac{\partial u_1}{\partial\psi}\Big) f_{x\dot{x}} + \frac{1}{2}\Big(A_1\cos\psi + \omega\frac{\partial u_1}{\partial\psi}\Big)^2 f_{\dot{x}^2}$$
$$= \frac{1}{2}b_1^2\big(\sin^2\psi \cdot f_{x^2} + 2\omega\sin\psi\cos\psi \cdot f_{x\dot{x}} + \omega^2\cos^2\psi \cdot f_{\dot{x}^2}\big) + \ldots$$
$$= \frac{1}{2}b_1^2 \widetilde{S}_1(a)\sin\psi + \ldots, \tag{4.19}$$

where non written terms do not contain b_1^2.

Assuming that $\widetilde{C}_1^{(3)}(a_0,\theta_0) \neq 0$, the constant b_1 is chosen as

$$b_1 = b_{10} = -\overline{C}_1^{(3)}(a_0,\theta_0)/\widetilde{C}_1^{(3)}(a_0,\theta_0). \tag{4.20}$$

Continuing the procedure presented, the result in higher approximation is obtained.

With regard that in the second approximation

$$A(a,\theta) = \varepsilon A_1(a) + \varepsilon^2 A_2(a,\theta),$$
$$B(a,\theta) = \varepsilon^2 B_2(a,\theta).$$

The first stability condition (by neglecting the terms of order ε^2) is

$$\Big(\frac{\partial A_1}{\partial a}\Big)_0 < 0 \quad \text{or} \quad S_1'(a_0) > 0. \tag{4.21}$$

and by neglecting the terms of order ε^4, the second stability is:

$$\frac{\partial B_2(a_0,\theta_0)}{\partial\theta} < 0. \tag{4.22}$$

5 Example

As an illustration, consider a self-excited system subjected to an external excitation in exact subharmonic resonance of order $1/3$:

$$\ddot{x} + x = \varepsilon\big\{h(1-x^2)\dot{x} + e\cos 3t\big\},$$

where 1 is the own-frequency; $h > 0$ is intensity of the self-excitation; $e > 0$ and 3 are intensity and frequency of the external excitation.

In the first approximation:

$$f(a,\theta,\psi) = h(1-a^2\cos^2\psi)(-a\sin\psi) + e\cos(3\psi-3\theta)$$
$$= -ha\left(1-\frac{a^2}{4}\right)\sin\psi + \frac{1}{4}ha^3\sin 3\psi + e\cos(3\psi - 3\theta),$$

$$S_1 = -ha\left(1-\frac{a^2}{4}\right), \quad A_1 = -\frac{ha}{2}\left(1-\frac{a^2}{4}\right), \quad C_1 = 0, \quad B_1 = \frac{-1}{2a}C_1 = 0,$$
$$u_1 = -\frac{ha^3}{32}\sin 3\psi - \frac{e}{8}\cos(3\psi-3\theta) + a_1\cos\psi + b_1\sin\psi, \quad a_0 = 2.$$

In the second approximation:

$$f^{(2)}(a,\theta,\psi) = S_1^{(2)}\sin\psi + C^{(2)}\cos\psi + \frac{3}{4}ha^2a_1\sin 3\psi - \frac{3}{4}ha^2b_1\cos 3\psi$$
$$-\frac{h^2a^3}{32}(3-2a^2)\cos 3\psi + \frac{5ha^2}{16}A_1\cos 3\psi$$
$$+\frac{3he}{8}\left(1-\frac{a^2}{2}\right)\sin(3\psi-\theta) + \frac{3h^2a^5}{128}\cos 5\psi - \frac{5hea^2}{32}\sin(5\psi-3\theta),$$

$$S_1^{(2)} = -h\left(1-\frac{3a^2}{4}\right)a_1 - \frac{hea^2}{32}\cos 3\theta, \quad A_2 = -\frac{1}{2}S_1^{(2)}, \quad B_2 = -\frac{1}{2a}C_1^{(2)},$$
$$C_1^{(2)} = \frac{hea^2}{32}\sin 3\theta + \frac{h^2a^5}{128} - A_1\frac{\partial A_1}{\partial a} + hA_1\left(1-\frac{3a^2}{4}\right) + hb_1\left(1-\frac{a^2}{4}\right),$$
$$\sin 3\theta_0 = -\frac{h^2a_0^5}{128}\Big/\frac{hea_0^2}{32} = -\frac{2h}{e}, \quad a_{10} = -\frac{hea_0^2}{32}\cos 3\theta_0\Big/h\left(1-\frac{3a_0^2}{4}\right) = \frac{e\cos 3\theta_0}{16}$$
$$u_2 = -\frac{3ha^2a_{10}}{32}\sin 3\psi + \frac{3ha^2b_1}{32}\cos 3\psi + \frac{h^2a^3(3-2a^2)}{256}\cos 3\psi - \frac{5ha^2}{128}A_1\cos 3\psi,$$
$$-\frac{3he}{64}\left(1-\frac{a^2}{2}\right)\sin(3\psi-3\theta) - \frac{3h^2a^5}{3072}\cos 5\psi$$
$$+\frac{5hea^2}{768}\sin(5\psi-3\theta) + a_2\cos\psi + b_2\sin\psi.$$

Substituting $a = a_0 = 2$, $\theta = \theta_0$, $a_1 = a_{10}$ into $f^{(3)}(a,\theta,\psi)$ and retaining only the first harmonic $\cos\psi$ we obtain the equation determining b_{10}:

$$-\frac{7}{2}ha_{10}b_1 - \frac{9h^2}{8}a_{10} = 0 \quad \text{or} \quad b_1 = b_{10} = -\frac{qh}{2e}.$$

The first stability condition is satisfied: $S_1'(a_0) = 2h > 0$.
The second stability condition

$$\frac{\partial B_2(a_0,\theta_0)}{\partial\theta} = \frac{-1}{2a_0}\frac{3hea_0^2}{32}\cos 3\theta_0 = -\frac{3he}{2}a_{10} < 0 \quad \text{or} \quad a_{10} > 0.$$

Thus, stationary oscillation with large amplitude $\sqrt{(a_0+\varepsilon a_{10}^+)^2 + (\varepsilon b_{10})^2}$, $a_{10}^+ = +\frac{e}{16}\sqrt{1-\frac{4h^2}{e^2}}$ is stable; that with small amplitude namely, that corresponding to $a_{10}^- = -\frac{e}{16}\sqrt{1-\frac{4h^2}{3^2}}$, is unstable.

6 Conclusion

The variant of the asymptotic procedure presented in [2, 3] can be used to examine oscillating systems in a special case. The structures of the equations of stationary oscillation in the first three approximations have been analyzed. The solution of order $0(\varepsilon)$ and $0(\varepsilon^2)$ are obtained in the second and third approximation, respectively. The stability conditions are simple enough.

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VỀ MỘT BIẾN THỂ CỦA TRÌNH TỰ TIỆM CẬN
(TRƯỜNG HỢP ĐẶC BIỆT)

Biến thể của tr` ınh .tưiệm cận trình bày ở [2, 3] được áp dụng để khảo sát hệ dao động ở trường hợp đặc biệt. Cấu trúc các phương trình dao động dừng được phân tích. Nghiệm sai kém $0(\varepsilon)$ và $0(\varepsilon^2)$ được tương ứng xác định ở xấp xỉ thứ hai và thứ ba. Các điều kiện ổn định khá đơn giản.