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INFLUENCE OF TRAJECTORIES ON THE JOINT TORQUES OF KINEMATICALLY REDUNDANT MANIPULATORS

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Abstract. This paper presents an algorithm for solving the inverse dynamics problem of redundant manipulators using MAPLE software. The method has the advantage of generating efficient symbolic solutions which reduces the computational cost. The influence of trajectories on the joint torques of redundant manipulators is considered. The theory is illustrated by the numerical simulation of a redundant four-link planar manipulator.
Keywords: Redundant manipulator, inverse dynamics, kinematic redundancy

1. INTRODUCTION

The operation tasks of today's robot manipulators become more sophisticated and require that manipulators possess more and more degrees of freedom (DOF) to offer greater flexibility. The kinematic redundancy occurs when the DOF of a manipulator is more than the minimum number necessary for executing a given operation task [1]. The extra DOF presented in redundant manipulators can be used to avoid obstacles and kinematic singularities, to increase the workspace or to optimize the motion of the manipulator relatively to a cost function. There is widespread interest in redundant manipulators due to such advantages.

A significant number of paper has been published concerning the problems of kinematic redundancy and much achievement has been reviewed by [2]. Recently, there have been several scientific papers focused upon kinematic analysis, motion planning and controls of redundant robot manipulators [1-9]. However, it should be pointed out that the development on the theory for solving the inverse dynamic problem of redundant manipulators is still limited and the literature on this respect therefore is little.

In this paper, the inverse problem of kinematics of redundant manipulators is briefly addressed. We have proposed an efficient calculating method to find the joint variables which give the desired workspace trajectory of the end-effector. The inverse dynamics problem is considered to study the influence of trajectories on the joint torques of redundant manipulators. In the example, the developed method is employed to the inverse dynamic analysis of a redundant four-link planar manipulator. A specialized program has been developed on the MAPLE computing environment for this study.

2. INVERSE DYNAMICS OF REDUNDANT MANIPULATORS

The robotic systems under study are n DOF serial manipulators. We consider the redundant systems which have more DOF than needed to accomplish the operation task, i.e. the dimension of the joint space n exceeds the dimension of the task space m. Let the configuration of the manipulator be represented by vector $\mathbf{q}$ of n joint positions, and the end-effector position and orientation by m-dimensional vector $\mathbf{x}$ of task positions and orientations. The joint and task positions are related by the following expression

$$\mathbf{x} = \mathbf{f}(\mathbf{q}), \tag{2.1}$$

where $\mathbf{f}$ is m-dimensional vector function representing the manipulator forward kinematics and the vectors $\mathbf{q}$ and $\mathbf{x}$ are defined by

$$\mathbf{x} = [x_1, \dots, x_m]^T, \quad \mathbf{q} = [q_1, \dots, q_n]^T. \tag{2.2}$$

Differentiating Eq. (2.1) with respect to time, we obtain the relation between velocities

$$\dot{\mathbf{x}} = \mathbf{J}\,\dot{\mathbf{q}}, \tag{2.3}$$

where $\mathbf{J} = \dfrac{\partial \mathbf{f}}{\partial \mathbf{q}}$ is the $m \times n$ manipulator Jacobian matrix.

In the case of redundant manipulators, there can exist also an internal motion which does not contribute to the motion of the end-effector. Hence, the general solution of Eq. (2.3) can be given as follows [4], [7]

$$\dot{\mathbf{q}} = \mathbf{J}^+\dot{\mathbf{x}} + \left(\mathbf{E}_n - \mathbf{J}^+\mathbf{J}\right)\,\mathbf{y}, \tag{2.4}$$

where $\mathbf{J}^+ \in \mathrm{R}^{n\times m}$ is the pseudo-inverse of matrix $\mathbf{J}$ [6], $\mathbf{y} \in \mathrm{R}^n$ is an arbitrary vector, and $\mathbf{E}_n \in \mathrm{R}^{n\times n}$ denotes an identity matrix. If the exact solution does not exist, Eq. (2.4) covers all the least-squares solutions that minimize $\|\dot{\mathbf{x}} - \mathbf{J}\dot{\mathbf{q}}\|$. Therefore, $\mathbf{J}(\mathbf{q})$ will always have a full rank, so that rank $(\mathbf{J}) = m$. Note that the solution according to Eq. (2.4) gives the minimum joint velocities for the desired workspace velocity [4].

Differentiating Eq. (2.3) again with respect to time, we obtain the relation between joint space and task space accelerations as

$$\ddot{\mathbf{x}} = \mathbf{J}\,\ddot{\mathbf{q}} + \dot{\mathbf{J}}\,\dot{\mathbf{q}}. \tag{2.5}$$

Hence, Eq. (2.5) becomes [4], [7]

$$\ddot{\mathbf{q}} = \mathbf{J}^+\left(\ddot{\mathbf{x}} - \dot{\mathbf{J}}\,\dot{\mathbf{q}}\right) + \left(\mathbf{E}_n - \mathbf{J}^+\mathbf{J}\right)\,\mathbf{z}, \tag{2.6}$$

where $\mathbf{z} \in \mathrm{R}^n$ is an arbitrary vector. The joint angles can then be calculated by the finite difference methods. For example, using the difference approximation produces

$$\dot{\mathbf{q}}_k = \frac{\mathbf{q}_{k+1} - \mathbf{q}_k}{\Delta t}, \tag{2.7}$$

$$\dot{\mathbf{x}}_k = \frac{\mathbf{x}_{k+1} - \mathbf{x}_k}{\Delta t}. \tag{2.8}$$

Substituting Eqs. (2.7) and (2.8) into Eq. (2.4), one obtains

$$\mathbf{q}_{k+1} = \mathbf{q}_k + \mathbf{J}^+(\mathbf{q}_k)\,(\mathbf{x}_{k+1} - \mathbf{x}_k) + \left[\mathbf{E}_n - \mathbf{J}^+(\mathbf{q}_k)\,\mathbf{J}(\mathbf{q}_k)\right]\,\mathbf{z}_k\Delta t. \tag{2.9}$$

Eqs. (2.4), (2.6) and (2.9) form a basis of the inverse kinematics of a redundant manipulator.

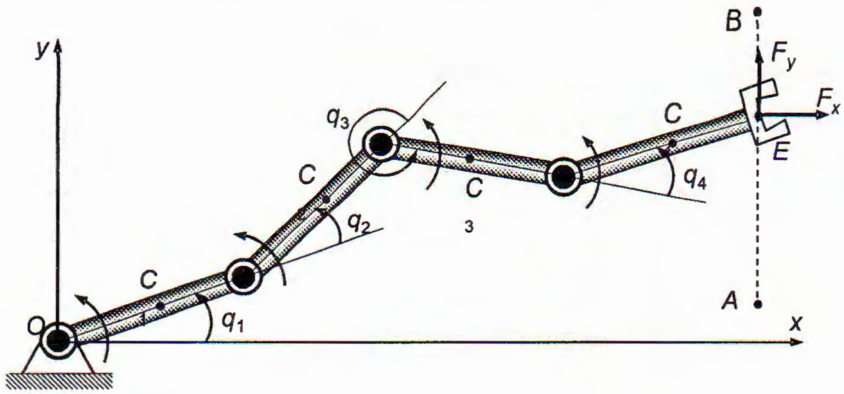

Fig. 1. A redundant four-link planar manipulator

The dynamics of robot manipulators is generally represented by the following equation [10-12]

$$\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \tag{2.10}$$

where $\mathbf{M}(\mathbf{q}) \in \mathrm{R}^{n\times n}$ denotes an inertia matrix, $\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) \in \mathrm{R}^{n}$ is a torque vector caused by centrifugal and Coriolis forces, $\mathbf{g}(\mathbf{q}) \in \mathrm{R}^{n}$ is a gravity torque vector, and $\boldsymbol{\tau} \in \mathrm{R}^{n}$ represents a joint torque vector.

Let $\mathbf{x} \in \mathrm{R}^{m}$ define the position and orientation of the end-effector in the task space. The joint angle, velocity and acceleration vectors can be determined by using Eqs. (2.4), (2.6) and (2.9). The joint torque vector $\boldsymbol{\tau}$ can then be calculated by Eq. (2.10).

3. ILLUSTRATING EXAMPLE FOR THE INFLUENCE OF TRAJECTORIES ON THE JOINT TORQUES

In the following example we introduce the application of the theory described above to a four-link planar robot manipulator shown in Fig. 1. The manipulator is connected directly to four high torque actuators. The first actuator drives link 1 and the fourth actuator drives link 4. As can be seen from the figure, the manipulator has two redundant DOF.

The configuration of the manipulator can be described by four relative rotation angles q_1, q_2, q_3 and q_4. The kinematic relationships for the links of the manipulator can then be expressed in the form

$$\begin{aligned} x_E &= l_1\cos(q_1) + l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) + l_4\cos(q_1+q_2+q_3+q_4),\\ y_E &= l_1\cos(q_1) + l_2\sin(q_1\ +q_2) + l_3\sin(q_1+q_2+q_3) + l_4\sin(q_1+q_2+q_3+q_4), \end{aligned} \tag{3.1}$$

where l_i denotes the length of the i^{th} link, x_E and y_E are workspace coordinates of the end-effector E in the fixed coordinate frame $\{Oxy\}$. Using Eq. (3.1) we obtain the relationship between velocities in matrix form as

$$\dot{\mathbf{x}} = \mathbf{J}\,\dot{\mathbf{q}}, \tag{3.2}$$

in which $\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_E & \dot{y}_E \end{bmatrix}^T$, $\mathbf{q} = [q_1,\ q_2,\ q_3,\ q_4]^T$ and the manipulator Jacobian matrix $\mathbf{J}$ is given by

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \end{bmatrix}, \tag{3.3}$$

where

$$
\begin{aligned}
J_{11} &= -l_4 \sin(q_1 + q_2 + q_3 + q_4) - l_3 \sin(q_1 + q_2 + q_3) - l_2 \sin(q_1 + q_2) - l_1 \sin q_1, \\
J_{12} &= -l_4 \sin(q_1 + q_2 + q_3 + q_4) - l_3 \sin(q_1 + q_2 + q_3) - l_2 \sin(q_1 + q_2), \\
J_{13} &= -l_4 \sin(q_1 + q_2 + q_3 + q_4) - l_3 \sin(q_1 + q_2 + q_3), \\
J_{14} &= -l_4 \sin(q_1 + q_2 + q_3 + q_4), \\
J_{21} &= l_4 \cos(q_1 + q_2 + q_3 + q_4) + l_3 \cos(q_1 + q_2 + q_3) + l_2 \cos(q_1 + q_2) + l_1 \cos q_1, \\
J_{22} &= l_4 \cos(q_1 + q_2 + q_3 + q_4) + l_3 \cos(q_1 + q_2 + q_3) + l_2 \cos(q_1 + q_2), \\
J_{23} &= l_4 \cos(q_1 + q_2 + q_3 + q_4) + l_3 \cos(q_1 + q_2 + q_3) \\
J_{24} &= l_4 \cos(q_1 + q_2 + q_3 + q_4).
\end{aligned} \tag{3.4}
$$

Based on the recursive rule expressed by Eq. (2.9) in the previous section, we can calculate the joint angles $\mathbf{q}$ for a given workspace position $\mathbf{x}$ of the end-effector. The angular velocities $\dot{\mathbf{q}}$ and angular accelerations $\ddot{\mathbf{q}}$ can then be numerically determined by using Eqs. (2.4) and (2.6). Hence, the inverse kinematics problem for the four-link planar manipulator is essentially resolved.

The next step will be the formulation of the differential equations of motion of the manipulator according to Eq. (2.10). Let $\mathbf{I}_i$ and $\boldsymbol{r}_i$ be, respectively, the inertia matrix of the i^{th} link referred to the center of mass C_i and the position vector of C_i in the fixed coordinate frame, and let m_i be the mass of the i^{th} link. The reduced inertia matrix $\mathbf{M}$ is given by [12]

$$
\mathbf{M} = \sum_{i=1}^{4} \left[\mathbf{J}_{Ti}^T m_i \, \mathbf{J}_{Ti} + \mathbf{J}_{Ri}^T \, \mathbf{I}_i \, \mathbf{J}_{Ri} \right], \tag{3.5}
$$

where $\boldsymbol{J}_{Ti}(\mathbf{q})$ and $\boldsymbol{J}_{Ri}(\mathbf{q})$ denote the Jacobian matrices that relate velocity $\dot{\mathbf{r}}_i$ and angular velocity $\boldsymbol{\omega}_i$ of the i^{th} link to the joint velocity $\dot{\mathbf{q}}$

$$
\boldsymbol{J}_{Ti}(\boldsymbol{q}) = \frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}}, \quad \boldsymbol{J}_{Ri}(\boldsymbol{q}) = \frac{\partial \omega_i}{\partial \dot{\boldsymbol{q}}}. \tag{3.6}
$$

For simplicity, we assume that the center of mass C_i of i^{th} link is positioned in the middle of the link line. Then, the matrix $\mathbf{M}$ can be determined without difficulty by using Eq. (3.5). For example, we obtain the following expressions with link 2

$$
\mathbf{r_2} = \begin{bmatrix} l_1 \cos(q_1) + \dfrac{l_2}{2} \cos(q_1 + q_2) \\ l_1 \sin(q_1) + \dfrac{l_2}{2} \sin(q_1 + q_2) \\ 0 \end{bmatrix}, \quad \boldsymbol{\omega_2} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}
$$

$$
\mathbf{J}_{T2} = \frac{\partial \mathbf{r_2}}{\partial \mathbf{q}} = \begin{bmatrix} -l_1 \sin(q_1) - \dfrac{1}{2} l_2 \sin(q_1 + q_2) & -\dfrac{1}{2} l_2 \sin(q_1 + q_2) & 0 & 0 \\ l_1 \cos(q_1) + \dfrac{1}{2} l_2 \cos(q_1 + q_2) & \dfrac{1}{2} l_2 \cos(q_1 + q_2) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
\mathbf{J}_{R2} = \frac{\partial \omega_2}{\partial \dot{\mathbf{q}}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.
$$

The total kinetic energy of the manipulator is given by

$$T = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}. \tag{3.7}$$

The total potential energy stored in the manipulator is

$$\Pi = -\sum_{i=1}^{4} m_i\mathbf{g}^T\mathbf{r}_i, \tag{3.8}$$

where $\boldsymbol{g} = [0,\ -g,\ 0\ ]^T$. We consider the case in which actuators exert torques $\tau_1 = M_{0,1},\ \tau_2 = M_{1,2},\ \tau_3 = M_{2,3},\ \tau_4 = M_{3,4}$ at the joints and an external force F is applied at the end-effector as shown in Fig. 1. Then the virtual work produced by these forces and torques is

$$\delta A \ = \ \tau_1\delta q_1 + \tau_2\delta q_1 + \tau_3\delta q_3 + \tau_4\delta q_4 + F_x\delta x_E + F_y\delta y_E. \tag{3.9}$$

Using Eqs. (3.1) and (3.9), we get the generalized forces as follows

$$\begin{aligned}
Q_1^* =&\tau_1 - F_x\left[l_1\sin q_1 + l_2\sin(q_1+q_2) + l_3\sin(q_1+q_2+q_3) + l_4\sin(q_1+q_2+q_3+q_4)\right]\\
&+ F_y\left[l_1\cos q_1 + l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) + l_4\cos(q_1+q_2+q_3+q_4)\right],\\
Q_2^* =&\tau_2 - F_x\left[l_2\sin(q_1+q_2) + l_3\sin(q_1+q_2+q_3) + l_4\sin(q_1+q_2+q_3+q_4)\right]\\
&+ F_y\left[l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) + l_4\cos(q_1+q_2+q_3+q_4)\right],\\
Q_4^* =&\tau_3 - F_x\left[l_3\sin(q_1+q_2+q_3) + l_4\sin(q_1+q_2+q_3+q_4)\right]\\
&+ F_y\left[l_3\cos(q_1+q_2+q_3) + l_4\cos(q_1+q_2+q_3+q_4)\right],\\
Q_3^* =&\tau_4 - F_x\left[l_4\sin(q_1+q_2+q_3+q_4)\right] + F_y\left[l_4\cos(q_1+q_2+q_3+q_4)\right].
\end{aligned} \tag{3.10}$$

The dynamic equations of motion of the redundant manipulator can be derived by using Lagrangian formulation

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_i}) - \frac{\partial T}{\partial q_i} = -\frac{\partial \Pi}{\partial q_i} + Q_i^*, \quad i = 1,\ ...,\ 4. \tag{3.11}$$

If the solution of the inverse kinematics problem is known, then the joint torques can be determined from the dynamic equations of motion as

$$\begin{aligned}
\tau_1 =&\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_1}) - \frac{\partial T}{\partial q_1} + \frac{\partial \Pi}{\partial q_1}\\
&+ F_x\left[l_1\sin q_1 + l_2\sin(q_1+q_2) + l_3\sin(q_1+q_2+q_3) + l_4\sin(q_1+q_2+q_3+q_4)\right]\\
&- F_y\left[l_1\cos q_1 + l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) + l_4\cos(q_1+q_2+q_3+q_4)\right],
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\tau_2 =&\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_2}) - \frac{\partial T}{\partial q_2} + \frac{\partial \Pi}{\partial q_2}\\
&+ F_x\left[l_2\sin(q_1+q_2) + l_3\sin(q_1+q_2+q_3) + l_4\sin(q_1+q_2+q_3+q_4)\right]\\
&- F_y\left[l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) + l_4\cos(q_1+q_2+q_3+q_4)\right],
\end{aligned} \tag{3.13}$$

$$\begin{aligned}\tau_3 =&\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_3}) - \frac{\partial T}{\partial q_3} + \frac{\partial \Pi}{\partial q_3}\\ &+ F_x\left[l_3 \sin(q_1+q_2+q_3) + l_4 \sin(q_1+q_2+q_3+q_4)\right]\\ &- F_y\left[l_3 \cos(q_1+q_2+q_3) + l_4 \cos(q_1+q_2+q_3+q_4)\right],\end{aligned} \tag{3.14}$$

$$\begin{aligned}\tau_4 =&\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_4}) - \frac{\partial T}{\partial q_4} + \frac{\partial \Pi}{\partial q_4}\\ &+ F_x\, l_4 \sin(q_1+q_2+q_3+q_4) - F_y\, l_4 \cos(q_1+q_2+q_3+q_4).\end{aligned} \tag{3.15}$$

Table 1. Parameters of the manipulator

m_1 (kg)	l_1 (m)	I_{z1} (kgm^2)	m_2	l_2	I_{z2}	m_3	l_3	I_{z3}	m_4	l_4	I_{z4}
4.0	0.3	0.03	3.0	0.5	0.06	2.0	0.4	0.03	2.5	0.5	0.05

A computer program on the MAPLE environment is developed to solve the inverse dynamics of the manipulator. The manipulator parameters are given in Table 1.

For the first case, the end-effector is assumed to move along the y-direction from point A to point B with a constant velocity (see Fig. 1) for a period of 10 seconds. A constant external force F is applied at the end-effector, that is, F_x = -5 (N), F_y = - 4 (N). The workspace coordinates of the end-effector $E$are given by

$$\begin{aligned}x_E &= 0.9,\\ y_E &= 0.2 + 0.1t,\end{aligned} \tag{3.16}$$

The following initial values are chosen for the joint angles $\mathbf{q}$: $q_1(0) = 0.524$, $q_2(0) =$ 1.047, $q_3(0)$ =3.516, $q_4(0)$=1.040 (rad). Fig. 2 shows the calculating results of the inverse dynamics corresponding to the given trajectory of the end-effector.

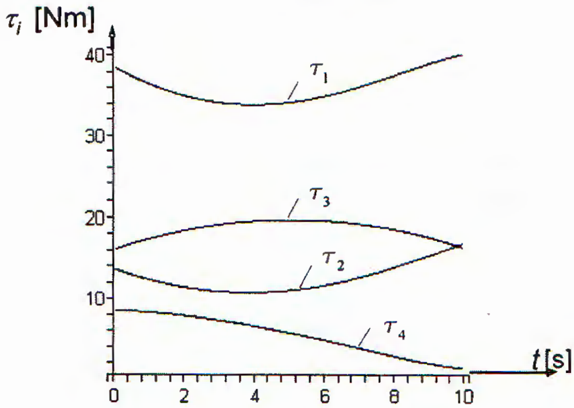

Fig. 2. Joint torques versus time for the first case

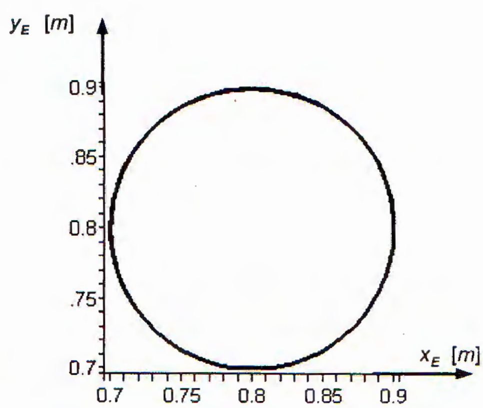

Fig. 3. Trajectory of the end-effector

For the second case, the end-effector moves along a circular trajectory as shown in Fig. 3. The workspace coordinates of the end-effector are given by

$$\begin{aligned}x_E =& 0.8 + 0.1\cos(t),\\ y_E =& 0.8 + 0.1\sin(t).\end{aligned} \tag{3.17}$$

The initial values of the joint angles $\mathbf{q}$ are chosen as

$q_1(0) = 1.048$, $q_2(0) = 0.582$, $q_3(0) = 4.118$, $q_4(0) = 1.048$ (rad). Fig. 4 displays calculating results of the joint torques for this case.

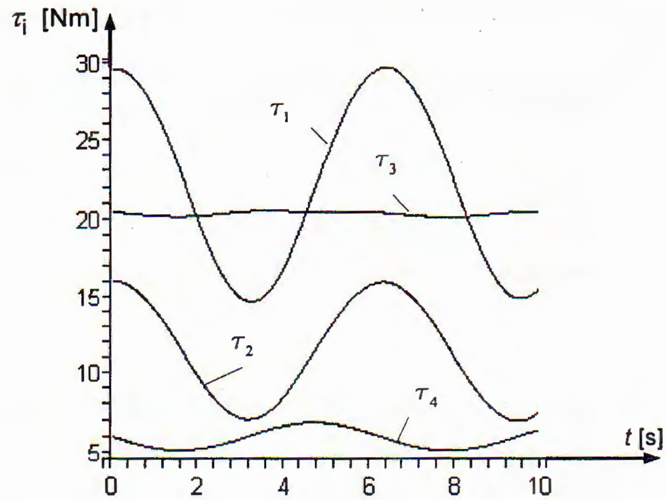

Fig. 4. Joint torques versus time for the second case

4. CONCLUSIONS

A kinematically redundant manipulator is a robot system that has more than the minimum number of DOF which are required for a specified task. In the paper, the influence of trajectories on the joint torques of redundant manipulators was considered. The manipulators under study are redundant with n DOF. The torques in the joint space and in the null space were defined corresponding to the desired trajectories of the end-effector.

The influence of trajectories on the joint torques is illustrated using the numerical simulation with a redundant four-link planar manipulator. The obtained results may be a base for motion controls of kinematically redundant manipulators.

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REFERENCES

1. R. M. Murray, Z. Li and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation.* CRC Press, Inc., Boca Raton 2000.
2. D. N. Nenchev, Redundancy Resolution through Local Optimization: a Review, *Journal of Robotic System* **6** (6) (1989) 769-798.
3. T. Shamir and Y. Yomdin, Repeatability of Redundant Manipulators: Mathematical Solution of the Problem, *IEEE Transactions on Automatic Control* **33** (11) (1988) 1004-1009.
4. Y. Nakamura, *Advanced Robotics: Redundancy and Optimization*, Addison. Wesley, 1991.
5. Y. Wang and G. S. Chirikjian, A Statistical Method for the Inverse Kinematics of Hyper-Redundant Manipulators, *Proceedings of the 11th World Congress in Mechanism and Machine Science*, April 1-4, 2004, Tianjin, China.

6. S. G. Yue, Redundant Robot Manipulators with Joint and Link Flexibility - I, Dynamic Motion Planning for Minimum End-effector Deformation, *Mech. Mach. Theory* **33** (22) (1998) 103-113.
7. Do Tuan Anh, *Inverse Problem of Dynamics of Redundant Robots* (in Vietnamese), Engineer-Thesis, Hanoi University of Technology, 2006.
8. L. Zlajpad, Influence of External Forces on the Behaviors of Redundant Manipulators, *Robotica* **17** (1999) 283-292.
9. R. G. Roberts and A. A. Maciejewski, Repeatable Generalized Inverse Control Strategies for Kinematically Redundant Manipulators, IEEE *Transactions on Automatic Control* **38** (5) (1993) 689-699.
10. M. W. Spong, M. Vidyasagar, *Dynamics and Control of Robot Manipulators*, John Wiley & Sons, Inc., New York 1989.
11. T. Yoshikawa, *Foundation of Robotics Analysis and Control*, MIT Press, Cambridge 1990.
12. Nguyen Van Khang, *Multibody Dynamics* (in Vietnamese), Science and Technique Publishing House, Hanoi 2007.

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ẢNH HƯỞNG CỦA CÁC DẠNG QUỸ ĐẠO ĐẾN MÔMEN PHÁT ĐỘNG CỦA TAY MÁY RÔBỐT DƯ DẪN ĐỘNG

Bài báo này trình bày một thuật toán giải bài toán động lực học ngược của Rôbốt dư dẫn động, trong đó có sử dụng phần mềm MAPLE. Phương pháp nêu ra có ưu điểm là giảm được khối lượng tính toán. Ảnh hưởng của các quỹ đạo lên các mômen phát động đã được khảo sát. Các kết quả lý thuyết được minh hoạ bằng một thí dụ tính toán mô phỏng số với Rôbốt phẳng 4 khâu dư dẫn động