Vietnam Journal of Mechanics, VAST, Vol. 30, No. 3 (2008), pp. 133 – 141

A NEW APPROACH FOR INVESTIGATING CORRUGATED LAMINATED COMPOSITE PLATES OF WAVE FORM

Dao Huy Bich
Vietnam National University, Hanoi
Khuc Van Phu
Military Logistic Academy

Abstract. Corrugated plates of wave form made of isotropic elastic material were considered as flat orthotropic plates with corresponding orthotropic constants determined empirically by the Seydel's technique. In some recent researches the extension of this technique was given for corrugated laminated composite plates.

In the present paper a new approach for investigating corrugated composite plate of wave form is proposed, regarding this plates as a combination of parts of shallow cylindrical shells with alternative curvatures. It reduces to no use of Seydel's empirical formulas and sufficiently apply to composite plates. Based on this approach governing equations of corrugated laminated composite plate of wave form are developed and application to the non-linear stability problem of this plate is considered. Obtained results are compared with those of Seydel's technique.

1. INTRODUCTION

Corrugated plates of wave form made of isotropic elastic material were considered as flat orthotropic plates with corresponding orthotropic constants determined empirically by Seydel's technique. This approach was acceptable to solve many bending and stability problems of corrugated isotropic elastic plates in practice [4, 5, 7]. However, the analysis of corrugated laminated composite plates has received comparitively little attention.

In [1] the authors developed the Seydel's technique to the bending problems of corrugated laminated composite plates and cylindrical shells. In the stability problem of corrugated laminated composite plates [2] besides bending stiffnesses to be extended it is necessary to formulate extensional stiffnesses and to define more exactly the strain expression by including the curvature of middle surface of corrugated plate. But the extension of Seydel's technique to corrugated composite plates meets with difficulties in determination of corresponding constants and experimental verification of these constants. Consequently, obtained calculation may not describe the real state of corrugated laminated plates.

In other hand, nowaday corrugated laminated composite plates and cylindrical shells are widely used and their static and dynamic problems with geometrical non-linearity are of significant practical interest, particularly stability and post-buckling behavior of composite plates and shells is more important. Therefore it needs more accuracy in investigating corrugated composite plates and shells.

In order to eliminate this restriction, a new approach for investigating corrugated composite plates of wave form is proposed naturally in the present paper, regarding this plate as a combination of shallow cylindrical shell parts with alternative curvatures. It reduces to no use of Seydel's empirical formulas and can sufficiently apply not only to an isotropic elastic corrugated plate, but to a composite corrugated plate as well.

Based on this approach governing equations of a corrugated laminated composite plate of wave form are developed and an application to the non-linear stability problem of this plate is considered. Obtained results are compared with those of Seydel's technique.

2. GOVERNING EQUATIONS

Consider a rectangular symmetrically laminated composite corrugated plate in the form of a sine wave (see Fig. 1), each layer of which is an unidirectional composite material. Suppose the portion of cross-section line of a corrugated plate in the plane (x, z) has the form of a sine wave

$$z = H \sin \frac{\pi x}{l} \quad \text{with} \quad H << l,$$

so that the alternative curvature of cross-section line is

$$k = \frac{z''}{(1 + z'^2)^{3/2}} \approx z'' = -H.\frac{\pi^2}{l^2}.\sin \frac{\pi x}{l}. \tag{1}$$

Based on the new approach the non-linear strain-displacement relationships in the middle surface and the changes of curvature and twist of a such corrugated plate now can be written in the form

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - kw, \qquad \chi_x = -\frac{\partial}{\partial x}\left(k.u + \frac{\partial w}{\partial x}\right) = -\left(\frac{\partial^2 w}{\partial x^2} + k\frac{\partial u}{\partial x} + u\frac{\partial k}{\partial x}\right),$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \qquad \chi_y = -\frac{\partial^2 w}{\partial y^2},$$

$$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y}, \quad \chi_{xy} = -\left(2\frac{\partial^2 w}{\partial x \partial y} + k\frac{\partial u}{\partial y}\right),$$

where u, v denote displacements of the middle surface point along x, y directions and

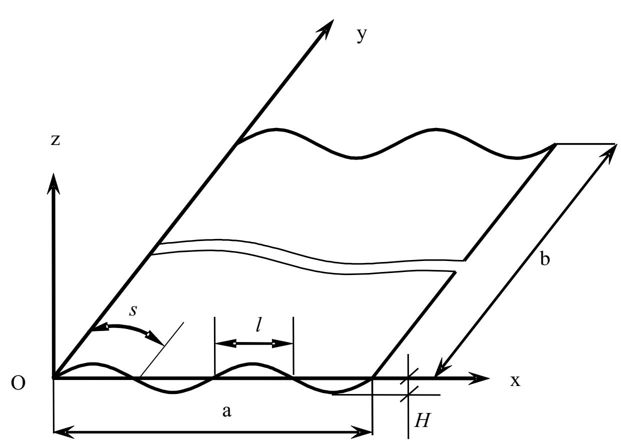

Fig. 1. Model of a corrugated plate

w - deflection of the plate respectively; $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ are strains in the middle surface and $\chi_x, \chi_y, \chi_{xy}$ are changes of curvatures and twist of the plate.

The constitutive stress-strain relations for the plate material are omitted here for brevity. Integrating the stress-strain equations through the thickness of the plate and taking into account that in a multilayered symmetrically laminated material the coupling stiffnesses are equal to zero, while the extensional stiffnesses A_{16}, A_{26} and the bending stiffnesses D_{16}, D_{26} are negligible compared to the others, we obtained the expressions for stress resultants and internal moment resultants of a corrugated composite plate of wave form

$$\begin{aligned}
N_x &= A_{11}\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - k\,w\right] + A_{12}\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right],\\
N_y &= A_{12}\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - k\,w\right] + A_{22}\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right],\\
N_{xy} &= A_{66}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y}\right],
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
M_x &= -D_{11}.\left(\frac{\partial^2 w}{\partial x^2} + k\frac{\partial u}{\partial x} + u\frac{\partial k}{\partial x}\right) - D_{12}.\frac{\partial^2 w}{\partial y^2},\\
M_y &= -D_{12}.\left(\frac{\partial^2 w}{\partial x^2} + k\frac{\partial u}{\partial x} + u\frac{\partial k}{\partial x}\right) - D_{22}.\frac{\partial^2 w}{\partial y^2},\\
M_{xy} &= -D_{66}.\left(2\frac{\partial^2 w}{\partial x\,\partial y} + k\frac{\partial u}{\partial y}\right),
\end{aligned} \tag{3}$$

where A_{ij}, D_{ij} $(i,\, j = 1,\, 2,\, 6)$ are extensional and bending stiffnesses of any laminated plate, i.e. for a flat composite plate such as a corrugated one. The geometry of a plate includes in the expressions of strains and curvature changes. Indeed, it is an advantage of the new approach.

3. FORMULATION OF EQUILIBRIUM EQUATIONS

The equilibrium equations of a corrugated plate of wave form subjected to uniformly distributed biaxial compressive loads of intensities p and q respectively according to [6, 8] when considering the non-linear geometry are of the form

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - k\left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}\right) = 0,\\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0,\\
&\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x} + N_{xy}\frac{\partial w}{\partial y}\right)\\
&\quad + \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x} + N_y\frac{\partial w}{\partial y}\right) + p\frac{\partial^2 w}{\partial x^2} + q\frac{\partial^2 w}{\partial y^2} = 0.
\end{aligned} \tag{4}$$

The substitution of equations (2) and (3) into equations (4) results a system of equilibrium equations in terms of displacements

$$\begin{aligned}
&A_{11}\frac{\partial^2 u}{\partial x^2}+k^2D_{11}\frac{\partial^2 u}{\partial x^2}+A_{66}\frac{\partial^2 u}{\partial y^2}+k^2D_{66}\frac{\partial^2 u}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\,\partial y}+2D_{11}k\frac{\partial u}{\partial x}\frac{\partial k}{\partial x}\\
&+D_{11}ku\frac{\partial^2 k}{\partial x^2}+D_{11}k\frac{\partial^3 w}{\partial x^3}+(D_{12}+2D_{66})\,k\frac{\partial^3 w}{\partial x\partial y^2}-A_{11}k\frac{\partial w}{\partial x}-A_{11}w\frac{\partial k}{\partial x}+A_{11}\frac{\partial w}{\partial x}.\frac{\partial^2 w}{\partial x^2}\\
&+(A_{12}+A_{66})\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}+A_{66}\frac{\partial w}{\partial x}.\frac{\partial^2 w}{\partial y^2}+(A_{12}+A_{66})\frac{\partial w}{\partial y}.\frac{\partial^2 w}{\partial x\,\partial y}-A_{11}\left(w\frac{\partial k}{\partial x}+k\frac{\partial w}{\partial x}\right)=0,
\end{aligned}$$

$$\begin{aligned}
&A_{22}\frac{\partial^2 v}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\,\partial y}+A_{22}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+A_{66}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}+\\
&(A_{12}+A_{66})\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\,\partial y}-A_{12}\left(w\frac{\partial k}{\partial y}+k\frac{\partial w}{\partial y}\right)=0,
\end{aligned}$$

$$\begin{aligned}
&D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+kD_{11}\frac{\partial^3 u}{\partial x^3}+k(D_{12}+2D_{66})\frac{\partial^3 u}{\partial x\partial y^2}+\\
&+2D_{11}\frac{\partial k}{\partial x}\frac{\partial^2 u}{\partial x^2}+(D_{12}+2D_{66})\frac{\partial k}{\partial x}\frac{\partial^2 u}{\partial y^2}+D_{11}\frac{\partial u}{\partial x}\frac{\partial^2 k}{\partial x^2}-A_{11}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}-2A_{66}\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x\partial y}\\
&-A_{12}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2}-A_{11}\frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x}-A_{66}\frac{\partial^2 u}{\partial y^2}\frac{\partial w}{\partial x}-A_{12}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2}-2A_{66}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\partial y}-A_{22}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial y^2}\\
&-(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}\frac{\partial w}{\partial x}-\frac{3}{2}A_{11}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}-A_{11}kw\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}A_{12}\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial x^2}-\\
&-(A_{12}+3A_{66})\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}-\frac{1}{2}(A_{12}+2A_{66})\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial y^2}-A_{12}kw\frac{\partial^2 w}{\partial y^2}-\\
&-\frac{1}{2}A_{22}\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2}+A_{11}w\frac{\partial k}{\partial x}\frac{\partial w}{\partial x}+A_{11}k\left(\frac{\partial w}{\partial x}\right)^2-p\frac{\partial^2 w}{\partial x^2}-q\frac{\partial^2 w}{\partial y^2}=0.
\end{aligned} \tag{5}$$

The system of equations (5) combined with boundary conditions are solving equations of the problem. For a simply supported plate the boundary conditions are

$$\begin{aligned}
&\text{at edges } x=0 \text{ and } x=a:\ w=v=0,\ M_x=0,\\
&\text{at edges } y=0 \text{ and } y=b: w=u=0,\ M_y=0,
\end{aligned} \tag{6}$$

Remark. With $k=0$ the system of equation (5) reduces to a system of partial differential equations of equilibrium of a flat composite plate considered in [3, 6].

An approximation is acceptable in representation of the buckling mode shape by using a single term of a double Fourier series. The boundary conditions (6) can be satisfied if a buckling mode shape is of the form

$$\begin{aligned}
u&=U_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},\\
v&=V_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b},\\
w&=W_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},
\end{aligned} \tag{7}$$

where m, n are natural numbers representing the number of half waves in the x and y directions respectively.

Substituting expressions (7) into the equation of eqilibrium (5) and applying the Bubnov-Galerkin procedure give the set of three algebraic equations with respect to amplitudes U_{mn}, V_{mn}, W_{mn}

$$\begin{aligned}
&a_{11}U_{mn} + a_{12}V_{mn} + a_{13}W_{mn} + a_{14}\,W_{mn}^2 = 0,\\
&a_{21}U_{mn} + a_{22}V_{mn} + a_{23}W_{mn} + a_{24}\,W_{mn}^2 = 0,\\
&a_{31}U_{mn}+a_{32}V_{mn}+a_{33}W_{mn}+a_{34}U_{mn}W_{mn}+a_{35}V_{mn}W_{mn}+a_{36}W_{mn}^2+a_{37}W_{mn}^3 = 0.
\end{aligned} \tag{8}$$

The first two equations of this set are linear algebraic equations for U_{mn}, V_{mn}, in turn which can be expressed in terms of W_{mn}. Then substitution of results into the remaining equation of equilibrium (8) yields a non-linear algebraic equation with respect to W_{mn} as following

$$b_1W_{mn}^3 + b_2W_{mn}^2 + b_3W_{mn} = 0, \tag{9}$$

where a_{ij} and b_i are coefficients depending on the material, geometry of plate and buckling mode shape

$$\begin{aligned}
a_{11} = &-\left\{\frac{1}{4}\left[\left(\frac{m\pi}{a}\right)^2 A_{11} + \left(\frac{n\pi}{b}\right)^2 A_{66}\right]ab + \frac{1}{8}\frac{bH^2\pi^5}{l^4\left(a^2-m^2l^2\right)}\times\right.\\
&\times\left\{\left[m^2l\sin\frac{2a\pi}{l} + \frac{\left(a^2+m^2l^2\right)}{a^2l^2}.\left(\frac{l}{2}\sin\frac{2a\pi}{l}\left(m^2l^2-2a^2\right) + a\pi\left(a^2-m^2l^2\right)\right)\right].D_{11}+\right.\\
&\left.\left.+\left[\frac{l}{2}\sin\frac{2a\pi}{l}\left(m^2l^2-2a^2\right) + \pi a\left(a^2-m^2l^2\right)\right]\left(\frac{n\pi}{b}\right)^2 D_{66}\right\}\right\},
\end{aligned}$$

$$a_{12} = -\frac{1}{4}mn\pi^2\left(A_{12}+A_{66}\right),$$

$$\begin{aligned}
a_{13} = &-\frac{1}{2}\frac{b.mH\pi^2}{\left(a^2-4m^2l^2\right).al^2}\left[\left(\frac{m\pi}{a}\right)^2 D_{11} + \left(\frac{n\pi}{b}\right)^2\left(D_{12}+2D_{66}\right) - 2m^2l^2A_{11}\right]\times\\
&\times\left(a^2-2m^2l^2\right)\left(\cos\frac{a\pi}{l}-1\right),
\end{aligned}$$

$$a_{14} = \frac{1}{9}\frac{\alpha\beta ab}{mn\pi^2}\left[\frac{mn^2\pi^3}{ab^2}\left(A_{12}+A_{66}\right) - 2\left(\frac{m\pi}{a}\right)^3 A_{11}\right], \quad a_{21} = -\frac{1}{4}m\,n\,\pi^2\left(A_{12}+A_{66}\right),$$

$$a_{22} = -\frac{1}{4}\left[\left(\frac{m\pi}{a}\right)^2 A_{66} + \left(\frac{n\pi}{b}\right)^2 A_{22}\right].ab, \quad a_{23} = \frac{H\,lm^2n\,\pi^2\left(\cos\frac{a\pi}{l}-1\right)}{\left(a^2-4m^2l^2\right)}A_{12},$$

$$a_{24} = \frac{1}{9}\frac{\alpha\beta ab}{mn\pi^2}\left[\frac{m^2n\pi^3}{a^2b}\left(A_{12}-A_{66}\right) - 2\left(\frac{n\pi}{b}\right)^3 A_{22}\right],$$

$$a_{31} = \frac{blm^3\pi^4H\left(\cos\frac{a\pi}{l}-1\right)}{a\left(a^2-4m^2l^2\right)}\left[\left(\frac{m}{a}\right)^2 D_{11} + \frac{\left(2l^2-a^2\right)}{2l^2}\left(\frac{n}{b}\right)^2\left(D_{12}+2D_{66}\right)\right],$$

$$a_{32} = 0, \quad a_{33} = a_{33}^* + \lambda,$$

$$a_{33}^* = -\frac{1}{4}\left[\left(\frac{m\pi}{a}\right)^4 D_{11} + 2\left(\frac{mn\pi^2}{ab}\right)^2\left(D_{12}+2D_{66}\right) + \left(\frac{n\pi}{b}\right)^4 D_{22}\right]ab,$$

$$\lambda = -\frac{1}{4}\left[\left(\frac{a}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \xi\right] ab,$$

$$a_{34} = \frac{2}{9}\frac{\alpha\beta ab}{mn\pi^2}\left[\left(\frac{m\pi}{a}\right)^3 A_{11} + 2\frac{mn^2\pi^3}{ab^2}A_{12}\right], \quad a_{35} = -\frac{2}{9}\frac{\alpha\beta ab}{mn\pi^2}\frac{m^2n\pi^3}{a^2b}A_{12},$$

$$a_{36} = 4\frac{(-1)^m \beta abl^2m^3H\sin\dfrac{a\pi}{l}}{n\left(a^2-9m^2l^2\right)\left(a^2-m^2l^2\right)}\left\{\left[\frac{1}{6}\frac{\pi^2}{l^2}\left(3m^2l^2+a^2\right) + \left(\frac{m\pi}{a}\right)^2\right]A_{11} + \left(\frac{n\pi}{b}\right)^2 A_{12}\right\},$$

$$a_{37} = -\frac{1}{32}\left[\frac{9}{4}\left(\frac{m\pi}{a}\right)^4 A_{11} + \left(\frac{mn\pi^2}{ab}\right)^2 A_{12}\right] ab,$$

$$\alpha, \beta = \begin{cases} 0 & \text{with } m, n \text{ are even numbers} \\ -2 & \text{with } m, n \text{ are odd numbers} \end{cases}$$

$$b_1 = a_{34}\left(a_{12}a_{24} - a_{14}a_{22}\right) + a_{35}\left(a_{21}a_{14} - a_{11}a_{24}\right) + a_{37}\left(a_{11}a_{22} - a_{12}a_{21}\right),$$

$$b_2 = a_{31}\left(a_{12}a_{24} - a_{14}a_{22}\right) + a_{34}\left(a_{12}a_{23} - a_{13}a_{22}\right) + a_{35}\left(a_{21}a_{13} - a_{11}a_{23}\right) + $$
$$+ a_{36}\left(a_{11}a_{22} - a_{12}a_{21}\right),$$
$$b_3 = a_{31}\left(a_{12}a_{23} - a_{13}a_{22}\right) + a^*_{33}\left(a_{11}a_{22} - a_{12}a_{21}\right) + \lambda\left(a_{11}a_{22} - a_{12}a_{21}\right).p. \tag{10}$$

Note that the parameter ξ occuring in (10) is a ratio of compressive loads q/p, since the plate is working in the elastic stage, so in the case of simultaneous action of loads p *and* $q$we can put here $q = \xi.p$.

Considering the plate after the lost of stability, i.e. $W_{mn} \neq 0$, from (9) it follows that

$$b_1 W_{mn}^2 + b_2 W_{mn} + b_3 = 0. \tag{11}$$

The upper critical load which coincides with the linear buckling load can be found from equation (11) by putting $W_{mn} = 0$

$$p_{up} = \frac{a_{31}\left(a_{12}a_{23} - a_{13}a_{22}\right) - a^*_{33}\left(a_{11}a_{22} - a_{12}a_{21}\right)}{\lambda\left(a_{11}a_{22} - a_{12}a_{21}\right)}. \tag{12}$$

The critical load q is determined by $q_{upper} = \xi p_{\text{upper}}$. From equation (11) the compressive load can be represented through W_{mn}

$$p = \mathrm{f}\left(\mathrm{W_{mn}}, \xi\right).$$

The lower buckling load of the corrugated composite plate can be determined by using the condition

$$\frac{\partial f\left(W_{\mathrm{mn}}, \xi\right)}{\partial W_{mn}} = 0,$$

finally we get

$$p_{lower} = \frac{b_2^2 - 4b_1\left[a_{31}\left(a_{12}a_{23} - a_{13}a_{22}\right) + a^*_{33}\left(a_{11}a_{22} - a_{12}a_{21}\right)\right]}{4b_1\lambda\left(a_{11}a_{22} - a_{12}a_{21}\right)}. \tag{13}$$

Numbers m, n must be chosen such that the absolute value of the critical buckling load p is minimum.

According to each value ξ, i.e. to each loading process, we can get from (12) and (13) upper and lower critical loads p respectively, then critical loads q is defined by $q = \xi p$.

The domain limited by the upper and lower buckling loads is called the unstable domain of a corrugated composite plate.

Particularly, for a flat composite plate with $k(x) = 0$, in this case $H = 0$, the coefficients in (10) $a_{13} = a_{23} = a_{31} = a_{36} = 0$ so that $b_2 = 0$, from the equations (12) and (13) we can see that the upper and lower critical loads coincide each other and are equal to the following value:

$$p_{cr} = \frac{a_{33}^*}{\lambda} = \frac{\left(\frac{m\pi}{a}\right)^4 D_{11} + 2\left(\frac{mn\pi^2}{ab}\right)^2 (D_{12} + 2D_{66}) + \left(\frac{n\pi}{b}\right)^4 D_{22}}{\left(\frac{m\pi}{a}\right)^2 + \xi\left(\frac{n\pi}{b}\right)^2}$$

For a flat composite plate there isn't an unstable domain, while for a corrugated composite plate that exists.

4. NUMERICAL EXAMPLES

For comparison of two approaches to investigate corrugated composite plates – Seydel's technique and proposed one – let's consider a simply supported rectangular corrugated symmetrically laminated plate in the form of a sine wave such as in [2] with $a = 0.99$ m, $b = 1.5$ m, $H = 0.03$ m, $1 = 0.09$ m; the skin of the plate has 6 plies $[45/-45/90/90/-45/45]$, each ply being 0.5 mm thick. The material of the plate had Thornel 300 graphite fibers and Narmco 5208 thermosetting epoxy resin with following properties $E_1 = 127.4$ GPa, $E_2 = 13.0$ GPa, $G_{12} = 6.4$ GPa, $\nu_{12} = 0.38$.

Some numerical results are shown in the Figs. 2, 3, 4 for the critical buckling load p of a corrugated composite plate subjected to biaxial compressive loads with $\xi = 1$. The effect of the plate thickness on critical load is illustrated in the Fig. 2.

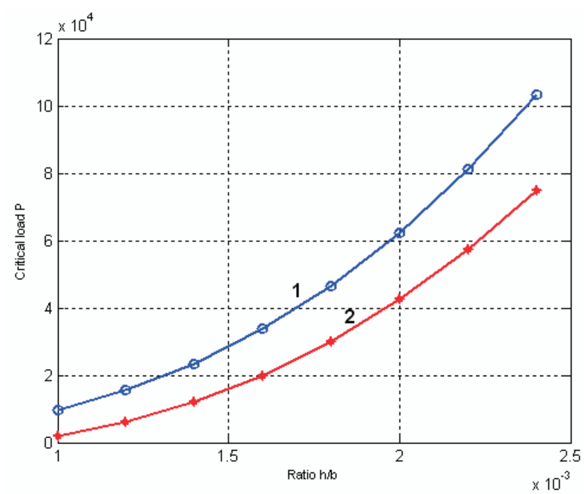

Fig. 2. Effect of plate thickness on buckling load
1. Seydel's technique, 2. Proposed approach

Fixing the values H, l and h, relation between the critical buckling load and the dimension ratio b/a is represented in the Fig. 3.

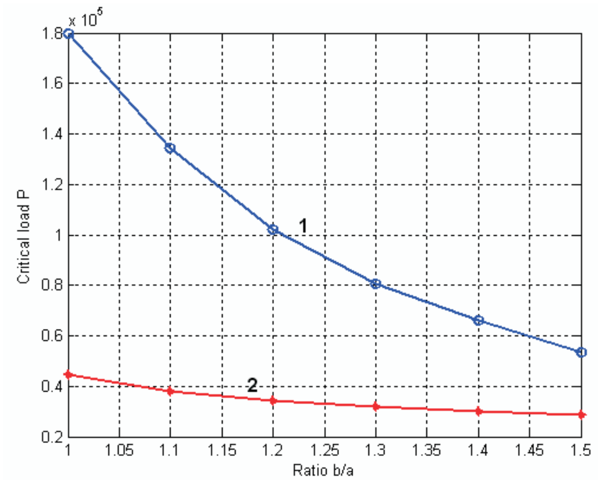

Fig. 3. Effect of plate dimension ratio on buckling load
1. Seydel's technique, 2. Proposed approach

Fixing the values a, b and $l$one can see the effect of the height H of portion line on critical buckling load in the Fig. 4.

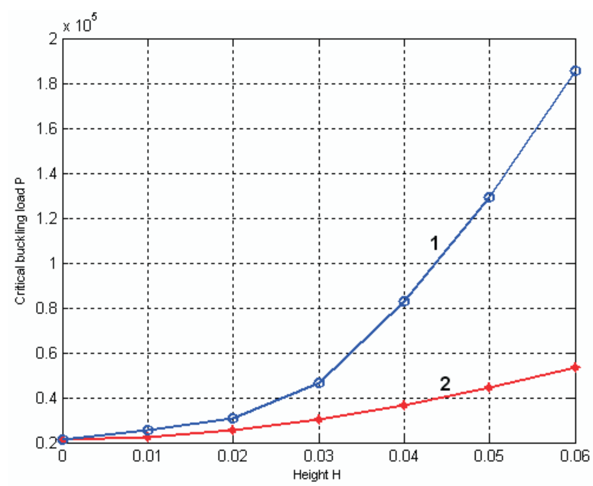

Fig. 4. Effect of the height on buckling load
1. Seydel's technique, 2. Proposed approach

From obtained results one can see that calculating results based on Seydel's technique for composite corrugated plates are greater than those based on proposed approach. It is possible the stiffnesses calculated by Seydel's technique may be greater than real ones of the corrugated plates. Consequently, for approving the accuracy of Seydel's technique and proposed approach to investigate corrugated composite plates it is necessary to establish experimental verification.

5. CONCLUSION

The proposed approach for investigating corrugated plates of wave form is quite natural without additional assumptions, because a corrugated plate of wave form exactly consists of cylindrical shell parts with alternative curvatures, so we can apply the shell theory in the consideration.

Based on the new approach the governing equations for corrugated plates of wave form are developed. These equations can be applied to consider static and dynamic problems of not only corrugated isotropic elastic plates but corrugated composite plates as well.

Applying obtained equations and using Bubnov-Galerkin method an approximated analytical solution to the non-linear stability problem of corrugated laminated composite plates subjected biaxial loads is investigated.

Obtained results are compared with those calculated by Seydel's technique and some conclusions are derived.

ACKNOWLEDGEMENT

This paper is completed with financial support by the National Council for Natural Sciences and project QGTD.08.07.

REFERENCES

1. Dao Huy Bich, Vu Khac Bay, Dao Van Dung, Tran Ich Thinh, Pham Chi Vinh, Calculation of the corrugated laminated composite cylindrical shells, *Vietnam Journal of Mechanics* **19** (1997) (3) 9-18, NCNST of Vietnam.
2. Dao Huy Bich, Khuc Van Phu, Non-linear Analysis on Stability of Corrugated Cross-Ply Laminated Composite Plates, *Vietnam Journal of Mechanics* **28** (2006) (4) 197-206.
3. Khuc Van Phu, Research on non-linear multilayered laminated CPS plates, *Ph.D. Thesis*, Hanoi, 2005.
4. S, G. Lechnitsky, *Anisotropic plates*, 2^{nd} edition, Moscow, 1957.
5. D. Mc.Farland, B. L. Smith, W. D. Bernhart, *Analysis of Plates*, Spartan Books, New York, 1972.
6. J. N. Reddy, *Mechanics of Laminated Composite Plates and Shells: Theory and analysis,* CRC Press, 2004.
7. E. Seydel, *Schubknickversuche mit Welblechtafeln*, DVL - Bericht, 1931.
8. S. Timoshenko, S. Krieger, *Theory of Plates and Shells.* Mc Graw-Hill Book Company, N.Y., 1959.

Received March 10, 2008

MỘT CÁCH TIẾP CẬN MỚI KHẢO SÁT TẤM COMPOSITE LỚP CÓ DẠNG LƯỢN SÓNG

Với quan điểm tấm lượn sóng là tập hợp của các mảnh vỏ trụ thoải được sắp xếp theo quy luật hình sin, bài báo đề cập đến việc thiết lập các hệ thức cơ sở của tấm composite lớp có dạng lượn sóng và áp dụng khảo sát bài toán ổn định phi tuyến cuả tấm composite lượn sóng chịu nén theo hai phương vuông góc với nhau. Kết quả số được so sánh với kết quả tính toán khi sử dụng kỹ thuật Seydel.