

THE ANALYSIS OF COMPOSITE BEAMS WITH PARTIAL INTERACTION USING THE GENERAL TECHNIQUE METHOD

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Abstract. This paper presents a modeling technique that derives from the work of Newmark to describe the behavior of steel-concrete composite beams with elastic shear connection. The model is used to derive expressions for beam curvature, rotation and deflection under monotonic load from which the stiffness matrix is derived and finite element analysis performed on a set of illustrative examples. Model results are compared to those obtained using other method

1. INTRODUCTION

Steel and concrete composite structural elements have been used in construction since the early 1920s. Mechanical connections lock together steel and concrete to cause the entity to behave as a single element. Composite beam deflections stress distribution and modes of failure are governed by the strength and reliability of the shear connection between the joined materials. To design effective composite sections, engineers must possess not only a clear understanding of the mechanical properties of steel and concrete but also the nature of the bond between them [2]. A topic of modern research into composite beam behavior addresses partial shear interaction between joined materials.

Newmark [1] derived a differential equation to describe general composite elements fabricated from dissimilar materials. Ranzi [6] used a direct stiffness formulation based on an element possessing 8DOF to describe vertical displacement, rotation and slip. Faella [4] derived an element stiffness matrix and fixed-end nodal forces as solutions of the Newmark equation.

This paper models the partial interaction between materials in a composite steel-concrete beam fabricated with elastic shear connections placed at discrete intervals along the length of a beam. The model permits the ready calculation of elemental curvature, rotation and deflection under monotonic loads from which the elemental stiffness element can be readily obtained.

The paper compares results obtained from the model with those computed using alternative forms of FEM analyses and other methods.

2. MODEL FORMULATION

2.1. General

Fig. 1 shows a composite beam in cross section and the assumed steel-concrete strain distribution as that found in [1]. The problem assumes:

1. The cross section is rigid and does not distort
2. The curvature is the same in both steel and concrete
3. The shear connection exhibits linear-elastic behavior as shown in Fig. 2
4. Concrete and steel exhibit linearly elastic stress/strain behavior

Denote properties section element as:

- A_c, A_r, A_s : area of the concrete, of the reinforcement and of the steel.
- A_1, A_2, A : area of the element 1, 2 and of the section
- $A_1 = A_c + A_r$; $A_2 = A_s$; $A = A_1 + A_2$
- S_c, S_r, S_s : first moment of area of the concrete, the reinforcement and steel joist, about the arbitrary reference axis.
- I_c, I_r, I_s : second moment of area of the concrete, reinforcement and steel joist, about the arbitrary reference axis.
- E_c, E_r, E_s : elastic moduli of concrete, reinforcement and structural steel
- $AE_1 = A_c E_c + A_r E_r$; $AE_2 = A_s E_s$; $AE = AE_1 + AE_2$; $SE_1 = S_c E_c + S_r E_r$
- $SE_2 = S_s E_s$; $SE = SE_1 + SE_2$; $IE_1 = I_c E_c + I_r E_r$; $IE_2 = I_s E_s$; $IE = IE_1 + IE_2$

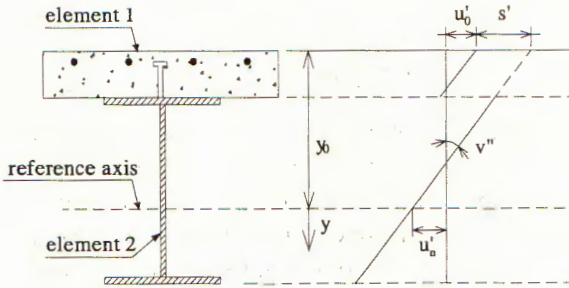


Fig. 1. Cross-section and strain diagram

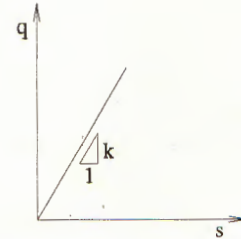


Fig. 2. Connector response

In Fig. 1 the reference y -axis is located a positive distance y^0 below the top of the cross-section. Axial displacements are referred to the reference axis, which without loss of generality can coincide with the top of the steel joist. u'_0 is the top fiber strain, u'_r is the strain at the reference axis, v'' is the curvature, and s' is the slip strain that characterizes the partial nature of the interaction.

Material properties can be expressed generically as:

$$\begin{aligned}
 \sigma_c &= E_c \varepsilon_c = E_c [u'_0 + (y + y_0)v''] , \\
 \sigma_r &= E_r \varepsilon_r = E_r [u'_0 + (y + y_0)v''] , \\
 \sigma_s &= E_s \varepsilon_s = E_s [u'_0 + (y + y_0)v'' + s'] .
 \end{aligned} \tag{1}$$

Fig. 2 illustrates the relationship between interface shear flow q per unit length and slip at the interface. From [1], [2] write:

$$\begin{aligned} q &= k \cdot s, \\ k &= n_c \frac{K}{i_c}, \end{aligned} \quad (2)$$

where: k - the shear connection stiffness (force per length²), K - the strength of the shear connection (in Pust - test), n_c - number of the shear connectors present at the cross-section, i_c - shear connector longitudinal spacing.

2.2. The model

Figs. 3, 4 illustrate an elemental beam and the upper material free body diagram. [5], [6].

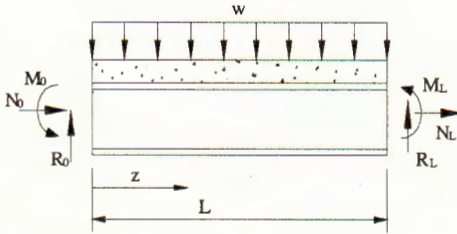


Fig. 3. General single span beam

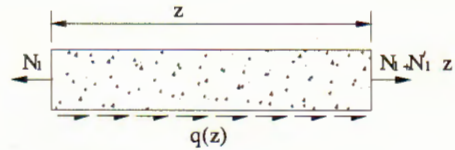


Fig. 4. Free body diagram of the top element

2.2.1. Horizontal equilibrium

Consider the composite beam illustrated in Fig. 3 and Fig. 4.

$$N = \sum_{i=1}^2 \int_{A_i} \sigma_i dA_i = N_1 + N_2; \quad M = \int_A \sigma y dA, \quad (3)$$

where N_1, N_2 are the axial forces in the element 1, 2 and M is the bending moment

From Eq. (1) and (3), we have:

$$N_1 = \int_{A_1} \sigma dA = AE_1 u'_0 + SE_1 v'' + y_0 AE_1 v'', \quad (4)$$

$$N_2 = \int_{A_2} \sigma dA = AE_2 u'_0 + SE_2 v'' + y_0 AE_2 v'' + AE_2 s',$$

$$N = N_1 + N_2 = AEu'_0 + SEv'' + y_0 AEv'' + AE_2 s', \quad (5)$$

$$M = \int_A y \sigma dA = SEu'_0 + IEv'' + y_0 SEv'' + SE_2 s'. \quad (6)$$

Solve (5) and (6) for u'_0, v'' :

$$u'_0 = a_1 M + a_2 N + a_3 s', \quad (7)$$

$$v'' = b_1 M + b_2 N + b_3 s'. \quad (8)$$

The expressions for the rotation and deflection are obtained by integrating the curvature, the strain along the length of the beam length denoted z . Hence:

$$v' = b_1 \int M dz + b_2 \int N dz + b_3 \int s' dz + D_1, \quad (9)$$

$$v = \int v' dz + D_2 = b_1 \iint M dz dz + b_2 \iint N dz dz + b_3 \iint s' dz dz + D_1 z + D_2. \quad (10)$$

Interface lip may be derived from the strain diagram as:

$$s = u_n - u_0 - y_0 v'. \quad (11)$$

From Eq. (11), u_n as:

$$u'_n = u'_0 + y_0 v'' + s'. \quad (12)$$

From (7), (8) and (12):

$$u'_n = l_1 M + l_2 N + l_3 s', \quad (13)$$

$$u_n = l_1 \int M dz + l_2 \int N dz + l_3 \int s' dz + D_3 \quad (14)$$

in which:

$$a_1 = -\frac{SE + y_0 AE}{AEIE - SE^2}, \quad a_2 = \frac{y_0 SE + AE}{AEIE - SE^2},$$

$$a_3 = \frac{SESE_2 + y_0(SE_2 AE_1 - SE_1 AE_2) - AE_2 IE}{AEIE - SE^2};$$

$$b_1 = \frac{AE}{AEIE - SE^2}, \quad b_2 = -\frac{SE}{AEIE - SE^2}, \quad b_3 = \frac{SE_1 AE_2 - SE_2 AE_1}{AEIE - SE^2};$$

$$l_1 = a_1 + y_0 b_1, \quad l_2 = a_2 + y_0 b_2, \quad l_3 = a_3 + y_0 b_3 + 1.$$

2.2.2. Shear connection

From Fig. 4 and using (4), (7) and (8), we obtain the internal axial force N_1 as [5]:

$$N_1 = q_1 M + q_2 N + q_3 s', \quad (15)$$

$$q_1 = \frac{SE_1 AE_2 - SE_2 AE_1}{AEIE - SE^2},$$

$$q_2 = \frac{AE_1 IE - SE_1 SE}{AEIE - SE^2},$$

$$q_3 = \frac{SE_1^2 AE_2 + SE_2^2 AE_1 - IE AE_1 AE_2}{AEIE - SE^2}.$$

Consider the horizontal equilibrium of the free body diagram,

$$\left(N_1 + \frac{dN_1}{dz} \delta z + q \delta z \right) - N_1 = 0. \quad (16)$$

From (2), (15) and (16),

$$\alpha \frac{d^2 s}{dz^2} - ks = \alpha_1 \frac{dM}{dz} + \alpha_2 \frac{dN}{dz} \quad (17)$$

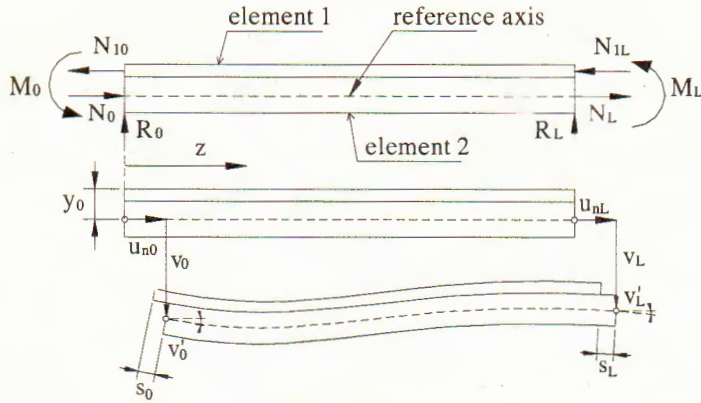


Fig. 5. Nodal Displacement, End Actions and Slip

in which:

$$\alpha = -\frac{SE_1^2AE_2 + SE_2^2AE_1 - IEAE_1AE_2}{AEIE - SE^2},$$

$$\alpha_1 = \frac{SE_1AE_2 - SE_2AE_1}{AEIE - SE^2},$$

$$\alpha_2 = \frac{AE_1IE - SE_1BE}{AEIE - SE^2}.$$

From (17),

$$s = C_1e^{\mu z} + C_2e^{-\mu z} + s_{0,p}, \tag{18}$$

where: $\mu^2 = \frac{k}{\alpha}$; $s_{0,p}$: depends on the nature of the applied load conditions

2.3. The stiffness matrix k ; the vector of the equivalent nodal loads

2.3.1. Equilibrium

Using the beam illustrated in Fig. 3, Fig. 5 defines 8DOF to encompass vertical displacement, rotation and material slip.

From [3], [6], obtain element equilibrium by direct evaluation.

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ & & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ & & & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ & & & & k_{55} & k_{56} & k_{57} & k_{58} \\ & & & & & k_{66} & k_{67} & k_{68} \\ & & & & & & k_{77} & k_{78} \\ SYM & & & & & & & k_{88} \end{bmatrix} \{q\} = \{g\} + \{g_{eq}\} \tag{19}$$

where: $\{q\} = [u_{n0}, v_0, v'_0, s_0, u_{nL}, v_L, v'_L, s_{1L}]^T$,
 $\{g\} = [N_0, R_0, M_0, N_{10}, N_L, R_L, M_L, N_{1L}]^T$,
 $\{q\}, \{g\}$ is the vector of nodal actions and displacements,

$\{g_{eq}\}$ is the vector of equivalent nodal.

2.3.2. The stiffness matrix K

The stiffness matrix K is obtained by restraining all DOF save the one related to the column considered, to which a unit displacement is applied.

The moment and the axial force along the beam can be expressed as

$$M = -M_0 + R_0z; \quad N = -N_0 \quad (20)$$

The slip can be expressed in the form

$$s = C_1 e^{\mu z} + C_2 e^{-\mu z} - \frac{\alpha_1}{k} R_0 \quad (21)$$

The first column of the stiffness matrix K

The load state is given by

$$\{q\} = [1, 0, 0, 0, 0, 0, 0, 0]_{(1)}^T \quad (22)$$

From (19) and (22),

$$\begin{aligned} & \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ & & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ & & & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ & & & & k_{55} & k_{56} & k_{57} & k_{58} \\ & & & & & k_{66} & k_{67} & k_{68} \\ & & & & & & k_{77} & k_{78} \\ & & & & & & & k_{88} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} N_0 \\ R_0 \\ M_0 \\ N_{10} \\ N_{10} \\ R_L \\ M_L \\ N_{1L} \end{Bmatrix}_{(1)} \quad (23) \\ & \Leftrightarrow \begin{Bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \\ k_{51} \\ k_{61} \\ k_{71} \\ k_{81} \end{Bmatrix} = \begin{Bmatrix} N_0 \\ R_0 \\ M_0 \\ N_{10} \\ N_{10} \\ R_L \\ M_L \\ N_{1L} \end{Bmatrix}_{(1)} \end{aligned}$$

The stiffness coefficients of the first column are derived by Eq. (23). The constant of integrations: C_1 ; C_2 ; D_1 ; D_2 and D_3 are determined by noting $s_0 = 0$, $s_L = 0$, $\nu_0 = 0$, $\nu_L = 0$, $u_{n0} = 1$. The actions N_0 , R_0 , M_0 can be determined after noting $\nu' = 0$, $\nu'_L = 0$, $u_{nL} = 0$.

From consideration of statics, obtain the nodal actions N_L , R_L , M_L are:

$$N_{10} = N_1 |_{z=0}; \quad N_{1L} = -N_1 |_{z=L} \quad (24)$$

2.3.3. Equivalent nodal loads due to uniform load w

The moment and the axial force along the beam can be expressed as

$$M = -M_0 + R_0z - \frac{wz^2}{2}; \quad N = -N_0 \quad (25)$$

The slip equilibrium can be expressed as

$$s = C_1 e^{\mu z} + C_2 e^{-\mu z} - \frac{\alpha_1}{k} R_0 + \frac{\alpha_1 w}{k} z \quad (26)$$

The enforcing the kinematic state is described as

$$\{d\} = [0, 0, 0, 0, 0, 0, 0, 0]^T_{(q)} \quad (27)$$

Similarly to 2.3.2, the vector of equivalent nodal loads is determined.

3. APPLICATIONS

3.1. Built in composite beam subject to a uniform load w

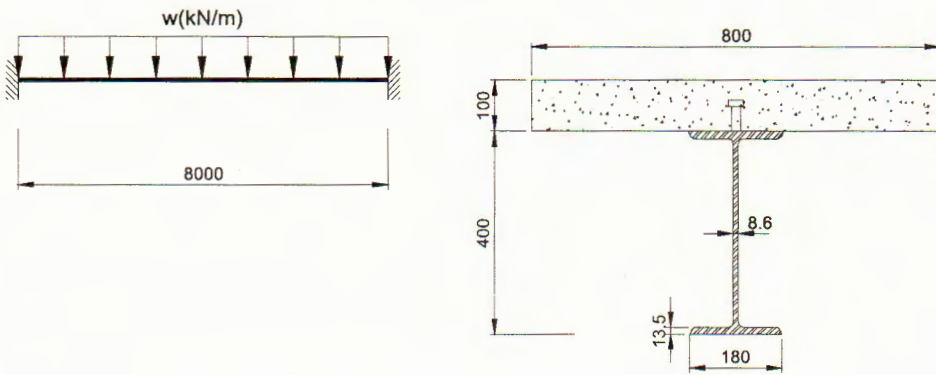


Fig. 6. Built-In beam and composite cross-section

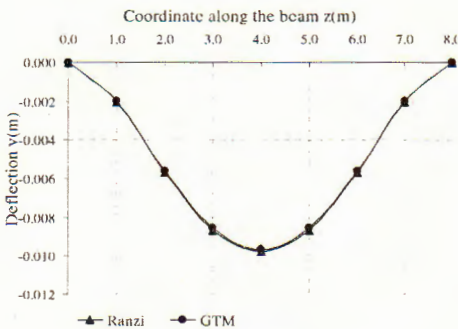


Fig. 7. Deflection $\mu L = 11.02$

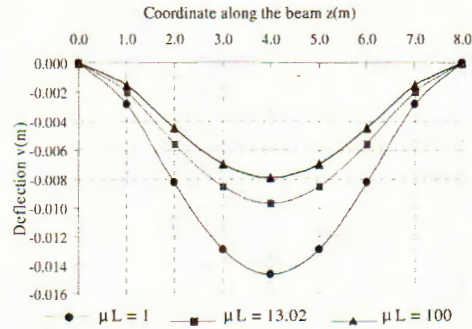


Fig. 8. Deflection versus μL

For the beam shown in Fig. 6, let $E_c = 2.1 \times 10^7$ KPa, $E_s = 2.1 \times 10^8$ KPa and the shear connection stiffness $k = 8.33 \times 10^5$ KPa. Using only one element to complete the analysis using the proposed model (GTM) yields a calculated deflection in perfect agreement with results presented in [5]. Fig. 8 predicts beam deflections for various values of shear connection stiffness (in terms of the dimensionless parameter μL).

3.2. Double Span Beam

Fig. 9 illustrates the 2 span continuous beam labeled CTB4 in Ansourian [7], in which $E_c = 1.565 \times 10^7$ KPa, $E_r = 1.565 \times 10^7$ KPa, $E_s = 1.565 \times 10^7$ KPa, $A_r = 160$ mm² and shear connection stiffness $k = 0.982 \times 10^6$ KPa.

Using twelve stiffness elements to analysis the left span deflection. The results shown in Table 1 and in Fig. 10. It can be seen good result with linear behaviour of material. This highlight ability demonstrated the advantages of the general technique method.

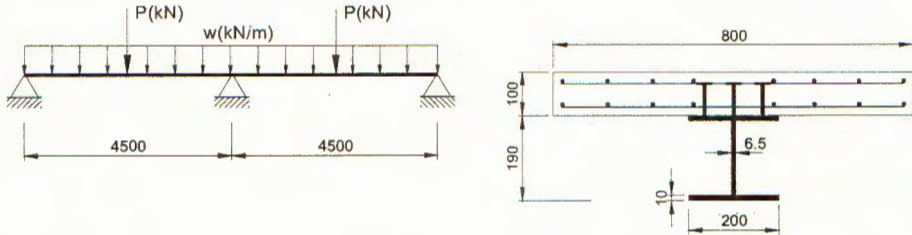


Fig. 9. 2 span beam and composite cross-section

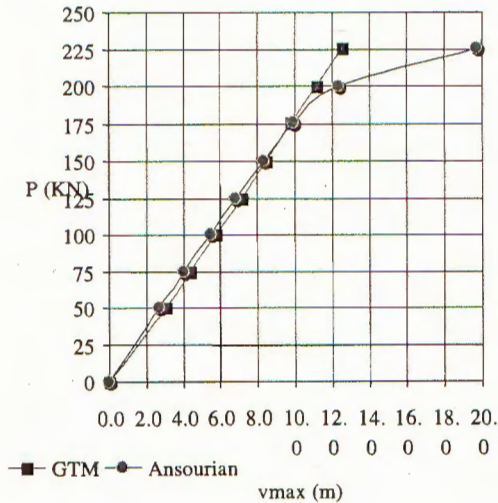


Fig. 10. Left Span Deflection)

Table 1. Left Span middle deflection

LoadP(KN)	Deflection v_{\max} (mm)		Comparision (1) & (2)(%)
	GTM(1)	Ansourian (2)	
0	0.000	0.000	0.000
50	3.037	2.720	11.654
75	4.399	4.017	9.510
100	5.762	5.461	5.512
125	7.125	6.801	4.764
150	8.488	8.346	1.701
175	9.815	9.941	-1.267
200	11.215	12.329	-9.036
225	12.577	19.741	-36.290

4. CONCLUSIONS

This paper describes a general modeling technique for the analysis of steel-concrete composite beams with elastic shear connection (partial interaction). The curvature, the rotation and the deflection formulation have obtained by using this model. Then derivation stiffness matrix K and application in the finite element method to analysis of the composite beams. (This is the 3rd time this same sentence appears in this paper!!)

The results obtained have demonstrated the advantages and the confidence of the GMT method in comparision with the other results and experimental test. Model is simply, this method need to research to analysis composite beam for various loading.

ACKNOWLEDGMENTS

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**PHÂN TÍCH DẦM THÉP-BÊ TÔNG LIÊN HỢP CÓ XÉT ẢNH HƯỞNG
TƯƠNG TÁC BÁN PHẦN Ở MẶT TIẾP XÚC BẰNG PHƯƠNG PHÁP KỸ
THUẬT TỔNG QUÁT**

Bài báo trình bày mô hình kỹ thuật tổng quát dựa trên mô hình kỹ thuật của Newmark để mô tả ứng xử của dầm thép-bê tông liên hợp với liên kết chịu cắt tuyến tính. Sử dụng mô hình này xây dựng các phương trình chuyển vị, biến dạng của phần tử dưới tác dụng của tải trọng tĩnh và sử dụng các phương trình này xác định ma trận độ cứng K . Áp dụng kết quả vào phương pháp phần tử hữu hạn để khảo sát các bài toán cơ bản. Kết quả thu được so sánh với kết quả khác.