NON-LINEAR ANALYSIS ON STABILITY OF CORRUGATED CROSS-PLY LAMINATED COMPOSITE PLATES

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Abstract. In the present paper the governing equations for corrugated cross-ply laminated composite plates in the form of a sine wave are developed based on the Kirchoff-Love's theory and the extension of Seydel's technique. By using Bubnov-Galerkin method approximated analytical solutions to the non-linear stability problem of corrugated laminated composite plates subjected to biaxial loads are investigated. The post buckling load-deflection curve of corrugated plates and analytical expressions of the upper and lower buckling loads are presented. The effectiveness of corrugated plates in enhancing the stability compared with corresponding flat plates is given.

1. INTRODUCTION

Laminated composite plates and shells are widely used as basic structure components and their static and dynamic problems with geometrical non-linearity are of significant practical interest, particularly stability and post-buckling behavior of composite plates and shells is more important. Investigated results in this problem of flat laminated composite plates have been received by many authors [1,3,6,7,9,10,11,13]. However, the analysis of corrugated laminated composite plates has received comparitively little attention.

Corrugated plates of wave form made of isotropic elastic material were considered as flat orthotropic plates with corresponding orthotropic constants determined by the Seydel's technique. This approach was acceptable to solve many bending and stability problems of corrugated plates in practice [5,8,12]. In paper [4] the authors developed the Seydel's technique to the bending problems of corrugated cross-ply laminated composite plates and cylindrical shells. But for the stability problems, besides bending stiffnesses we need to formulate extensional stiffnesses of corrugated cross-ply laminated composite plates.

Consequently, the purpose of the present paper is to formulate the constitutive equations of corrugated cross-ply laminated composite plates of wave form by using the Seydel's technique and investigate the non-linear stability problems of corrugated composite plates subjected to biaxial compressive loads.

2. GOVERNING EQUATIONS

Consider a rectangular symmetrically laminated composite corrugated plate in the form of a sine wave (see Fig. 1), each layer of which is an unidirectional composite material. The plate is subjected to biaxial compressive loads of intensities p and q respectively, where

p and q vary arbitrarily. Suppose the portion of cross-section line of the corrugated plate in the plane (x, z) has the form of a sine wave (Fig. 1)
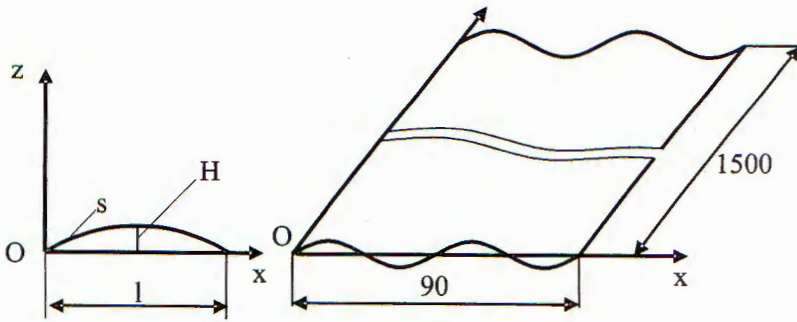
$$z = H \sin \frac{\pi x}{l}$$

Fig. 1. Model of corrugated plate

Non-linear strain-displacement relationships in the middle surface for a such corrugated plate are:

$$\varepsilon_1^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - kw,$$

$$\varepsilon_2^0 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \tag{2.1}$$

$$\varepsilon_6^0 = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

where u, v and w denote displacements of the middle surface point along x, y and z directions respectively, ε_i^o (i = 1, 2 and 6) are strains in the middle surface; k is the curvature of the portion line in (x, z) plane, which is defined as:

$$k = \frac{z''}{(1 + z'^2)^{\frac{3}{2}}} \approx z'' = -H\frac{\pi^2}{l^2}\sin\frac{\pi x}{l} \tag{2.2}$$

The constitutive stress – strain relations for the plate material are omitted here for brevity [see Reddy 11]. However note that in a multilayered symmetrically laminated material the coupling stiffnesses are equal to zero, while the extensional stiffnesses A_{16} and A_{26} are negligible compared to the others. This means that the constitutive equations are indentical to those for a specially orthotropic material. Then integrating the stress –strain equations through the thickness of the plate we obtain the expressions for stress

resultants:

$$N_1 = A_{11} \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - k.w \right] + A_{12} \left[\frac{\partial u}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right],$$

$$N_2 = A_{12} \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - k.w \right] + A_{22} \left[\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right], \qquad (2.3)$$

$$N_6 = A_{66} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right)$$

where A_{ij} $(i, j = 1, 2$ and $6)$ are extensional stiffnesses of the plate.

About the changes of curvature and twist of a corrugated plate in reality it is a very complicated problem, but with the condition restrained on the form of corrugated plate, where $H \ll l$ we can apply the Seydel's technique into consideration. Suppose that the changes of curvature and twist of a corrugated plate are the same as of a flat one, i.e.

$$\chi_x = -\frac{\partial^2 w}{\partial x^2}, \quad \chi_y = -\frac{\partial^2 w}{\partial y^2}, \quad \chi_{xy} = -2\frac{\partial^2 w}{\partial x \partial y},$$

but the bending stiffnesses of a corrugated composite plate $D_{ij}^*(i, j = 1, 2$ and $6)$ are determined by the extension of Seydel's technique [4] as follows:

$$D_{11}^* = \frac{l}{s} D_{11}; \qquad D_{22}^* = E_2 I; \qquad D_3^* = (D_{12}^* + 2D_{66}^*) = \frac{l}{s} (D_{12} + 2D_{66})$$

where D_{ij} $(i, j = 1, 2$ and $6)$ are bending stiffnesses of the corresponding flat plate;

$$I = \frac{hH^2}{2} \left[1 - \frac{0.81}{1 + 2.5 \left(\frac{H}{2l} \right)^2} \right]$$

where:

E_2 - the effective modulus in the y direction,

h - the plate thickness,

s - the length of a portion of corrugated line (Fig. 1),

$$s = \int_0^l \sqrt{1 + \frac{\pi^2 H^2}{l^2} \cos^2 \frac{\pi x}{l}} dx \approx l \sqrt{1 + \frac{\pi^2 H^2}{2 l^2}} \approx l \left(1 + \frac{\pi^2 H^2}{4 l^2} \right)$$

It is recommended [5] that these constants be determined by experimentation whenever possible, but can be used successfully in practice.

In the results the internal moment resultants of a corrugated composite plate of wave form are:

$$M_1 = -\left(D_{11}^* \frac{\partial^2 w}{\partial x^2} + D_{12}^* \frac{\partial^2 w}{\partial y^2} \right),$$

$$M_2 = -\left(D_{12}^* \frac{\partial^2 w}{\partial x^2} + D_{22}^* \frac{\partial^2 w}{\partial y^2} \right),$$

$$M_{66} = -2D_{66}^* \frac{\partial^2 w}{\partial x \partial y}$$

The equations of equilibrium of a plate subjected to uniformly distributed biaxial compressive loads of intensities p and q respectively, according to [2], when considering the non-linear geometry are of the form:

$$\frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0,$$

$$\frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0,$$

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + N_1 \frac{\partial^2 w}{\partial x^2} + 2N_6 \frac{\partial^2 w}{\partial x \partial y} + N_2 \frac{\partial^2 w}{\partial y^2} + p\frac{\partial^2 w}{\partial x^2} + q\frac{\partial^2 w}{\partial y^2} = 0 \quad (2.4)$$

The substitution of equations (2.3) and (2.4) into (2.4) yields the system of equations of equilibrium in terms of displacements

$$A_{11} \frac{\partial^2 u}{\partial x^2} + A_{66} \frac{\partial^2 u}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v}{\partial x \partial y} + A_{11} \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + A_{66} \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}$$

$$+ (A_{12} + A_{66}) \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - A_{11} \left(w\frac{\partial k}{\partial x} + k\frac{\partial w}{\partial x} \right) = 0,$$

$$A_{22} \frac{\partial^2 v}{\partial y^2} + A_{66} \frac{\partial^2 v}{\partial x^2} + (A_{12} + A_{66}) \frac{\partial^2 u}{\partial x \partial y} + A_{22} \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2} + A_{66} \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}$$

$$+ (A_{12} + A_{66}) \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y} - A_{12} \left(w\frac{\partial k}{\partial y} + k\frac{\partial w}{\partial y} \right) = 0, \quad (2.5)$$

$$D_{11}^* \frac{\partial^4 w}{\partial x^4} + 2\left(D_{12}^* + 2D_{66}^* \right) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}^* \frac{\partial^4 w}{\partial y^4} - \frac{1}{2}A_{11} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2}$$

$$- \frac{1}{2}A_{12} \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial x^2} - \frac{1}{2}A_{12} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial y^2} - \frac{1}{2}A_{22} \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial y^2}$$

$$- A_{11} \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} - A_{12}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2} - 2A_{66}\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2A_{66}\frac{\partial^2 w}{\partial x \partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

$$- A_{66}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2} - A_{22}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial y^2} - 2A_{66}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x \partial y}$$

$$+ A_{11}k\, w\frac{\partial^2 w}{\partial x^2} + A_{12}k\, w\frac{\partial^2 w}{\partial y^2} - p\frac{\partial^2 w}{\partial x^2} - q\frac{\partial^2 w}{\partial y^2} = 0$$

The plate considered in the following analysis is simply supported, thus the boundary conditions are:

$$w = 0, \quad v = 0, \quad M_1 = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = a$$
$$w = 0, \quad u = 0, \quad M_2 = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad y = b \tag{2.6}$$

where a and b are lengths of plate edges.

An approximation is acceptable in the vicinity of the buckling loads, so that the buckling mode shape is represented by a single term of a double Fourier series. The boundary conditions (2.6) discussed here can be satisfied if the buckling mode shape is represented by:

$$u = U_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b},$$
$$v = V_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}, \tag{2.7}$$
$$w = W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

where m, n are natural numbers representing the number of half waves in the x and y directions respectively.

Substituting expressions (2.7) into the equations of equilibrium (2.5) and applying the Bubnov-Galerkin procedure yield the set of three algebraic equations with respect to the amplitudes U_{mn}, V_{mn}, W_{mn}. The first two equations of this set are linear algebraic equations for U_{mn}, V_{mn}:

$$a_1 U_{mn} + a_2 V_{mn} = a_3 W_{mn} + a_4 W_{mn}^2,$$
$$a_5 U_{mn} + a_6 V_{mn} = a_7 W_{mn} + a_8 W_{mn}^2 \tag{2.8}$$

so that the amplitudes of the in-plane displacements can be expressed in terms of W_{mn}. When these expressions are substituted into the remaining equation of equilibrium, the result is a non-linear algebraic equation with respect to W_{mn} that can be represented in the form

$$a_9 W_{mn}^3 + a_{10} W_{mn}^2 + (a_{11} + R) W_{mn} = 0 \tag{2.9}$$

where $a_i \ (i = 1, 2, ..., 11)$ and R are coefficients which depend on the material, geometry and buckling mode shape:

$$a_1 = \frac{1}{4} \frac{\left(m^2\pi^2 b^2 A_{11} + n^2\pi^2 a^2 A_{66}\right)}{ab},$$

$$a_2 = \frac{1}{4} mn\pi^2 \left(A_{12} + A_{66}\right),$$

$$a_3 = \frac{H\pi^2 m \, a \, b A_{11}}{l \left(4m^2 l^2 - a^2\right)} + \frac{H\pi^2 m \, b \left(a^2 - 2m^2 l^2\right) A_{66}}{al \left(a^2 - 4m^2 l^2\right)},$$

$$a_4 = - \left[\frac{8}{9} \frac{b\pi m^2 A_{11}}{a^2 n} + \frac{8}{9} \frac{n\pi A_{66}}{b} - \frac{4}{9} \frac{n\pi \left(A_{12} + A_{66}\right)}{b}\right],$$

$$a_5 = \frac{1}{4} m \, n \, \pi^2 \left(A_{12} + A_{66}\right),$$

$$a_6 = \frac{1}{4} \left[\frac{n^2\pi^2 a^2 A_{22} + m^2\pi^2 b^2 A_{66}}{ab}\right],$$

$$a_7 = \frac{H\,n\,\pi^2\left(a^2 - 2m^2 l\right) A_{12}}{l\left(a^2 - 4m^2 l^2\right)},$$

$$a_8 = -\left[\frac{8}{9}\frac{a\,n^2\pi A_{22}}{m\,b^2} + \frac{8}{9}\frac{m\pi A_{66}}{a} - \frac{4}{9}\frac{m\pi\left(A_{12} + A_{66}\right)}{a}\right],$$

$$a_9 = \frac{3\pi^4}{64a^3b^3}\left[m^4b^4 A_{11} + 2m^2 n^2 a^2 b^2 A_{12} + n^4 a^4 A_{22}\right]$$
$$- \frac{16\pi}{9a^2 nb}\left[m^2 b^2 A_{11} + a^2 n^2\left(A_{12} + 2A_{66}\right)\right]\frac{\left(a_4 a_6 - a_2 a_8\right)}{a_1 a_6 - a_2 a_5}$$
$$- \frac{16\pi}{9ab^2 m}\left[b^2 m^2\left(A_{12} + 2A_{66}\right) + a^2 n^2 A_{22}\right]\frac{\left(a_1\,a_8 - a_4 a_5\right)}{a_1 a_6 - a_2 a_5},$$

$$a_{10} = -\frac{16\pi}{9a^2 nb}\left[m^2 b^2 A_{11} + a^2 n^2\left(A_{12} + 2A_{66}\right)\right]\frac{\left(a_3\,a_6 - a_2 a_7\right)}{a_1 a_6 - a_2 a_5}$$
$$- \frac{16\pi}{9ab^2 m}\left[b^2 m^2\left(A_{12} + 2A_{66}\right) + a^2 n^2 A_{22}\right]\frac{\left(a_1 a_7 - a_3 a_5\right)}{a_1 a_6 - a_2 a_5},$$

$$a_{11} = \frac{1}{4}\frac{\pi^4}{a^3 b^3}\left[m^4 b^4 D_{11}^* + 2m^2 n^2 a^2 b^2\left(D_{12}^* + 2D_{66}^* + n^4 a^4 D_{22}^*\right)\right]$$

$$R = \frac{1}{4}\frac{\pi^2}{ab}\left(m^2 b^2 p + n^2 a^2 q\right) \tag{2.10}$$

Note that a similar equation can be obtained for other boundary conditions as long as the plate is modeled as a single degree of freedom system, i.e. a single Fourier term is retained in each displacement component, of course, with other appropriate functions.

Taking $W_{mn} \neq 0$, i.e. considering the plate after the lost of stability we obtain

$$a_9 W_{mn}^2 + a_{10} W_{mn}^+ a_{11} + R = 0 \tag{2.11}$$

Because the plate is working in the elastic stage, so in the case of simultaneous action of forces p and q we can set $q = \alpha p$, the equation (2.11) can be rewritten

$$f = a_9\,W_{mn}^2 + a_{10} W_{mn}^+ a_{11} + \frac{1}{4}\frac{\pi^2}{ab}\left(m^2 b^2 + \alpha n^2 a^2\right) p = 0 \tag{2.12}$$

Substituting $W_{mn} = 0$ in equation (2.12) yields the value of critical load p

$$p = -\frac{4ab.a_{11}}{\pi^2\left(m^2 b^2 + \alpha n^2 a^2\right)} \tag{2.13}$$

or $\quad p_{\text{upper}} = \dfrac{\pi^2\left(m^4 D_{11}^* \lambda^4 + 2m^2 n^2 \lambda^2\left(D_{12}^* + 2D_{66}^*\right) + n^4 D_{22}^*\right)}{b^2\left(m^2 \lambda^2 + n^2\alpha\right)},$ where $\quad \lambda = \dfrac{b}{a},$ that is called the upper buckling load, which coincides with the linear buckling load. Then the critical load q is determined by $q_{\text{upper}} = \alpha p_{\text{upper}}$.

The lower buckling load of the corrugated composite plate can be obtained from equation (2.12) by using the condition

$$\frac{df}{dW_{mn}} = 0$$

The value of W_{mn} corresponding to the lower buckling load is found from this equation

$$W_{mn}^0 = -\frac{a_{10}}{2a_9}$$

and substituted into equation (2.12) yielding

$$p_{\text{lower}} = -\frac{4ab}{\pi^2 \left(m^2 b^2 + \alpha n^2 a^2\right)} \left(a_{11} - \frac{a_{10}^2}{4a_9}\right) \tag{2.14}$$

The sign "minus" indicates that the load p is compressive. Numbers m, n must be chosen such that the absolute value of the critical buckling load p is minimum.

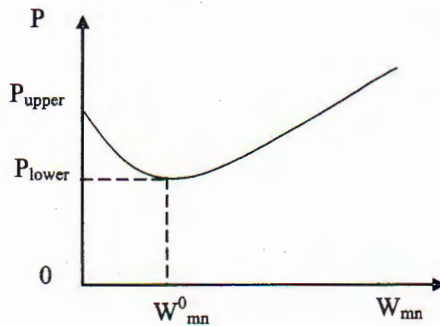The post-buckling load-deflection curve (2.12) is illustrated in the Fig. 2

Fig. 2. Post-buckling load-deflection curve of the plate

The domain limited by the upper and lower buckling loads is called the unstable domain of the corrugated plate. Thus permissible loads have to be chosen such that the safety is provided according to the lower critical load.

Remark. From the equation (2.12) we can see that post-buckling behaviour of flat and corrugated composite plates are different. A corrugated composite plate becomes a flat one when $l = s$, $H \to 0$, stiffnesses D_{ij}^* will tend to D_{ij}, the coefficients in (2.10) $a_3 = a_7 = 0$ *and* $a_{10} = 0$, at that time the equation (2.12) representing a post-buckling load deflection curve has following character: gradually increasing the active load leads to increasing the deflection, the critical load is

$$p_{cr} = \frac{\pi^2 \left(m^4 D_{11} \lambda^4 + 2m^2 n^2 \lambda^2 \left(D_{12} + 2D_{66}\right) + n^4 D_{22}\right)}{b^2 \left(m^2 \lambda^2 + n^2 \alpha\right)}$$

and there isn't an unstable domain, while for a corrugated composite plate there exists an unstable domain. When analysing reinforced composite plates [7] also occurs such behavior.

3. NUMERICAL EXAMPLES

Let's consider a simply supported rectangular corrugated symmetrically laminated plate in the form of a sine wave with $a = 0.9$ m, $b = 1.5$ m, $H = 0.03$ m, $l = 0.09$ m and $s = 0.112$ m. The skin of the plate had 6 plies [45/-45/90/90/-45/45], each ply being 0.5 mm thick. The material of the plates considered in the following examples had Thornel 300 graphite fibers and Narmco 5208 thermosetting epoxy resin [6]. The properties of this material are $E_1 = 127.4$GPa, $E_2 = 13.0$ GPa, $G_{12} = 6.4$ GPa, $\nu_{12} = 0.38$.

Some numerical results are shown in the Fig. 3, 4 and 5. Relation between the critical buckling load and the dimension ratio of corrugated composite plate subjected to a compressive load in the y direction is illustrated in Fig. 3.
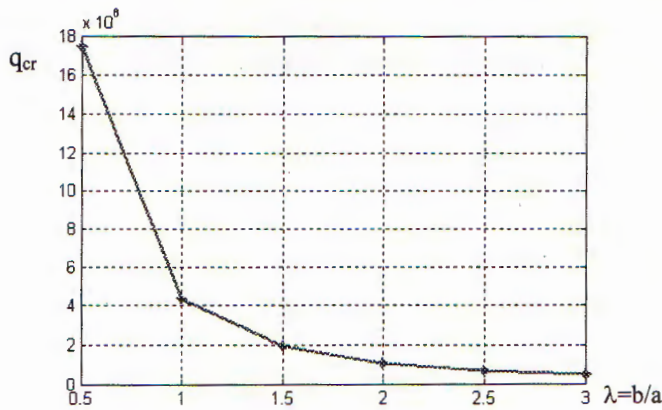

Fig. 3. Effect of the plate dimension ratio on bucking loads

The effect of the plate thickness on critical load for a corrugated composite plate subjected to uniaxial and biaxial loads (with $\alpha = 1$) is shown in Fig. 4a and for a flat one in Fig. 4b.
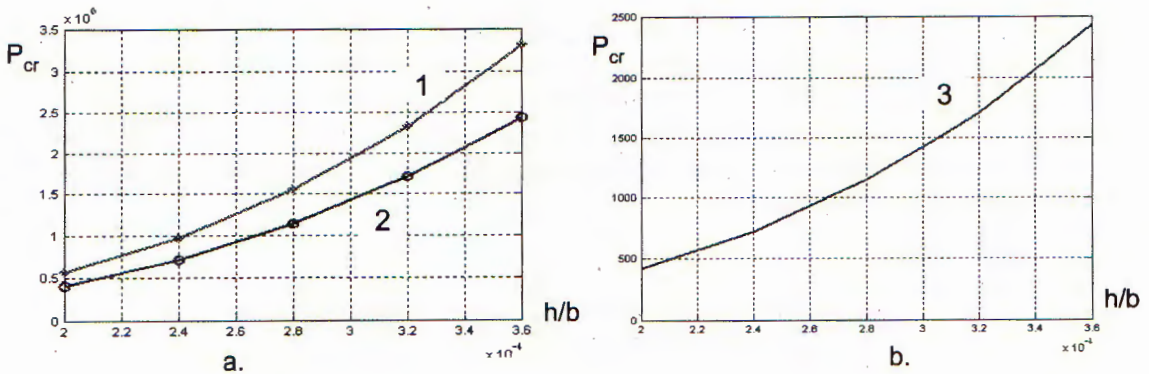

Fig. 4. Effect of the plate thickness on buckling load 1.uniaxial load; 2. biaxial load; 3. flat plate

The effect of the height H of portion line on critical loads of a corrugated composite plate subjected to compressive load in the x direction is presented in Fig. 5.
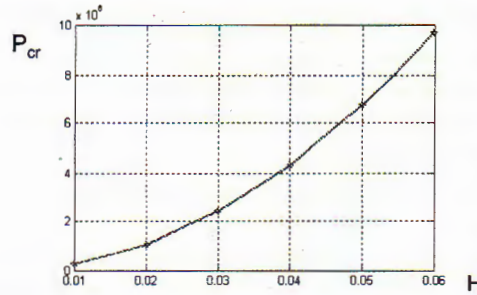
Fig. 5. Effect of the height H on buckling loads

From these figures it is clear that the corrugated composite plates significantly increase the buckling loads. A similar conclusion follows for corrugated plates of wave form with greater height H. But the buckling loads decrease for longer plate in the y direction, when they are subjected to axial compressive load in the same direction. The wave form of corrugated laminated composite plate provides a higher stability of plates, it is why corrugated composite plates of wave form are more useful in practice.

4. CONCLUSION

The governing equations for corrugated cross-ply laminated composite plates have been derived based on the extension of Seydel's technique. These equations can be used to non-linear analysis of static and dynamic problems of corrugated laminated composite plates. The approach to considering stability problems presented in this paper allows to obtain analytical expressions of critical buckling loads of corrugated plates subjected to biaxial compressive loads.

The effectiveness of corrugated composite plates in enhancing the stability compared with corresponding flat plates is illustrated.

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PHÂN TÍCH PHI TUYẾN VỀ ỔN ĐỊNH
CỦA TẤM COMPOSITE LỚP LƯỢN SÓNG

Trong bài báo này đã thiết lập các phương trình cơ sở của tấm composite lớp lượn sóng dựa trên lý thuyết Kirchff-Love và mở rộng cách tiếp cận Seydel. Sử dụng phương pháp Bubnov-Galerkin vào xác định nghiệm giải tích gần đúng của bài toán ổn định phi tuyến của tấm composite lượn sóng chịu nén theo hai phương. Đã nhận được đường tải - độ võng sau tới hạn và biểu thức giải tích của lực tới hạn trên và dưới. Đã chỉ ra hiệu qủa tăng ổn định của tấm lượn sóng so với tấm phẳng tương ứng.