

AN EXTENSION OF THE IDENTIFICATION ALGORITHM FOR FEEDBACK ACTIVE CONTROL OF INCOMPLETE MEASURED SYSTEMS

LA DUC VIET, NGUYEN DONG ANH

Institute of Mechanics

Abstract. In a previous paper [1], the identification algorithm is presented for feedback active controlled systems. However, this method can only be applied to complete measured systems. The aim of this paper is to present a combination of the identification algorithm and the modal superposition method to control the incomplete measured systems. The system response is expanded by modal eigenfunction technique. The external excitation acting on some first modes is identified with a time delay and with a small error depending on the sensor locations. Then the control forces will be generated to balance the identified excitations. A numerical simulation is applied to an eight story building subjected to base acceleration.

1. INTRODUCTION

The active control method can be applied to many problems such as robot control, ship autopilot, airplane autopilot, vibration control of vehicles or structures ... Fig. 1 provides a schematic diagram of an active control system.

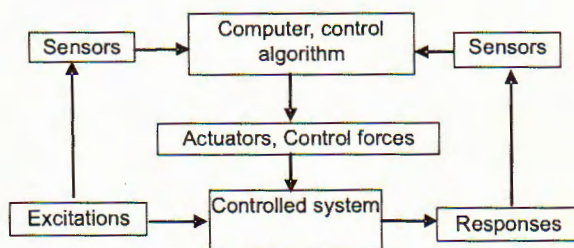


Fig. 1. Diagram of a structural control system

It consists of 3 main parts: sensors to measure either external excitations or system responses or both; computer controller to process the measured information and to compute necessary control force based on a given control algorithm; actuators to produce the required forces. When only the responses can be measured, the method is called feedback active control. In recent years, the active control method has been widely used to reduce the excessive vibrations of civil structures due to environmental disturbances ([2]). One of the basic tasks of active structural control problem is to determine a control strategy that

uses the measured structural responses to calculate an appropriate control signal to send to the actuator. Many control strategies have been proposed, such as LQR/LQG control [3], H_2/H_∞ control [4,5], sliding mode control [6], saturation control [7], reliability-based control [8], fuzzy control [9], neural control [10]... We also proposed two control algorithms for structural control. The first applies to structures, in which some components of excitation can be known [11] and the second applies to structures, in which the control forces are bounded [12]. In fact, it is usually that one is unable to measure the external excitation while the structural response can often be measured. The identification algorithm presented in [1] is a method, which identifies the external excitation from the structural response measured. Although this version of identification algorithm can be applied even for the nonlinear structures, it requires knowledge of the entire displacement vector of the structure, which is not possible for large structures. Thus, the aim of this paper is to combine the identification algorithm and the modal superposition method for the linear structures with incomplete measurement, i.e only some components of response vector can be measured.

2. PROBLEM FORMULATION

Consider a multi-degree-of-freedom system described by the second order linear differential equation

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = u(t) + f(t), \quad x(0) = x_0, \quad (1)$$

where $x(t)$ is the n -dimensional displacement vector, $f(t)$ is the n -dimensional external force vector, $u(t)$ is the n -dimensional control vector, three $n \times n$ matrices M, D and K are mass, damping and stiffness matrices, respectively. Let $y(t)$ be the p -dimensional measurement (output) vector ($p \leq n$) with:

$$y(t) = Cx(t), \quad (2)$$

where C is a $p \times n$ measurement matrix. The control force vector $u(t)$ is selected as a function of the measurement vector $y(t)$. The control problem is to find the active control force $u(t)$ necessary to reduce the norm state vector. It is seen obviously that the best control law is that

$$u(t) = -f(t). \quad (3)$$

Indeed with control law (3), the external excitation is totally eliminated. However, it is usually that one is unable to measure the external excitation, so the control law (3) cannot be realized practise. The idea involved in the control law (3) may be used in a modified way, in which the history of the external excitation can be identified with a time delay by a so called identification process. The process identifying the entire external excitation is presented in [1] and here and after is called the original identification algorithm. The original identification algorithm requires the knowledge of the entire state vector to identify the entire excitation. However, when only the measurement vector in (2) can be measured, the excitation can not be identified all. In this paper, by using modal superposition method, the identification algorithm will be extended to identify some most important excitations based on the measurement vector $y(t)$. The detail of this extension is presented in section 4.

3. IDENTIFICATION ALGORITHM

The original identification algorithm developed in [1] is shortly presented here. Let T be the time duration of the action of external excitation. Let all the components of the displacement vector $x(t)$ can be measured and all components of its first and second order derivatives can be calculated in a short time. The interval $[0, T]$ is divided into n small equal intervals of the length Δ where Δ is a small positive number whose value depends on computation speed and accuracy of computer. Thus one has:

$$T = q\Delta$$

For any given function vector $m(t)$, the following notation is introduced:

$$m^{(k)}(t) = \begin{cases} m(t) & (k-1)\Delta \leq t \leq k\Delta \\ 0 & \text{otherwise} \end{cases} \quad k = 1, 2, \dots, q \quad (4)$$

In $T_k = [(k-1)\Delta \leq t \leq k\Delta]$, the system response is described by the following equation:

$$M\ddot{x}^{(k)}(t) + D\dot{x}^{(k)}(t) + Kx^{(k)}(t) = u^{(k)}(t) + f^{(k)}(t). \quad (5)$$

In this subinterval, we assume that the control force $u^{(k)}(t)$ can be known (by the control law (7) below), the displacement vector $x^{(k)}(t)$ is measured, the velocity and acceleration vectors are calculated. Thus, the external disturbance $f^{(k)}(t)$ can be calculated as

$$f^{(k)}(t) = M\ddot{x}^{(k)}(t) + D\dot{x}^{(k)}(t) + Kx^{(k)}(t) - u^{(k)}(t). \quad (6)$$

So, at the end of the subinterval T_k , one can know all about $f(t)$ in this subinterval. Because the subinterval T_k ended, this information can be used only in the next subinterval T_{k+1} to calculate $u^{(k+1)}(t)$. This means that the information about $f(t)$ has a time delay Δ . Using the information of the delayed external excitation $f(t)$, the control algorithm is proposed as:

$$\begin{cases} u^{(1)}(t) = 0 \\ u^{(k)}(t) = -f^{(k-1)}(t - \Delta) \\ \quad = -[M\ddot{x}^{(k-1)}(t - \Delta) + D\dot{x}^{(k-1)}(t - \Delta) + Kx^{(k-1)}(t - \Delta) - u^{(k-1)}(t - \Delta)] \\ k = 2, 3, \dots, q \end{cases} \quad (7)$$

As we see, the control law (7) is established in the inductive way. With control law (7), the delayed external excitation $f(t - \Delta)$ is totally eliminated. Substituting (7) into (1), the differential equation of the controlled system has form:

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = -f(t - \Delta) + f(t)$$

One has:

$$\|-f(t - \Delta) + f(t)\| = \Delta \|\dot{f}(\zeta)\|$$

where ζ is a value between t and $t - \Delta$. Therefore, it is easy to see that the norm of excitation is sufficiently small if the time delay Δ is small. However, as mentioned above, the disadvantage of the original identification algorithm is the requirement of the knowledge of entire displacement vector $x(t)$.

4. IDENTIFICATION ALGORITHM USING MODAL SUPERPOSITION METHOD

Incomplete measurement leads to incomplete excitation identification. Two questions need to be addressed: which excitation is important and how to identify it? These questions are not easy to answer if the system is nonlinear. However, in case of the linear system modeled in (1), the answer can be found by well-known modal eigenfunction technique. The decoupled modal equations take the form:

$$\ddot{z}_i(t) + 2\xi_i\omega_i\dot{z}_i(t) + \omega_i^2 z_i(t) = \Phi_i^T u(t) + \Phi_i^T f(t), i = 1, \dots, n, \quad (8)$$

where ξ_i and ω_i ($i = 1, \dots, n$) are respectively the i^{th} damping ratio and the i^{th} natural frequency. Φ_i and z_i ($i=1, \dots, n$) are corresponding eigenvectors and modal coordinates. The measurement vector $y(t)$ is also rewritten in modal space:

$$y = \sum_{i=1}^n C\Phi_i z_i. \quad (9)$$

As one knows, the vibrational modes corresponding to large natural frequencies often contribute little to the response [13], so attention needs to be paid only to a few vibrational modes. Denote z_c as the vector containing the first p modal coordinates and z_r as the vector containing the remaining $n - p$ modal coordinates. This means:

$$z_c = [z_1 \quad z_2 \quad \dots \quad z_p]^T; \quad z_r = [z_{p+1} \quad z_{p+2} \quad \dots \quad z_n]^T$$

Then (8) and (9) are rewritten as

$$\ddot{z}_c(t) + 2\Psi_c\Omega_c\dot{z}_c(t) + \Omega_c^2 z_c(t) = \Phi_c^T u(t) + \Phi_c^T f(t), \quad (10)$$

$$\ddot{z}_r(t) + 2\Psi_r\Omega_r\dot{z}_r(t) + \Omega_r^2 z_r(t) = \Phi_r^T u(t) + \Phi_r^T f(t), \quad (11)$$

$$y = C_c z_c + C_r z_r, \quad (12)$$

where Ψ_c, Ψ_r, Ω_c and Ω_r are diagonal matrices are defined as:

$$\Psi_c = \text{diag} [\xi_1 \quad \xi_2 \quad \dots \quad \xi_p], \quad \Psi_r = \text{diag} [\xi_{p+1} \quad \xi_{p+2} \quad \dots \quad \xi_n]$$

$$\Omega_c = \text{diag} [\omega_1 \quad \omega_2 \quad \dots \quad \omega_p], \quad \Omega_r = \text{diag} [\omega_{p+1} \quad \omega_{p+2} \quad \dots \quad \omega_n]$$

and Φ_c, Φ_r, C_c and C_r are defined as:

$$\Phi_c = [\Phi_1 \quad \Phi_2 \quad \dots \quad \Phi_p]; \quad \Phi_r = [\Phi_{p+1} \quad \Phi_{p+2} \quad \dots \quad \Phi_n]$$

$$C_c = C\Phi_c; \quad C_r = C\Phi_r$$

Because attention needs to be paid only to the first p modes, we need to identify and eliminate the excitation $\Phi_c^T f(t)$. The identification process here is implemented in the same manner as the process in section 3. The interval $[0, T]$ is also divided into n small equal intervals of the length Δ . Using the notation (4), in $T_k = [(k-1)\Delta \leq t \leq k\Delta]$, the equation (10) has form:

$$\ddot{z}_c^{(k)}(t) + 2\Psi_c\Omega_c\dot{z}_c^{(k)}(t) + \Omega_c^2 z_c^{(k)}(t) = \Phi_c^T u^{(k)}(t) + \Phi_c^T f^{(k)}(t)$$

Using (12), we have

$$\begin{aligned}\Phi_c^T f^{(k)}(t) &= \ddot{z}_c^{(k)}(t) + 2\Psi_c \Omega_c \dot{z}_c^{(k)}(t) + \Omega_c^2 z_c^{(k)}(t) - \Phi_c^T u^{(k)}(t) \\ &= C_c^{-1}(\ddot{y}^{(k)}(t) - C_r \ddot{z}_r^{(k)}(t)) + 2\Psi_c \Omega_c C_c^{-1}(\dot{y}^{(k)}(t) - C_r \dot{z}_r^{(k)}(t)) + \\ &\quad + \Omega_c^2 C_c^{-1}(y^{(k)}(t) - C_r z_r^{(k)}(t)) - \Phi_c^T u^{(k)}(t).\end{aligned}$$

Then

$$\Phi_c^T f^{(k)}(t) + E^{(k)}(t) = C_c^{-1} \ddot{y}^{(k)}(t) + 2\Psi_c \Omega_c C_c^{-1} \dot{y}^{(k)}(t) + \Omega_c^2 C_c^{-1} y^{(k)}(t) - \Phi_c^T u^{(k)}(t), \quad (13)$$

where

$$E^{(k)}(t) = C_c^{-1} C_r \ddot{z}_r^{(k)}(t) + 2\Psi_c \Omega_c C_c^{-1} C_r \dot{z}_r^{(k)}(t) + \Omega_c^2 C_c^{-1} C_r z_r^{(k)}(t). \quad (14)$$

In the subinterval T_k , we assume that the control force $u_c^{(k)}(t)$ can be known (by the control law (15) below), the measurement vector $y^{(k)}(t)$ is known, and the first and second derivatives are calculated. The error term $E^{(k)}(t)$ introduced through the truncation process is still unknown. Thus, from (13), we can not know the exact excitation $\Phi_c^T f(t)$, but only an estimate of $\Phi_c^T f(t)$ to with an error $E(t)$. To attenuate this error term, the sensors should be located to obtain a significant contribution of the information of x_c . This means a large norm of C_c in comparison with the norm of C_r . Because the subinterval T_k ended, the information known can be used only in the next subinterval T_{k+1} to calculate $u^{(k+1)}(t)$. Using the delayed information, the control force $\Phi_c^T u(t)$ acting on the significant modes z_c is proposed as:

$$\left\{ \begin{array}{l} \Phi_c^T u^{(1)}(t) = 0 \\ \Phi_c^T u^{(k)}(t) = -\{\Phi_c^T f^{(k-1)}(t - \Delta) + E^{(k-1)}(t - \Delta)\} \\ \quad = -[C_c^{-1} \ddot{y}^{(k-1)}(t - \Delta) + 2\Psi_c \Omega_c C_c^{-1} \dot{y}^{(k-1)}(t - \Delta) \\ \quad \quad + \Omega_c^2 C_c^{-1} y^{(k-1)}(t - \Delta) - \Phi_c^T u^{(k-1)}(t - \Delta)]; \quad k > 1 \end{array} \right. \quad (15)$$

To ensure the stability of the controlled system, the control force $u(t)$ must be chosen to make the error $E(t)$ in (14) independent the control process. This means that the control force $\Phi_r^T u(t)$ acting on the insignificant vibrational mode z_r must be set to zero for the entire time duration

$$\Phi_r^T u(t) = 0. \quad (16)$$

At last, we determine the physical control force $u(t)$ by transformation from modal space to state space:

$$u = \begin{bmatrix} \Phi_c^T \\ \Phi_r^T \end{bmatrix}^{-1} \begin{bmatrix} \Phi_c^T \\ \Phi_r^T \end{bmatrix} u = \begin{bmatrix} \Phi_c^T \\ \Phi_r^T \end{bmatrix}^{-1} \begin{bmatrix} \Phi_c^T u \\ 0 \end{bmatrix}, \quad (17)$$

where $\Phi_c^T u(t)$ is calculated from (15). The control law using the combination of the identification algorithm and the modal superposition method is described by (15) and (17). Substituting (15) and (16) into (10) and (11), the modal differential equations of the controlled system take the form:

$$\begin{aligned}\ddot{z}_c(t) + 2\Psi_c \Omega_c \dot{z}_c(t) + \Omega_c^2 z_c(t) &= -\Phi_c^T f(t - \Delta) - \Phi_c^T E(t - \Delta) + \Phi_c^T f(t) \\ \ddot{z}_r(t) + 2\Psi_r \Omega_r \dot{z}_r(t) + \Omega_r^2 z_r(t) &= \Phi_r^T f(t)\end{aligned}$$

One has:

$$\|-\Phi_c^T f(t - \Delta) - \Phi_c^T E(t - \Delta) + \Phi_c^T f(t)\| \leq \Delta \|\dot{f}(\zeta)\| + \|\Phi_c^T E(t)\|.$$

It can be seen that the norm of the excitation acting on the significant modes z_c is small if the time delay and the error term E are small. Because the error term E defined in (14) depends only on the insignificant modes z_r , we assume this error is sufficiently small. However, the effect of this term on the control process should be the subject for further investigations.

5. NUMERICAL SIMULATION

Consider an eight-story building subject to base acceleration as shown in Fig. 2.

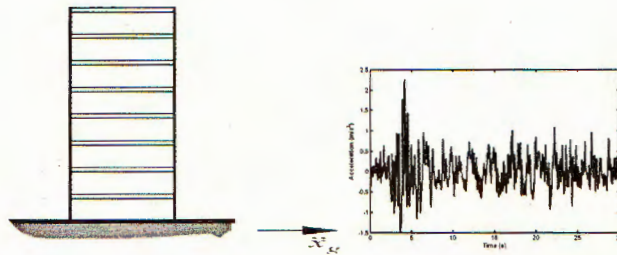


Fig. 2. Model of an eight story building subjected to base acceleration

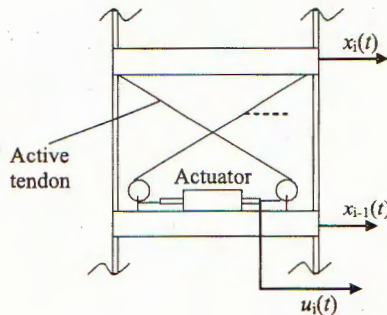


Fig. 3. Active tendon system between two floors

The history of base acceleration is taken from the N-S component recorded at Hachinohe City during the Tokachioki earthquake of May 16, 1968. The absolute peak acceleration of the earthquake record is 2.25m/s^2 . The characteristics of the building are the same for each story: floor mass m , elastic stiffness k and internal damping coefficient d . The control force is applied to the structure through a set of tendons connected to an actuator placed between each two floors. The active tendon generally consists of a set of prestressed tendons whose tensions are controlled by electrohydraulic servomechanism [13]. Fig. 3 shows an active tendon mechanism between two floors:

It is not difficult to derive the structural motion equation as:

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = u(t) + f(t)$$

where the mass, damping and stiffness matrices have form

$$M = \begin{bmatrix} 8 & 7 & 6 & \dots & 1 \\ 7 & 7 & 6 & \dots & 1 \\ 6 & 6 & 6 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} m; \quad D = \text{diag}(d, d, \dots, d); \quad K = \text{diag}(k, k, \dots, k)$$

The displacement vector x , the control vector u and the excitation vector f have the form:

$$\begin{aligned} x &= [x_1 \quad x_2 - x_1 \quad x_3 - x_2 \quad \dots \quad x_8 - x_7]^T \\ u &= -2k_c \cos \alpha [u_1 \quad u_2 \quad u_3 \quad \dots \quad u_8]^T \\ f &= -[8 \quad 7 \quad 6 \quad \dots \quad 1]^T m \ddot{x}_g \end{aligned}$$

in which, x_i ($i=1,..8$) are the relative displacement of the i^{th} floor with respect to the foundation, u_i ($i=1,..8$) are the corresponding actuator displacements, k_c , α and \ddot{x}_g denote, respectively the tendon stiffness, the tendon angle and the base acceleration. Let the parameters take values as in [14]: $m= 345.6$ metric tons, elastic stiffness $k= 3.404 \times 10^5$ kN/m, internal damping coefficient $d= 2937$ metric tons/sec, $k_c=10^5$ kN/m and $\alpha=36^\circ$. The natural frequencies of this building are 5.79, 17.18, 27.98, 37.83, 46.39, 53.37 and 61.70 rad/s. The damping ratios of some first modes are 2.4%, 7.4%, 12%, 16.3%. We assume only one sensor measures the relative displacement of a certain floor with respect to the below floor. If the sensor is located on the i^{th} floor, the measurement matrix C in (2) has the form:

$$C = [0 \quad 0 \quad \dots \quad 1 \quad \dots \quad 0]$$

i

The displacement vector of the structure has 8 components, of which only the first mode is controlled by the identification algorithm. The numerical simulation is taken with different time delays and with different sensor locations. As we see from (14), the magnitude of the error term depends mainly on the magnitude of the matrix $C_c^{-1}C_r$. Table 1 shows the norms of $C_c^{-1}C_r$ for different sensor locations

Table 1. The norms of $C_c^{-1}C_r$ in different cases

Sensor Floor Location	1	2	3	4	5	6	7	8
Norm of $C_c^{-1}C_r$	11.2	16.4	17.6	19.8	23.4	30.0	43.7	86.0

It can be seen that the error produced through the truncation process in (14) might be larger if the sensor location is higher. To see this more clearly, we plot the history of the error term $E(t)$ in (14). The histories of $E(t)$ are plotted in Fig. 4 for each case of the sensor location. It can be seen that, the 3rd floor is the best sensor location and the 8th floor is the worst sensor location. However, more investigate need to be done in the future to find the method seeking the optimal sensor locations.

The time delay is taken as 1/500 and 1/800 of the total duration time. The displacements of the top floor relative to foundation and actuator displacements are compared in Table 2

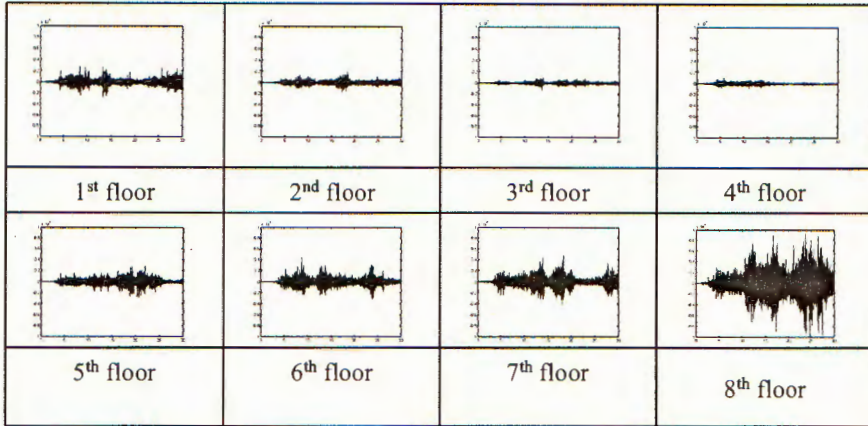


Fig. 4. Error term $E(t)$ histories for 8 choices of sensor location

Table 2. The maxima displacements of the top floor in the numerical simulation

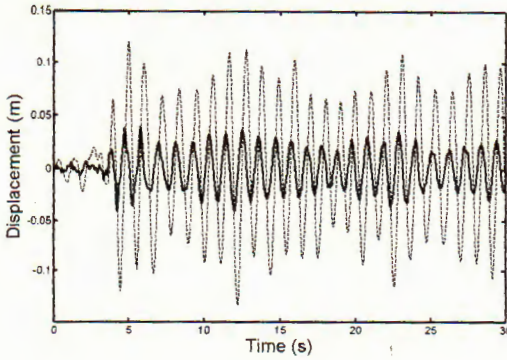
Sensor location	Controlled, Time delay $\Delta=0.2\%T$ (cm)	Controlled, Time delay $r\Delta=0.125\%T$ (cm)	Uncontrolled (cm)
1 st floor	6.76	5.13	13.4
2 nd floor	5.61	4.34	
3 rd floor	4.10	2.85	
4 th floor	4.79	2.61	
5 th floor	4.90	3.61	
6 th floor	7.26	5.70	
7 th floor	8.31	7.53	
8 th floor	10.23	8.89	

The histories of the top floor displacement in two cases of sensor location: 3rd floor (the best case) and 8th floor (the worst case) are shown in Fig. 5 and 6. In the figures, bold lines trace the controlled response, and light dotted lines the uncontrolled response thin and dotted lines are uncontrolled responses

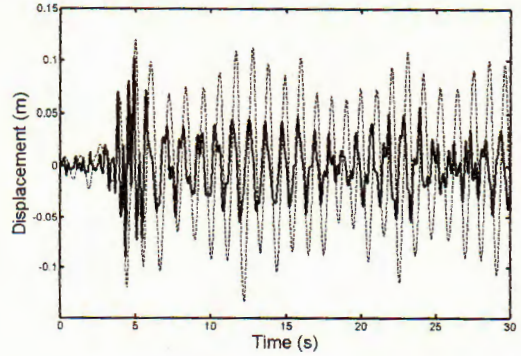
As we see, the effect of identification algorithm depends on the error term $E(t)$ and the time delay Δ . The more small $E(t)$ and Δ are, the better identification control algorithm is.

6. CONCLUSION

The aim of this paper is to extend the identification algorithm for feedback active control of incomplete measured systems. The system is expanded to the modal space. A limited number of sensors is used to measure some components of the displacement vector. Using this incomplete information, an algorithm is presented to identify the external excitation acting on some first modes. The excitation is identified with a time delay and a small error term. The magnitude of the error term depends on sensor numbers and locations. To illustrate the algorithm, the numerical simulation is applied to an eight story

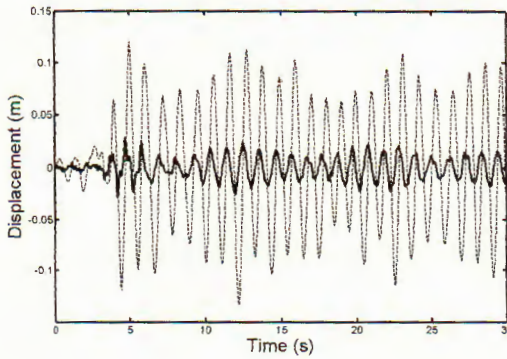


(a)

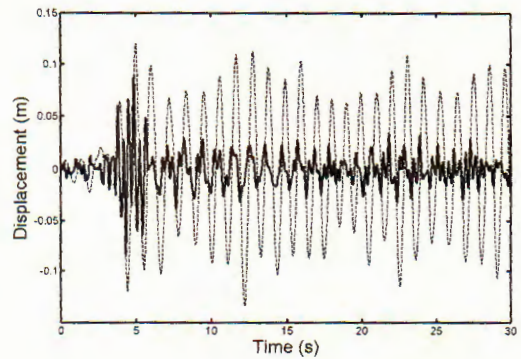


(b)

Fig. 5. The histories of the top floor displacement, $\Delta=0.2\%T$, sensor is located on the 3rd floor (a) and on the 8th floor (b)



(a)



(b)

Fig. 6. The histories of the top floor displacement, $\Delta=0.125\%T$, sensor is located on the 3rd floor (a) and on the 8th floor (b)

building subjected to base acceleration. The effects of the time delay and sensor location are considered.

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MỞ RỘNG THUẬT TOÁN NHẬN DẠNG CHO BÀI TOÁN ĐIỀU KHIỂN TÍCH CỰC PHẢN HỒI CỦA CÁC HỆ BỊ ĐO HẠN CHẾ

Trong bài báo đã được công bố [1], thuật toán nhận dạng đã được trình bày cho các hệ được điều khiển tích cực phản hồi. Tuy nhiên, phương pháp này mới chỉ áp dụng cho các hệ được đo hoàn toàn. Mục đích của bài báo là trình bày một sự kết hợp giữa thuật toán nhận dạng và phương pháp chồng chất các dạng riêng để điều khiển các hệ bị đo hạn chế. Đáp ứng của hệ được tách bằng kỹ thuật dạng riêng. Kích động ngoài tác động vào một số dạng riêng đầu tiên được nhận dạng với một thời gian trễ và một thành phần sai số phụ thuộc vào vị trí đặt đầu đo. Sau đó lực điều khiển được sinh ra để cân bằng với kích động ngoài đã được nhận dạng. Mô phỏng số được thực hiện cho mô hình nhà 8 tầng chịu tải gia tốc nền