

NON-LINEAR VIBRATION OF ECCENTRICALLY STIFFENED LAMINATED COMPOSITE SHELLS

Dao Huy Bich, Vu Do Long
Vietnam National University, Hanoi

Abstract. The present paper deals with a non-linear vibration of eccentrically stiffened laminated composite doubly curved shallow shells. The calculations of internal forces and displacements of the shell are based upon the thin shell theory considering the geometrical non-linearity and the Lekhnitsky's smeared stiffeners technique. From the deformation compatibility equation and the motion equation a system of partial differential equations for stress function and deflection of shell is obtained. The Bubnov-Galerkin's method and iterative procedure in conjunction with Newmark constant acceleration scheme are used for dynamical analysis of shells to give the frequency- amplitude relation of free non-linear vibration and non-linear transient responses. Numerical results show the influence of boundary conditions and Gauss curvature on the non-linear vibration of shells.

1. INTRODUCTION

Reinforced laminated structures like plates and shallow shells are widely used in air-industry and ship-industry. The stiffening member provides the benefit of added load-carrying static and dynamic capability. When structures subjected to external loads may be appear a large deflection then the geometrical non-linearity of shell must be considered; of course it meets with mathematics difficulty. To solve problem we are concerned with two aspects: to seek an approximated analytical solution which allows to investigate the motion characteristics and to seek solution by numerical methods. The research results for nonlinear vibration of composite plates have been represented in [4, 11] and for cylindrical shells in [2, 5, 7, 8]. Approximated analytical solutions for the vibration problem of doubly curved unstiffened composite shells were given in [1, 6, 9, 10]. In [3] the authors carried out the non-linear dynamic analysis of doubly curved stiffened composite shells by the displacement approach.

The aim of this paper is to search an approximated analytical solution for the dynamic problem of doubly curved eccentrically stiffened laminated shells with negative and positive Gauss curvature and different boundary conditions by using stress function and Bubnov-Galerkin methods to give non-linear vibration equations of shell. Numerical solutions are given by the iterative method and Newmark constant acceleration scheme. The frequency-amplitude relation in the non-linear free vibration and the influence of boundary conditions and Gauss curvature on the solution of dynamic problem of shells are examined.

2. GOVERNING EQUATIONS

Consider a symmetrically laminated composite doubly curved shallow shells of thickness h and in-plane edges a and b . The shell is reinforced by eccentrically longitudinal and transversal composite stiffeners and subjected to the transverse load of intensity $q(x_1, x_2, t)$.

Using Kirchoff-Love theory non-linear strain-displacement relations for doubly curved shallow shells are formulated

$$\begin{aligned}\varepsilon_1^o &= \frac{\partial u}{\partial x_1} - k_1 w + \frac{1}{2} \left(\frac{\partial w}{\partial x_1} \right)^2; & \phi_1 &= -\frac{\partial^2 w}{\partial x_1^2}; \\ \varepsilon_2^o &= \frac{\partial v}{\partial x_2} - k_2 w + \frac{1}{2} \left(\frac{\partial w}{\partial x_2} \right)^2; & \phi_2 &= -\frac{\partial^2 w}{\partial x_2^2}; \\ \varepsilon_6^o &= \frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1} + \frac{\partial w}{\partial x_1} \cdot \frac{\partial w}{\partial x_2}; & \phi_6 &= -2 \frac{\partial^2 w}{\partial x_1 \partial x_2},\end{aligned}\quad (1)$$

where $k_1 = \frac{1}{R_1}$, $k_2 = \frac{1}{R_2}$ are principal curvatures of the shell; R_1, R_2 are radii of curvature; u, v, w are displacements of the middle surface point along $x_1, x_2, x_3 \equiv z$ directions respectively; ε_i^o and ϕ_i ($i = 1, 2, 6$) are strain of the middle surface point and curvature variations satisfying the deformation compatibility equation:

$$\frac{\partial^2 \varepsilon_1^o}{\partial x_2^2} + \frac{\partial^2 \varepsilon_2^o}{\partial x_1^2} - \frac{\partial^2 \varepsilon_6^o}{\partial x_1 \partial x_2} = \left(\frac{\partial^2 w}{\partial x_1 \partial x_2} \right)^2 - \frac{\partial^2 w}{\partial x_1^2} \cdot \frac{\partial^2 w}{\partial x_2^2} - k_1 \frac{\partial^2 w}{\partial x_2^2} - k_2 \frac{\partial^2 w}{\partial x_1^2}. \quad (2)$$

Internal forces of an unstiffened composite shell are calculated by Reddy [10]. Note that in a symmetrically laminated shell the coupling stiffnesses B_{ij} are equal to zero and the extensional A_{16}, A_{26} and bending D_{16}, D_{26} stiffnesses are negligible compared to the other stiffnesses. Using Lekhnitsky's smeared stiffeners technique we get governing internal force resultants and moments as follows

$$\begin{aligned}N_1 &= \left(A_{11} + \frac{EA_1}{s_1} \right) \varepsilon_1^o + A_{12} \varepsilon_2^o - \frac{EA_1 z_1}{s_1} \cdot \frac{\partial^2 w}{\partial x_1^2}, \\ N_2 &= A_{12} \varepsilon_1^o + \left(A_{22} + \frac{EA_2}{s_2} \right) \varepsilon_2^o - \frac{EA_2 z_2}{s_2} \cdot \frac{\partial^2 w}{\partial x_2^2}, \\ N_6 &= A_{66} \varepsilon_6^o,\end{aligned}\quad (3)$$

$$\begin{aligned}M_1 &= - \left(D_{11} + \frac{EI_1}{s_1} \right) \frac{\partial^2 w}{\partial x_1^2} - D_{12} \frac{\partial^2 w}{\partial x_2^2} + \frac{EA_1 z_1}{s_1} \varepsilon_1^o, \\ M_2 &= - D_{12} \frac{\partial^2 w}{\partial x_1^2} - \left(D_{22} + \frac{EI_2}{s_2} \right) \frac{\partial^2 w}{\partial x_2^2} + \frac{EA_2 z_2}{s_2} \varepsilon_2^o, \\ M_6 &= -2D_{66} \frac{\partial^2 w}{\partial x_1 \partial x_2},\end{aligned}\quad (4)$$

where

$$(A_{ij}, D_{ij}) = \sum_{k=1}^{n_1} \int_{h_{k-1}}^{h_k} \bar{Q}_{ij}^{(k)}(1, z^2) dz, \quad (i, j = 1, 2, 6)$$

