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THE INFLUENCE OF NONLINEAR TERMS IN MECHANICAL SYSTEMS HAVING TWO DEGREES OF FREEDOM

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Abstract. For many years the higher order stochastic averaging method has been widely used for investigating nonlinear systems subject to white and coloured noises to predict approximately the response of the systems. In the paper the method is further developed for two-degree-of-freedom systems subjected to white noise excitation. Application to Duffing oscillator is considered.

1. INTRODUCTION

It is well-known, the stochastic averaging method (SAM) is widely used in different problems of stochastic mechanics, such as vibration, stability and reliability problems (see e.g. Mitropolskii et al, 1992; Red-Horse and Spanos, 1992; Zhu and Lin, 1994; Zhu et al, 1997). However, the effect of some nonlinear terms cannot be investigated by using the classical first order SAM. In order to overcome this insufficiency the different procedures to obtain approximate solutions have been developed for the nonlinear systems with one degree of freedom under white and coloured noise excitations (see e.g. Anh, 1993; Anh and Tinh, 1995; Tinh, 1999). In the present paper this procedure is further developed for two-degree-of-freedom nonlinear systems subjected to white noise excitation. An application to Duffing system is considered and the effect of nonlinear terms can be detected in the approximate solutions of Fokker-Planck (FP) equation while it cannot be investigated by using the classical first order SAM.

2. HIGHER SAM IN TWO-DEGREE-OF-FREEDOM SYSTEMS

Consider the motion equations of a mechanical system with two degrees of freedom

$$\begin{aligned} \ddot{x}_1 + \omega_1^2 x_1 &= \varepsilon f_{11}(x_1, x_2, \dot{x}_1, \dot{x}_2) + \varepsilon^2 f_{12}(x_1, x_2, \dot{x}_1, \dot{x}_2) + \sqrt{\varepsilon}\sigma_1 \dot{\xi}(t), \\ \ddot{x}_2 + \omega_2^2 x_2 &= \varepsilon f_{21}(x_1, x_2, \dot{x}_1, \dot{x}_2) + \varepsilon^2 f_{22}(x_1, x_2, \dot{x}_1, \dot{x}_2) + \sqrt{\varepsilon}\sigma_2 \dot{\xi}(t), \end{aligned} \tag{2.1}$$

where ω_1, ω_2, σ_1, σ_2 are positive constants and ε is a small positive parameter.

According to the averaging method we transform the state coordinates $x = (x_1, x_2)$ into the variables $a = (a_1, a_2)$ and φ=(φ_1, φ_1) by the change

$$\begin{aligned} x_j &= a_j \cos\varphi_j, \\ \dot{x}_j &= -\omega_j \sin\varphi_j, \ (j = 1, 2). \end{aligned} \tag{2.2}$$

By using Ito differentiation formula [7] the system of equations (2.1) is transformed into the following system of equations

$$\begin{aligned} \dot a_j &= \varepsilon A_{j1}(a,\varphi) + \varepsilon^2 A_{j2}(a,\varphi) - \sqrt{\varepsilon}\sigma_j \frac{\sin\varphi_j}{\omega_j}\dot\xi(t), \\ \dot\varphi_j &= \omega_j + \varepsilon B_{j1}(a,\varphi) + \varepsilon^2 B_{j2}(a,\varphi) - \sqrt{\varepsilon}\sigma_j \frac{\cos\varphi_j}{\omega_j a_j}\dot\xi(t)\ ,\ (j=1,2) \end{aligned} \tag{2.3}$$

where it is denoted

$$\begin{aligned} A_{j1}(a,\varphi) &= -\frac{f_{j1}(a,\varphi)}{\omega_j}\sin\varphi_j + \frac{\sigma_j^2\cos^2\varphi_j}{2\omega_j^2 a_j}, \\ B_{j1}(a,\varphi) &= -\frac{f_{j1}(a,\varphi)}{\omega_j a_j}\cos\varphi_j + \frac{\sigma_j^2\cos\varphi_j\sin\varphi_j}{\omega_j^2 a_j^2}, \qquad (j=1,2), \\ A_{j2}(a,\varphi) &= -\frac{f_{j2}(a,\varphi)}{\omega_j}\sin\varphi_j\ ,\ B_{j2}(a,\varphi) = -\frac{f_{j2}(a,\varphi)}{\omega_j a_j}\cos\varphi_j. \end{aligned} \tag{2.4}$$

The Fokker-Planck (FP) equation for the stationary probability density function $W(a,\varphi)$ takes the form (Anh, 1995)

$$\sum_{j=1}^{2}\omega_j\frac{\partial W}{\partial\varphi_j} = -\varepsilon[A_1,B_1]L[W] - \varepsilon^2[A_2,B_2]L[W], \tag{2.5}$$

where the operators $[A_j, B_j]L[.]$, j=1, 2 are defined as follows

$$\begin{aligned} [A_1,B_1]L[W] = &\sum_{j=1}^{2}\left[\frac{\partial}{\partial a_j}(A_{j1}W) + \frac{\partial}{\partial\varphi_j}(B_{j1}W)\right] \\ &-\sum_{j=1}^{2}\sum_{s=1}^{2}\left\{\frac{\partial^2}{\partial a_j\partial a_s}\left(\frac{\sigma_j\sigma_s\sin\varphi_j\sin\varphi_s}{2\omega_j\omega_s}W\right)\right. \\ &\left.+\frac{\partial^2}{\partial a_j\partial\varphi_s}\left(\frac{\sigma_j\sigma_s\sin\varphi_j\cos\varphi_s}{2a_s\omega_j\omega_s}W\right) + \frac{\partial^2}{\partial\varphi_j\partial\varphi_s}\left(\frac{\sigma_j\sigma_s\cos\varphi_j\cos\varphi_s}{2a_j a_s\omega_j\omega_s}W\right)\right\} \end{aligned} \tag{2.6}$$

$$[A_2,B_2]L[W] = \sum_{j=1}^{2}\left[\frac{\partial}{\partial a_j}(A_{j2}W) + \frac{\partial}{\partial\varphi_j}(B_{j2W})\right]. \tag{2.7}$$

We seek the solution of (2.5) in the form

$$W(a,\varphi) = W_0(a,\varphi) + \varepsilon W_1(a,\varphi) + \varepsilon^2 W_2(a,\varphi) + \ldots \tag{2.8}$$

Substituting (2.8) into (2.5) and comparing the coefficients of like power of ε we obtain

$$\varepsilon^0 :\ \sum_{j=1}^{2}\omega_j\frac{\partial W_0}{\partial\varphi_j} = 0, \tag{2.9}$$

$$\varepsilon^1 :\ \sum_{j=1}^{2}\omega_j\frac{\partial W_1}{\partial\varphi_j} = -[A_1,B_1]L[W_0], \tag{2.10}$$

$$\varepsilon^2 : \quad \sum_{j=1}^{2} \omega_j \frac{\partial W_2}{\partial \varphi_j} = -\{[A_2, B_2]L[W_0] + [A_1, B_1]L[W_1]\}, \tag{2.11}$$

....

From (2.9) we get

$$W_0 = W_0(a). \tag{2.12}$$

The arbitrary integration function $W_0(a)$ must be chosen from the condition for the function $W_1(a, \varphi)$ to be periodic to φ.

Thus, we get from (2.10)

$$< [A_1, B_1]L[W_0(a)] >= 0, \tag{2.13}$$

where $< . >$ is the averaging operator with respect to φ

$$< . >= \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} (.) d\varphi_1 d\varphi_2. \tag{2.14}$$

Substituting (2.6) into (2.13) yields

$$\sum_{j=1}^{2} \left\{ \frac{\partial}{\partial a_j} (< A_{j1} > W_0(a)) - \frac{\sigma_j^2}{4\omega_j^2} \frac{\partial^2 W_0(a)}{\partial a_j^2} \right\} = 0. \tag{2.15}$$

The second term $W_1(a, \varphi)$ in (2.8) is determined from (2.10) using Fourier expansion

$$[A_1, B_1]L[W_0(a)] = W_0(a) \sum_{l_1} \sum_{l_2} C_{l_1 l_2}(a) \exp[i(l_1\varphi_1 + l_2\varphi_2)], \tag{2.16}$$

where

$$C_{l_1 l_2}(a) = \frac{1}{(2\pi)^2 W_0(a)} \int_0^{2\pi} \int_0^{2\pi} [A_1, B_1]L[W_0(a)] \exp[-i(l_1\varphi_1 + l_2\varphi_2)] d\varphi_1 d\varphi_2. \tag{2.17}$$

Substituting (2.16) into (2.10) we get

$$W_1(a, \varphi) = W_0(a) \left\{ W_{10}(a) + \sum_{l_1} \sum_{l_2} \frac{C_{l_1 l_2}}{l_1\omega_1 + l_2\omega_2} \exp[i(l_1\varphi_1 + l_2\varphi_2) \right\}, \tag{2.18}$$

where l_1, l_2 are integers and

$$l_1\omega_1 + l_2\omega_2 \neq 0. \tag{2.19}$$

The arbitrary integration function $W_{10}(a)$ must be chosen from the condition for the function $W_2(a,\varphi)$ to be periodic to φ. Similarly, we can find the third term $W_2(a,\varphi)$ in (2.11).

3. APPLICATION

3.1. SAM of coefficients in FP equation

Now we apply the proposed procedure to Duffing system whose motion equations take the form

$$\begin{aligned} m_1\ddot{x}_1 + c_1x_1 &= -\varepsilon[2h_1\dot{x}_1 + \beta_1x_1^3] - \varepsilon^2[c_{12}(x_1 - x_2) + 2h_{12}(\dot{x}_1 - \dot{x}_2)] + \sqrt{\varepsilon}\delta_1\dot{\xi}(t), \\ m_2\ddot{x}_2 + c_2x_2 &= -\varepsilon[2h_2\dot{x}_2 + \beta_2x_2^3] + \varepsilon^2[c_{12}(x_1 - x_2) + 2h_{12}(\dot{x}_1 - \dot{x}_2)] + \sqrt{\varepsilon}\delta_2\dot{\xi}(t). \end{aligned} \tag{3.1}$$

We represent the physical model of this system in Fig. 1. Where m_1, m_2 are masses, c_1, c_2, c_{12} are spring constants, h_1, h_2, h_{12} are damping coefficients, β_1 , β_2 , δ_1, δ_2 are positive constants and

$$R_1 = -\varepsilon\beta_1x_1^3 + \sqrt{\varepsilon}\delta_1\dot{\xi}(t)\,, \quad R_2 = -\varepsilon\beta_2x_2^3 + \sqrt{\varepsilon}\delta_2\dot{\xi}(t). \tag{3.2}$$

The system of equations (3.1) can be written in the form

$$\begin{aligned} \ddot{x}_1 + \omega_1^2x_1 &= -\varepsilon[2k_1\dot{x}_1 + \gamma_1x_1^3] - \varepsilon^2[q_1(x_1 - x_2) + 2k_{11}(\dot{x}_1 - \dot{x}_2)] + \sqrt{\varepsilon}\sigma_1\dot{\xi}(t), \\ \ddot{x}_2 + \omega_2^2x_2 &= -\varepsilon[k_2\dot{x}_2 + \gamma_2x_2^3] + \varepsilon^2[q_2(x_1 - x_2) + 2k_{12}(\dot{x}_1 - \dot{x}_2)] + \sqrt{\varepsilon}\sigma_2\dot{\xi}(t), \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} &\omega_1^2 = \frac{c_1}{m_1},\ \omega_2^2 = \frac{c_2}{m_2},\ k_1 = \frac{h_1}{m_1},\ k_2 = \frac{h_2}{m_2},\ \gamma_1 = \frac{\beta_1}{m_1},\ \gamma_2 = \frac{\beta_2}{m_2}, \\ &q_1 = \frac{c_{12}}{m_1},\ q_2 = \frac{c_{12}}{m_2},\ k_{11} = \frac{h_{12}}{m_1},\ k_{12} = \frac{h_{12}}{m_2},\ \sigma_1 = \frac{\delta_1}{m_1},\ \sigma_2 = \frac{\delta_2}{m_2}. \end{aligned} \tag{3.4}$$

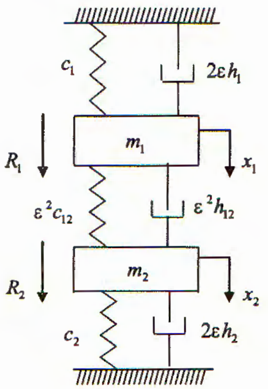

Fig. 1. Physical model of a mechanical system with two degrees of freedom

In this case we have

$$\begin{aligned} f_{11} &= -2k_1\dot{x}_1 - \gamma x_1^3; \quad f_{12} = -q_1(x_1 - x_2) - 2k_{11}(\dot{x}_1 - \dot{x}_2); \\ f_{21} &= -2k_2\dot{x}_2 - \gamma x_2^3; \quad f_{22} = q_2(x_1 - x_2) + 2k_{12}(\dot{x}_1 - \dot{x}_2). \end{aligned} \tag{3.5}$$

From (2.4), using (2.2) and (3.5), after calculations we obtain

$$\begin{aligned} A_{11} &= -2k_1a_1\sin^2\varphi_1 + \frac{\gamma_1}{\omega_1}a_1^3\sin\varphi_1\cos^3\varphi_1 + \frac{\sigma_1^2}{2\omega_1^2a_1}\cos^2\varphi_1, \\ A_{21} &= -2k_2a_2\sin^2\varphi_2 + \frac{\gamma_2}{\omega_2}a_2^3\sin\varphi_2\cos^3\varphi_2 + \frac{\sigma_2^2}{2\omega_2^2a_2}\cos^2\varphi_2, \end{aligned}$$

$$B_{11} = -k_1 \sin 2\varphi_1 + \frac{\gamma_1 a_1^2}{\omega_1} \cos^4 \varphi_1 - \frac{\sigma_1^2}{2\omega_1^2 a_1^2} \sin 2\varphi_1,$$
$$B_{21} = -k_2 \sin 2\varphi_2 + \frac{\gamma_2 a_2^2}{\omega_2} \cos^4 \varphi_2 - \frac{\sigma_2^2}{2\omega_2^2 a_2^2} \sin 2\varphi_2, \tag{3.6}$$

$$A_{12} = \frac{q_1}{\omega_1}(a_1 \cos\varphi_1 - a_2 \cos\varphi_2) \sin\varphi_1 + \frac{2k_{11}}{\omega_1}(-\omega_1 a_1 \sin^2\varphi_1 + \omega_2 a_2 \sin\varphi_1 \sin\varphi_2),$$
$$A_{22} = \frac{q_2}{\omega_2}(-a_1 \cos\varphi_1 + a_2 \cos\varphi_2) \sin\varphi_2 + \frac{2k_{12}}{\omega_2}(-\omega_2 a_2 \sin^2\varphi_2 + \omega_1 a_1 \sin\varphi_1 \sin\varphi_2),$$

$$B_{12} = \left[\frac{q_1}{\omega_1 a_1}(a_1 \cos\varphi_1 - a_2 \cos\varphi_2) + \frac{2k_{11}}{\omega_1 a_1}(-\omega_1 a_1 \sin\varphi_1 + \omega_2 a_2 \sin\varphi_2)\right] \cos\varphi_1,$$
$$B_{22} = \frac{q_2}{\omega_2 a_2}\left[(-a_1 \cos\varphi_1 + a_2 \cos\varphi_2) + \frac{2k_{12}}{\omega_2 a_2}(-\omega_2 a_2 \sin\varphi_2 + \omega_1 a_1 \sin\varphi_1)\right] \sin\varphi_2, \tag{3.7}$$

From (3.6) and (3.7), using (2.14) we get

$$< A_{11} >= -k_1 a_1 + \frac{\sigma_1^2}{4\omega_1^2 a_1}; \quad < A_{21} >= -k_2 a_2 + \frac{\sigma_2^2}{4\omega_2^2 a_2};$$
$$< A_{12} >= -k_{11} a_1; \qquad < A_{22} >= -k_{12} a_2. \tag{3.8}$$

In this case, from (2.5) the averaged FP equation takes the form

$$[< A_1, B_1 >]L[W(a)] + \varepsilon[< A_2, B_2 >]L[W(a)] = 0, \tag{3.9}$$

where it is denoted

$$[< A_1, B_1 >]L[W(a)] = \sum_{j=1}^{2}\left[\frac{\partial}{\partial a_j}(< A_{j1} > W(a)) + \frac{\partial}{\partial \varphi_j}(< B_{j1} > W(a))\right]$$
$$-\sum_{j=1}^{2}\sum_{s=1}^{2}\left\{\frac{\partial^2}{\partial a_j \partial a_s}\left(\left\langle\frac{\sigma_j\sigma_s \sin\varphi_j \sin\varphi_s}{2\omega_j\omega_s}\right\rangle W(a)\right)\right.$$
$$\left.+\frac{\partial^2}{\partial a_j \partial \varphi_s}\left(\left\langle\frac{\sigma_j\sigma_s \sin\varphi_j \cos\varphi_s}{2a_s\omega_j\omega_s}\right\rangle W(a)\right) + \frac{\partial^2}{\partial \varphi_j \partial \varphi_s}\left(\left\langle\frac{\sigma_j\sigma_s \cos\varphi_j \cos\varphi_s}{2a_j a_s\omega_j\omega_s}\right\rangle W(a)\right)\right\},$$

$$[< A_2, B_2 >]L[W(a)] = \sum_{j=1}^{2}\left[\frac{\partial}{\partial a_j}(< A_{j2} > W(a)) + \frac{\partial}{\partial \varphi_j}(< B_{j2} > W(a))\right]. \tag{3.10}$$

From (3.9), noting (3.8) and (3.10) we have the FP equation for the probability density function $W(a)$ in the form

$$\sum_{j=1}^{2}\left\{\frac{\partial}{\partial a_j}\left[\left(-k_j a_j + \frac{\sigma_j^2}{4\omega_j^2 a_j} - \varepsilon k_{1j} a_j\right)W(a)\right] - \frac{\sigma_j^2}{4\omega_j^2}\frac{\partial^2 W(a)}{\partial a_j^2}\right\} = 0. \tag{3.11}$$

This equation gives the solution

$$W(a) = Ca_1a_2\exp\left\{-\frac{2k_1\omega_1^2}{\sigma_1^2}a_1^2 - \frac{2k_2\omega_2^2}{\sigma_2^2}a_2^2 - \varepsilon\Big(\frac{2k_{11}\omega_1^2}{\sigma_1^2}a_1^2 + \frac{2k_{12}\omega_2^2}{\sigma_2^2}a_2^2\Big)\right\}, \tag{3.12}$$

$(C = \text{const})$.

The condition

$$\int_0^{2\pi}\int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} W(a)da_1da_2d\varphi_1d\varphi_2 = 1,$$

gives

$$C = \frac{(p_1+\varepsilon\alpha_1)(p_2+\varepsilon\alpha_2)}{\pi^2}, \tag{3.13}$$

where

$$p_1 = \frac{2k_1\omega_1^2}{\sigma_1^2},\ p_2 = \frac{2k_2\omega_2^2}{\sigma_2^2},\ \alpha_1 = \frac{2k_{11}\omega_1^2}{\sigma_1^2},\ \alpha_2 = \frac{2k_{12}\omega_2^2}{\sigma_2^2}. \tag{3.14}$$

It can be seen from (3.12) that the effect of the nonlinear terms $\varepsilon\beta_1x_1^3$ and $\varepsilon\beta_2x_2^3$ is lost during the conventional averaging procedure and we can not show their effect in the approximation (3.12) of the density probability function W(a,φ). So, in order to overcome this insufficiency we need to determine the higher approximate solutions to the FP equation by using the higher SAM.

3.2. Higher approximate solutions to FP equation

We determine the second approximate solution of FP equation (2.5) for the Duffing system (3.1).

Substituting (3.8) into (2.15) yields

$$W_0(a) = Ca_1a_2\exp\left\{-\frac{2k_1\omega_1^2}{\sigma_1^2}a_1^2 - \frac{2k_2\omega_2^2}{\sigma_2^2}a_2^2\right\},\qquad (C=\text{const}). \tag{3.15}$$

Substituting (3.15) into (2.10) we get

$$\sum_{j=1}^{2}\omega_j\frac{\partial W_1}{\partial\varphi_j} = -[A_1,B_1]L[W_0(a)]. \tag{3.16}$$

Using (2.6), (3.6) and expanding the right-hand side of (3.16) into the double Fourier series of φ_1, φ_2, after calculations we have the equation for $W_{11}(a,\varphi)$

$$\begin{aligned}\omega_1\frac{\partial W_{11}}{\partial\varphi_1} + \omega_2\frac{\partial W_{11}}{\partial\varphi_2} &= 2k_1\cos\varphi_1 + 2k_2\cos\varphi_2 + \frac{k_1\omega_1\gamma_1}{2\sigma_1^2}a_1^4(2\sin\varphi_1+\sin4\varphi_1)\\ &+\frac{k_2\omega_2\gamma_2}{2\sigma_2^2}a_2^4(2\sin\varphi_2+\sin4\varphi_2) + \frac{s(a)}{2}\cos(\varphi_1-\varphi_2) + \frac{s(a)}{2}\cos(\varphi_1+\varphi_2).\end{aligned} \tag{3.17}$$

Using the principle of superposition we get the solution of equation (3.17)

$$\begin{aligned} W_{11}(a,\varphi) =& \frac{k_1}{\omega_1}\sin 2\varphi_1 + \frac{k_2}{\omega_2}\sin 2\varphi_2 - \frac{k_1\gamma_1}{8\sigma_1^2}a_1^4(4\cos 2\varphi_1 + \cos 4\varphi_1) \\ &- \frac{k_2\gamma_2}{8\sigma_2^2}a_2^4(4\cos 2\varphi_2 + \cos 4\varphi_2) + \frac{\omega_1}{\omega_2^2-\omega_1^2}s(a)\cos\varphi_1\sin\varphi_2 \\ &- \frac{\omega_2}{\omega_2^2-\omega_1^2}s(a)\sin\varphi_1\cos\varphi_2, \end{aligned} \tag{3.18}$$

where

$$s(a) = \frac{\sigma_1\sigma_2}{\omega_1\omega_2}\frac{1}{a_1a_2} - \frac{2k_1\omega_1\sigma_2}{\omega_2\sigma_1}\frac{a_1}{a_2} - \frac{2k_2\omega_2\sigma_1}{\omega_1\sigma_2}\frac{a_2}{a_1} + \frac{16k_1k_2\omega_1\omega_2}{\sigma_1\sigma_2}a_1a_2. \tag{3.19}$$

Substituting (3.15) and (3.18) into (2.11) we have the equation for the arbitrary function $W_{10}(a)$ in the form

$$\begin{aligned} &\sum_{j=1}^{2}\frac{\partial}{\partial a_j}(< A_{j1} > W_0W_{10}) - \sum_{j=1}^{2}\frac{\partial^2}{\partial a_j^2}\Big(\frac{\sigma_j^2}{4\omega_j^2}W_0W_{10}\Big) = \sum_{j=1}^{2}\frac{\partial^2}{\partial a_j^2}\Big(\frac{\sigma_j^2}{2\omega_j^2} < W_0W_{11}\sin^2\varphi_j > \Big) \\ &- \sum_{j=1}^{2}\frac{\partial}{\partial a_j}(< A_{j1}W_0W_{11} >) - \sum_{j=1}^{2}\frac{\partial}{\partial a_j}(< A_{j2} > W_0). \end{aligned} \tag{3.20}$$

Substituting A_{j1} in (3.6), A_{j2} in (3.7), ($j = 1, 2$) and $W_{11}(a,\varphi)$ in (3.18) into (3.20), after calculations we have the equation (3.20) in the form

$$\begin{aligned} &\sum_{j=1}^{2}\frac{\partial}{\partial a_j}[\frac{\sigma_j^2}{4\omega_j^2}\frac{\partial W_{10}}{\partial a_j}W_0(a)] = \\ &-\sum_{j=1}^{2}\Big\{\frac{\partial^2}{\partial a_j^2}(\frac{k_j\gamma_j}{16\omega_j^2}a_j^4W_0(a) + \frac{\partial}{\partial a_j}\Big[\Big(\frac{k_j^2\gamma_j}{4\sigma_j^2} + \frac{k_j\gamma_j}{16\omega_j^2}\Big)W_0(a)\Big] + \frac{\partial}{\partial a_j}[k_{1j}a_jW_0(a)]\Big\}. \end{aligned} \tag{3.21}$$

The solution $W_{10}(a)$ of equation (3.21) can be found in the form

$$W_{10}(a) = W_{01}(a_1) + W_{02}(a_2), \tag{3.22}$$

Substituting (3.22) into (3.21), after calculations we get

$$W_{10}(a) = -\alpha_1a_1^2 - \alpha_2a_2^2 - \alpha_{11}a_1^4 - \alpha_{22}a_2^4, \tag{3.23}$$

where α_1, α_2 are defined in (3.14) and

$$\alpha_{11} = \frac{3k_1\gamma_1}{8\sigma_1^2},\ \alpha_{22} = \frac{3k_2\gamma_2}{8\sigma_2^2}. \tag{3.24}$$

Hence, using the second approximate solution to the FP equation (2.5) for the Duffing system (3.1) takes the form

$$W(a,\varphi) = W_0(a)\{1 + \varepsilon[W_{10}(a) + W_{11}(a,\varphi)]\}, \tag{3.25}$$

where $W_0(a)$, $W_{11}(a,\varphi)$ and $W_{10}(a)$ are defined in (3.15), (3.18) and (3.23), respectively. It is seen from (3.18) and (3.23) that the effect of the nonlinear terms $\varepsilon\beta_1x_1^3$ and $\varepsilon\beta_2x_2^3$ is shown in the formula (3.25).

From the normalization condition

$$\int_0^{2\pi}\int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} W(a,\varphi)da_1da_2d\varphi_1d\varphi_2 = 1,$$

we get

$$C = \frac{p_1^3p_2^3}{\pi^2[p_1^2p_2^2 - \varepsilon(\alpha_1p_1p_2^2 + \alpha_2p_1^2p_2 + \alpha_{11}p_2^2 + \alpha_{22}p_1^2)]}. \tag{3.26}$$

Now we find the approximate mean squares $E[x_1^2]$ and $E(x_2^2)$ for the cases of the classical and higher SAM.

In the case of the classical SAM we have

$$E_{cl}[x_j^2] = \int_0^{2\pi}\int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} x_j^2W(a)da_1da_2d\varphi_1d\varphi_2. \tag{3.27}$$

Substituting x_j, (j=1,2) in (2.2), $W(a)$ in (3.12) and C in (3.13) into (3.27), after calculations we have

$$E_{cl}[x_1^2] = \frac{1}{2(p_1+\varepsilon\alpha_1)} = \frac{1}{2p_1} - \varepsilon\frac{\alpha_1}{p_1^2} + \varepsilon^2\ldots \tag{3.28}$$

Similarly, we have

$$E_{cl}[x_2^2] = \frac{1}{2(p_2+\varepsilon\alpha_2)} = \frac{1}{2p_2} - \varepsilon\frac{\alpha_2}{p_2^2} + \varepsilon^2\ldots \tag{3.29}$$

where p_1, p_2, α_1 and α_2 are defined in (3.14).

In the case of the higher SAM we have

$$E[x_j^2] = \int_0^{2\pi}\int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} x_j^2W(a,\varphi)da_1da_2d\varphi_1d\varphi_2, \quad (j=1,\,2). \tag{3.30}$$

Substituting x_j, $(j=1,2)$ in (2.2), $W(a,\varphi)$ in (3.25) and C in (3.26) into (3.30), noting (3.24), after calculations we have

$$E[x_1^2] = \frac{p_1^2p_2^2 - \varepsilon(2\alpha_1p_1p_2^2 + \alpha_2p_1^2p_2 + 10\alpha_{11}p_2^2 + 2\alpha_{22}p_1^2)}{2p_1[p_1^2p_2^2 - \varepsilon(\alpha_1p_1p_2^2 + \alpha_2p_1^2p_2 + \alpha_{11}p_2^2 + \alpha_{22}p_1^2)]}. \tag{3.31}$$

Expanding the right-hand side of (3.31) into the power series of ε we get

$$E[x_1^2] = \frac{1}{2p_1} - \varepsilon\left(\frac{\alpha_1}{2p_1^2} + \frac{9\alpha_{11}}{2p_1^3} + \frac{\alpha_{22}}{2p_1p_2^2}\right) + \varepsilon^2\ldots \tag{3.32}$$

Similarly, we have

$$E[x_2^2] = \frac{1}{2p_2} - \varepsilon\left(\frac{\alpha_2}{2p_2^2} + \frac{9\alpha_{22}}{2p_2^3} + \frac{\alpha_{11}}{2p_1^2p_2}\right) + \varepsilon^2\ldots \tag{3.33}$$

Hence, the second approximate solution to the FP equation, the effect of the nonlinear terms $\varepsilon\beta_1x_1^3$ and $\varepsilon\beta_2x_2^3$ is obtained in the formulae (3.32) and (3.33). It is seen from (3.32) and (3.33) that the nonlinear terms $\varepsilon\beta_1x_1^3$ and $\varepsilon\beta_2x_2^3$ reduce the mean squares $E[x_1^2]$ and $E[x_2^2]$.

In the case $\beta_1 = \beta_2 = 0$ (linear system) we have

$$E[x_1^2] = \frac{1}{2p_1} - \varepsilon\frac{\alpha_1}{2p_1^2} + \varepsilon^2\ldots, \quad E[x_2^2] = \frac{1}{2\alpha_2} - \varepsilon\frac{\alpha_2}{2p_2^2} + \varepsilon^2\ldots \tag{3.34}$$

which equal to the mean squares in (3.28), (3.29) obtained by the classical SAM.

We consider the case where $m_1 = m_2 = c_1 = c_2 = \delta_1 = \delta_2 = 1$, $\beta_1 = \beta_2 = \beta$, $h_1 = h_2 = h_{12} = c_{12} = 0.5$. The mean square responses corresponding to some values of the coefficients β are given in Table 1. It is seen that the mean square responses decrease when increasing the coefficient β.

Table 1. Mean square responses to Duffing system
(Effect of non-linear coefficient β)

N	β	$E[x_1^2] = E[x_2^2]$
1	0	$0.5 - 0.5\varepsilon + \varepsilon^2 \ldots$
2	0.2	$0.5 - 0.6875\varepsilon + \varepsilon^2 \ldots$
3	0.5	$0.5 - 0.9687\varepsilon + \varepsilon^2 \ldots$
4	1.0	$0.5 - 1.4375\varepsilon + \varepsilon^2 \ldots$
5	2.0	$0.5 - 2.375\varepsilon + \varepsilon^2 \ldots$

4. CONCLUSION

For many years the higher order stochastic averaging method has been widely used for investigating single-degree-of-freedom nonlinear systems subject to white and coloured random noises. In this paper, the method is applied to the nonlinear vibration systems having two degrees of freedom under white noise excitations. The application to the Duffing system is considered and shows the effect of the non-linear terms to the mean square responses of the system.

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ẢNH HƯỞNG CỦA SỐ HẠNG PHI TUYẾN TRONG CÁC HỆ CƠ HỌC HAI BẬC TỰ DO

Phương pháp trung bình ngẫu nhiên bậc nhất được áp dụng rộng rãi đối với các hệ dao động phi tuyến chịu kích động ngẫu nhiên dạng ồn trắng và ồn màu. Tuy nhiên, hiệu ứng của nhiều số hạng phi tuyến bị biến mất do kết quả của phép lấy trung bình. Để khắc phục nhược điểm trên phương pháp trung bình ngẫu nhiên bậc cao được phát triển. Trong bài báo, phương pháp được trình bày đối với hệ phi tuyến yếu hai bậc tự do chịu kích động ngẫu nhiên dạng ồn trắng. Sau đó phương pháp được áp dụng để xác định nghiệm xấp xỉ bậc hai của phương trình Fokker-Planck đối với hệ dạng Duffing.