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PARAMETRIC VIBRATION OF MECHANICAL SYSTEM WITH SEVERAL DEGREES OF FREEDOM UNDER THE ACTION OF ELECTROMAGNETIC FORCE

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1. SYSTEMS WITH *n* DEGREES OF FREEDOM

Let us consider a vibrating system with n degrees of freedom which consists of a weightless cantilever beam carrying n concentrated masses $m_1, m_2, \dots, m_n$ (Fig. 1). The elastic elements of the vibrating system have stiffness $k_1, k_2, \dots, k_n$.

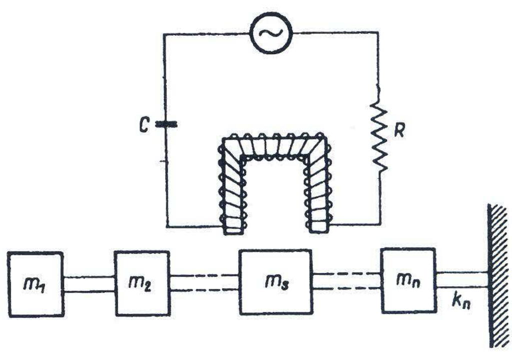

Fig. 1

Supposing that some s^{th} mass is subjected to electromagnetic force, the differential equations of motion of the system considered can be written, in accordance with [1] in the form:

$$\frac{d}{dt}(L\dot{q}) + R\dot{q} + \frac{1}{C}q = E\sin\nu t,$$

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) = -h_1\dot{x}_1 - \beta_1(x_1 - x_2)^3,$$

$$m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2(x_2 - x_3) = -h_2\dot{x}_2 - \beta_1(x_2 - x_1)^3 - \beta_2(x_2 - x_3)^3,$$

$$
\begin{aligned}
&\dots \quad \dots \quad \dots \quad \dots \\
&m_s\ddot{x}_s + k_{s-1}(x_s - x_{s-1}) + k_s(x_s - x_{s+1}) = -h_s\dot{x}_s - \beta_{s-1}(x_s - x_{s-1})^3 \\
&\qquad\qquad - \beta_s(x_s - x_{s+1})^3 + \frac{1}{2}\dot{q}^2\frac{\partial L}{\partial x_s}, \\
&\dots \quad \dots \quad \dots \quad \dots \\
&m_n\ddot{x}_n + k_{n-1}(x_n - x_{n-1}) + k_n x_n = -h_n\dot{x}_n - \beta_{n-1}(x_n - x_{n-1})^3 - \beta_n x_n^3. \qquad (1.1)
\end{aligned}
$$

We assume that

$$L = L(x_s) = L_0(1 - \alpha_1 x_s + \alpha_2 x_s^2),$$

and that the friction forces and the non-linear terms in (1.1) are small with respect to the remaining terms. Then, Eqs. (1.1) can be rewritten as:

$$
\begin{aligned}
&L_0\ddot{q} + \frac{1}{C}q = E\sin\nu t - \mu\big[L_0\ddot{q}(-\alpha_1 x_2 + \alpha_2 x_s^2) + \dot{q}L_0(-\alpha_1\dot{x}_s + 2\alpha_2 x_s\dot{x}_x)\big], \\
&m_1\ddot{x}_1 + k_1(x_1 - x_2) = \mu F_1, \\
&m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2(x_2 - x_3) = \mu F_2, \\
&\dots \quad \dots \quad \dots \quad \dots \qquad (1.2) \\
&m_s\ddot{x}_s + k_{s-1}(x_s - x_{s-1}) + k_s(x_s - x_{s+1}) = -\frac{1}{2}\dot{q}^2 L_0\alpha_1 + \mu_s F_s, \\
&\dots \quad \dots \quad \dots \quad \dots \\
&m_n\ddot{x}_n + k_{n-1}(x_n - x_{n-1}) + k_n x_n = \mu F_n
\end{aligned}
$$

where

$$
\begin{aligned}
&\mu F_1 = -h_1\dot{x}_1 - \beta_1(x_1 - x_2)^3, \\
&\mu F_2 = -h_2\dot{x}_2 - \beta_1(x_2 - x_1)^3 - \beta_2(x_2 - x_3)^3, \\
&\dots \quad \dots \quad \dots \quad \dots \qquad (1.3) \\
&\mu F_s = -h_s\dot{x}_s + \dot{q}^2 L_0\alpha_2 x_s - \beta_{s-1}(x_s - x_{s-1})^3 - \beta_s(x_s - x_{s+1})^3, \\
&\dots \quad \dots \quad \dots \quad \dots \\
&\mu F_n = -h_n\dot{x}_n - \beta_{n-1}(x_n - x_{n-1})^3 - \beta_n x_n^3.
\end{aligned}
$$

We suppose that the characteristic equation of the homogeneous system

$$
\begin{aligned}
&m_1\ddot{x}_1 + k_1(x_1 - x_2) = 0, \\
&m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2(x_2 - x_3) = 0, \\
&\dots \quad \dots \quad \dots \quad \dots \qquad (1.4) \\
&m_n\ddot{x}_n + k_{n-1}(x_n - x_{n-1}) + k_n x_n = 0,
\end{aligned}
$$

has no multiple roots and that its roots $\omega_1, \dots, \omega_n$ are linearly independent. Then, to study the system (1.2), we shall analyze its particular solution corresponding to the one-frequency regime of vibrations [2]. To that end, we introduce the normal coodinates $\xi_1, \dots, \xi_n$ by means of the formulae:

$$x_s = \sum_{\sigma=1}^{n} c_s^{(\sigma)}\xi_\sigma, \quad s = 1, 2, \dots, n, \qquad (1.5)$$

where $c_s^{(\sigma)}$ is algebraic supplement of the element placed in the s-th column and the lasrt line of the characteristic determinant of the system (1.4).

We can easily verify that the normal coordinates $\xi_1, \dots, \xi_n$, satisfy the following equations:

$$\begin{aligned} &L_0\ddot{q} + \frac{1}{C}q = E\sin\nu t - \mu F_0\Big(\dot{q}, \ddot{q}, \sum_{\sigma=1}^{n} c_s^{(\sigma)}\xi_\sigma, \sum_{\sigma=1}^{n} c_s^{(\sigma)}\dot{\xi}_\sigma\Big), \\ &\ddot{\xi}_j + \omega_j^2\xi_j = \mu\Phi_j\Big(\sum_{\sigma=1}^{n} c_1^{(\sigma)}\xi_\sigma, \dots, \sum_{\sigma=1}^{n} c_n^{(\sigma)}\xi_s, \sum_{\sigma=1}^{n} c_1^{(\sigma)}\dot{\xi}_s, \dots, \sum_{\sigma=1}^{n} c_n^{(\sigma)}\dot{\xi}_\sigma, \dot{q}\Big). \end{aligned} \tag{1.6}$$

Here

$$\begin{aligned} &F_0(\dot{q}, \ddot{q}, x_s, \dot{x}_s) = L_0\ddot{q}(-\alpha_1 x_s + \alpha_2 x_s^2) + L_0\dot{q}\dot{x}_s(-\alpha_1 + 2\alpha_2\alpha_s) + R\dot{q}, \\ &\Phi_j = \frac{1}{M_j}\sum_{k=1}^{n} c_k^{(j)}F_k - \frac{\alpha_1 L_0\dot{q}^2}{2\mu M_j}c_s^{(j)}, \\ &M_j = \sum_{i=1}^{n} m_i c_1^{2(j)}. \end{aligned}$$

In the first approximation, the investigation of one-frequency regime in the system considered can be reduced to a study of two equations: the first of (1.6) and one of remaining n equations. The choice of the appropriate equation depends on the value of natural frequency ω in the neighbourhood of which the parametric vibrations are examined. Supposing that the frequency ν of external force is near the nartural frequency ω_j. Then we shall investigate the equations:

$$\begin{aligned} &L_0\ddot{q} + \frac{1}{C}q = E\sin\nu t - \mu F_0^*, \\ &\ddot{\xi}_j + \omega_j^2\xi_j = -\mu h^*\dot{\xi}_j - \mu\beta^*\xi_j^3 - \frac{\alpha_1 L_0 c_s^{(j)}}{2M_j}\dot{q}^2 + \frac{\mu}{M_j}L_0\alpha_2 c_s^{2(j)}\dot{q}^2\xi_j, \end{aligned} \tag{1.7}$$

where

$$\mu h^* = \frac{1}{M_j}\sum_{s=1}^{n} h_s c_s^{2(j)},$$

$$\begin{aligned} \mu\beta^* = \frac{1}{M_j}\big[&\beta_1 c_1^{(j)}(c_1^{(j)} - c_2^{(j)})^3 + \beta_1 c_1^{(j)}(c_2^{(j)} - c_1^{(j)})^3\beta_2 c_2^{(j)}(c_2^{(j)} - c_3^{(j)})^2 + \dots \\ &+ \beta_{s-1}c_s^{(j)}(c_s^{(j)} - c_{s-1}^{(j)})^3 + \beta_s c_s^{(j)}(c_s^{(j)} - c_{s+1}^{(j)}) + \dots \\ &+ \beta_{n-1}c_n^{(j)}(c_n^{(j)} - c_{n-1}^{(j)})^3 + \beta_n c_n^{4(j)}\big], \end{aligned}$$

$$\mu F_0^* = L_0\ddot{q}(-\alpha_1 c_s^{(j)}\xi_j + \alpha_2 c_s^{2(j)}\xi_j^2) + L_0\dot{q}c_s^{(j)}\dot{\xi}_j(-\alpha_1 + 2\alpha_2 c_s^{(j)}\xi_j).$$

The remaining $n-1$ normal coordinates $\xi_1, \dots, \xi_{j-1}, \xi_{j+1}, \dots, \xi_n$ are far from the resonance, their vibration will be small in comparison with the resonant vibration considered of the coordinate ξ_j, and in the first approximation they may be disregarded.

Equations (1.7) describing the one-frequency regime of vibrations have the same structure as the equations of motion of the system with single degree of freedom [1]. This gives reason to expect that in each resonant region the same peculiarities of motion will be displayed were found in the system with a single degree of freedom.

Introducing the notations

$$h = \frac{h^*}{\omega_j}, \quad \tau = \omega_j t, \quad \beta = \frac{\beta^*}{\omega_j^2}, \quad \alpha_1^* = \frac{\alpha_1 L_0 c_s^{(j)}}{2M_j},$$
$$\alpha_2^* = \frac{1}{M_j} L_0 \alpha_2 c_s^{2(j)}, \quad \Omega_j = \frac{\Omega_0}{\omega_j}, \quad \Omega_0^2 = \frac{1}{L_0 C}, \quad e_j = \frac{L_0}{E_0 \omega_j^2}, \quad \gamma_j = \frac{\gamma}{\omega_j}, \tag{1.8}$$

Eqs. (1.7) assume the form:

$$q'' + \nabla_j^2 q = e_j \sin \gamma_j \tau - \frac{\mu}{L_0 \omega_j^2} F_0^*,$$
$$\xi_j'' + \xi_j = -\mu h \xi_j' - \mu \beta \xi_j^3 - \alpha_1^* q'^2 + \mu \alpha_2^* q'^2 \xi_j. \tag{1.9}$$

Now, we transform the system (1.9) by means of the formulae:

$$q = e_j^* \sin \gamma_j \tau + B \sin \varphi,$$
$$q' = \gamma_j e_j^* \cos \gamma_j \tau + \Omega_j B \cos \varphi,$$
$$\xi_j = -b - \frac{b}{1 - 4\gamma_j^2} \cos 2\gamma_j \tau + A_j \sin \theta_j, \tag{1.10}$$
$$\xi_j' = \frac{2\gamma_j b}{1 - 4\gamma_j^2} \sin 2\gamma_j \tau + A_j \gamma_j \cos \theta_j,$$

where

$$e_j^* = \frac{e_j}{\Omega_j^2 - \gamma_j^2}, \quad b = \frac{\alpha_j^* \gamma_j^2 e_j^{*2}}{2}, \quad \varphi = \Omega_j \tau, \quad \theta_j = \gamma_j \tau + \psi_j.$$

The transformed equations have the form:

$$\Omega_j \frac{dB}{d\tau} = -\frac{\mu}{L_0 \omega_j} F_0^* \cos \varphi,$$
$$\Omega_j B \frac{d\Phi}{d\tau} = \frac{\mu}{L_0 \omega_j^2} = F_0^* \sin \varphi,$$
$$\gamma_j \frac{dA_j}{d\tau} = -A_j (1 - \gamma_j^2) \sin \theta_j \cos \theta_j - \mu S \cos \theta_j + \ldots, \tag{1.11}$$
$$A_j \gamma_j \frac{d\psi_j}{d\tau} = A_j (1 - \gamma_j^2) \sin^2 \theta_j + \mu S \sin \theta_j + \ldots,$$
$$S = h \xi_j' + \beta \xi_j^3 - \alpha_2^* q'^2 \xi,$$

where the non-written terms vanish when $B = 0$.

We suppose that γ_j is in the neighbourhood of 1 and that γ_j and Ω_j are linearly indepenent. Then, in the first approximation the solution of the system (1.9) satisfies the equations obtained from (1.9) by averaging in time its right-hand part

$$[1 + 0(\mu)] \frac{dB}{d\tau} = -\mu \frac{R}{2} B,$$

$$\begin{aligned}
\frac{d\Phi}{d\tau} &= G(B,\Phi,A_j,\psi_j), \\
\gamma_j\frac{dA_j}{d\tau} &= -\mu\frac{h}{2}\gamma_jA_j + \frac{\mu}{2}c_1A_j\sin 2\psi_j + \dots, \\
\gamma_jA_j\frac{d\psi_j}{d\tau} &= \frac{1}{2}(1-\gamma_j^2+\mu\Delta)A_j + \frac{3}{8}\mu\beta A_j^2 + \frac{\mu}{2}c_1A_j\cos 2\psi_j + \dots
\end{aligned} \tag{1.12}$$

where

$$c_1 = \frac{q}{2} + \frac{3\beta b^2}{1-4\gamma_j^2},$$

$$A = q + 3\beta b^2 + \frac{3}{2}\beta\frac{b^2}{(1-4\gamma_j^2)^2},$$

$$b = \frac{\alpha_j^*\gamma_j^2c_j^{*2}}{2}, \quad q = \frac{\alpha_2^*}{2}\gamma_j^2e_j^{*2}.$$

Since $B \to 0$ when $t \to \infty$, then below we shal take into account only the equations:

$$\begin{aligned}
\gamma_j\frac{dA_j}{d\tau} &= -\mu\frac{h}{2}\gamma_jA_j + \frac{\mu}{2}c_1A_j\sin 2\psi_j, \\
\gamma_jA_j\frac{d\psi_j}{d\tau} &= \frac{1}{2}(1-r_j^2+\mu\Delta_1)A_j + \frac{3}{8}\mu\beta A_j^3 + \frac{\mu}{2}c_1A_j\cos 2\psi_j,
\end{aligned} \tag{1.13}$$

from which we obtain the amplitude A_j of vibrations:

$$A_j^2 = \frac{4}{3\beta}\Big(\frac{\gamma_j^2-1}{\mu} - \Delta \pm \sqrt{c_1^2-h^2\gamma_j^2}\Big), \tag{1.14}$$

and the phase

$$\sin 2\psi_j = \frac{h}{c_1}\gamma_j, \quad \cos 2\psi_j = \mp\frac{1}{c_1}\sqrt{c_1^2-h^2\gamma_j^2}. \tag{1.15}$$

Equations (1.12)-(1.15) are different from the corresponding ones in the system with a single degree of freedom [1] only by the values of the constant coefficients. The method used enabled us to reduce the more complicated problem to the whole complex of n problems of the type considered earlier. In spite of this, in the first approximation each of such problems can be investigated independently of the others, because according to the conditions of the problem, the resonant processes cannot be developed at the same time in more than on resonant region.

The stability of stationary regimes of vibrations may be found by analysing Eqs. (1.12). The criteria of stability formed in [1] are:

$$\frac{\partial W}{\partial A_j} > 0 \quad \text{for} \quad A_j \neq 0,$$

$$W = \Big(\frac{3}{4}\mu\beta A_j^2 + 1 - \gamma_j^2 + \mu\Delta\Big)^2 - \mu^2(c_1^2 - h^2\gamma_j^2),$$

and

$$\mu^2(h^2\gamma_j^2 - c_1^2) + (\gamma_j^2 - 1 - \mu\Delta)^2 > 0 \quad \text{for} \quad A_j = 0.$$

The study made in [1] concerning the stability of stationary regimes of motion will be suitable for the character of resonant processes described by Eqs. (1.12) in qualitative relation. This removes the necessity of analysis in detail the criteria of stability. Here we note only that for very slow change of frequency ν in the system considered, n resonant peaks corresponding to the values $\nu = \omega_1$ $(\gamma_1 = 1)$, $\nu = \omega_2$ $(\gamma_2 = 1)\ldots$ are observed (Fig. 2).

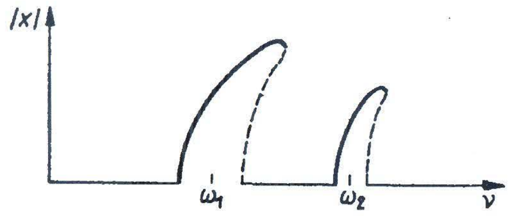

Fig. 2

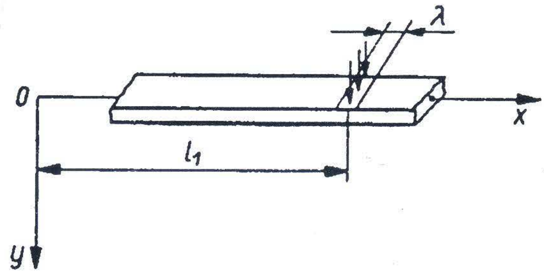

Fig. 3

2. PARAMETRIC RESONANCE IN A SYSTEM WITH INFINITE NUMBER OF DEGREES OF FREEDOM

We investigate in the Cartesian coordinates x, y, z a prismatic beam with length ℓ whose cross-section is symmetrical with respect to two mutually perpendicular axes. We assume that the axis of the beam in the underformed state coincides with the axis x and that the symmetrical axes are parallel to the axes y and z (Fig. 3).

The beam under certain conditions of strengthning of its end is subjected to the action of electromagnetic force which is ℓ_1 distant from the origin of the coordinates and directed to the axis y. We assume that the inductance L is a function of distance $y_1 = y(\ell_1, t)$,

$$L = L(y_1) = L_0(1 - \alpha_1 y_1 + \alpha_2 y_1^2), \tag{2.1}$$

and therefore the electromagnetic force depends on the location of the electromagnet and on the vibrations of the beam, and has intensity $\dfrac{1}{2}\dot{q}^2\dfrac{\partial L}{\partial \ell_1}$.

We assume that the material of the beam follows the law [3]

$$\sigma_x = f(\varepsilon_x) = E(1 - dE^2\varepsilon_x^2)\varepsilon_x,$$

where σ_x is the longitudinal force and ε_x is the longitudinal elongation. Then, the equation of motion of the beam is:

$$\frac{\partial^2 M}{\partial x^2} = -\rho\frac{\partial^2 y}{\partial t^2} - H\frac{\partial y}{\partial t} + P(x,t), \tag{2.2}$$

where ρ is the intensity of mass of the beam, $y = y(x,t)$-the deflection, $P(x,t)$-the intensity of external load, $M(x,t)$-the bending moment:

$$M = \iint f\Big(y\frac{\partial^2 y}{\partial x^2}\Big)ydydz = E\iint \Big[1 - dE^2y^2\Big(\frac{\partial^2 y}{\partial x^2}\Big)^2\Big]y^2\frac{\partial^2 y}{\partial x^2}dydz.$$

Substituting this expression into (2.2), we obtain:

$$\rho\frac{\partial^2 y}{\partial t^2} + EJ\frac{\partial^4 y}{\partial x^4} = 3dE^3J_1\Big[\frac{\partial^4 y}{\partial x^4}\frac{\partial^2 y}{\partial x^2} + 2\Big(\frac{\partial^3 y}{\partial x^3}\Big)^2\Big]\frac{\partial^2 y}{\partial x^2} - H\frac{\partial y}{\partial t} + P(x,t),$$

where,

$$J_1 = \iint y^4 dydz, \quad J = \iint y^2 dydz.$$

We assume that the non-linear terms and the terms characterizibng friction are small in comparison with the linear terms. Then the equation of motion of the system considered can be represented in the form:

$$\begin{aligned} &\ddot{q} + \Omega_0^2 q = e \sin \nu t + \mu F_1 \Big(y_1, \frac{\partial y_1}{\partial t}, \dot{q}, \ddot{q} \Big), \\ &\frac{\partial^2 y}{\partial t^2} + b^2 \frac{\partial^4 y}{\partial x^4} = \mu F_2, \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} \Omega_0^2 &= \frac{1}{L_0 C}, \quad e = \frac{E}{L_0}, \quad b = \frac{EJ}{\rho}, \quad \beta = \frac{3dE^3 J_1}{\rho}, \\ \mu F_1 &= -\frac{R}{L_0}\dot{q} + \alpha_1 \Big(\dot{q}\frac{\partial y_1}{\partial t} + y_1 \ddot{q} \Big) - \alpha_2 \Big(2y_1 \dot{q}\frac{\partial y_1}{\partial t} + \ddot{q} y_1^2 \Big), \\ \mu F_2 &= -\frac{H}{\rho}\frac{\partial y}{\partial t} + \beta \Big[\frac{\partial^4 y}{\partial x^4}\frac{\partial^2 y}{\partial x^2} + 2\Big(\frac{\partial^3 y}{\partial x^3} \Big)^2 \Big] \frac{\partial^2 y}{\partial x^2} + \frac{1}{\rho} P(x,t). \end{aligned} \tag{2.4}$$

The external load $P(x,t)$ has the form:

$$P(x,t) = \begin{cases} 0 & \text{for } 0 \leqslant x < \ell_1 - \dfrac{\lambda}{2}, \\ \dfrac{L_0 \alpha_2}{\lambda} \dot{q}^2 y_1 - \dfrac{L_0 \alpha_1}{2\lambda} \dot{q}^2 & \text{for } \ell_1 - m\dfrac{\lambda}{2} \leqslant x \leqslant \ell_1 + \dfrac{\lambda}{2}, \\ 0 & \text{for } \ell_1 + \dfrac{\lambda}{2} < x \leqslant \ell, \end{cases} \tag{2.5}$$

where λ is the length of that element of the beam is directly subjected to the action of electromagnetic force.

To solve the system (2.3), we note first that the generative equations ($\mu = 0$)

$$\ddot{q} + \Omega_0^2 q = e \sin \nu t, \quad \frac{\partial^2 y}{\partial t^2} + b^2 \frac{\partial^4 y}{\partial x^4} = 0 \tag{2.6}$$

have the solution:

$$\begin{aligned} &q = e^* \sin \nu t + B \sin \varphi, \\ &y = \sum_{n=1}^{\infty} X_n(x) c_n \cos \Big(\frac{m_n^2}{\ell^2} bt + \gamma_n \Big), \end{aligned} \tag{2.7}$$

where B, Φ, c_n, γ_n are arbitrary constants, X_n are the eigenfunctions which define the natural modes of vibrations of the beam and depend on the boundary conditions.

Equations (2.3) are different from the corresponding ones of the systems (2.6) only by small terms μF_1, μF_2. Consequently, it is natural to propose the following form of solution of the system (2.3):

$$q = e^* \sin \nu t + B \sin \varphi, \quad y = \sum_{n=1}^{\infty} X_n(x) s_n(t), \tag{2.8}$$

where $\varphi = \Omega_0 t + \Phi$ and B, Φ, s_n are functions of time.

Now, instead of determining the functions q and y, we determine the functions B, Φ, s_n. To find the different equations for these variables, we represent F_2 in the form of a series:

$$F_2 = \sum_{n=1}^{\infty} X_n V_n(s_1, s_2, \ldots, \dot{s}_1, \dot{s}_2, \ldots, t). \tag{2.9}$$

To seek the functions of time V_n, we multiply both sides of the equality (2.9) by X_i, and integrate the result over the total length of the beam; due to the orthogonality of the eigenfunctions there remains only term on the right-hand side which corresponds to the number n, so that

$$V_n = \int_0^\ell F_2 X_n dx \Big/ \int_0^\ell X_n^2 dx. \tag{2.10}$$

Substituting (2.8), (2.9) into (2.3) and equating the coefficients X_n, we arrive at:

$$\begin{aligned} &\ddot{q} + \Omega_0^2 q = e \sin \nu t + \mu F_1, \\ &\ddot{s}_n + \omega_n^2 s_n = \mu r_n L_0 \alpha_2 \dot{q}^2 S_n + \mu K_n(s_1, s_2, \ldots, \dot{s}_1, \dot{s}_2, \ldots, t), \end{aligned} \tag{2.11}$$

where K_n are polynomials of third degree, relatively of $s_1, s_2, \ldots$

We consider now parametric resonance when the frequency of the electric circuit ν is in the neighbourhood of ω_j assuming that the natural frequencies $\omega_1, \omega_2, \ldots$ are independent. Then we retain in (1.1) only the coordinates s_j. The remaining coordinates $s_1, \ldots, s_{j-1}, s_{j+1}, \ldots$ are far from the resonance and their values will be small in comparison with s_j and in the first approximation we can disregard them. Thus, following the expressions (2.4), (2.8), we have:

$$\mu F_2 = -\frac{H}{\rho} \dot{s}_j X_j + \beta [X_j^{IV} X_j''^2 + 2 X_j'''^2 X_j''] s_j^3 + \frac{P}{\varrho}$$

Therefore, from (2.10), (2.11) we obtain the equations for q, s_n:

$$\begin{aligned} &\ddot{q} + \Omega_0 q = e \sin \nu t + \mu F_1, \\ &\ddot{s}_j + \omega_j^2 s_j = -\frac{H}{\rho} \dot{s}_j + \beta a_j s_j^3 + b_j \dot{q}^2 s_j - c \dot{q}^2, \end{aligned} \tag{2.12}$$

where

$$a_j = \int_0^\ell (X_j^{IV} X_j''^2 + X_j'''^2 X_j'') X_j dx \Big/ \int_0^\ell X_j^2 dx,$$

$$b_j = L_0 \alpha_2 X_j(\ell_1) \int_{\ell_1 - \frac{\lambda}{2}}^{\ell_1 + \frac{\lambda}{2}} X_j dx \Big/ \lambda \rho \int_0^\ell X_j^2 dx, \tag{2.13}$$

$$c_j = \frac{L_0 \alpha_1}{2} \int\limits_{\ell_1 - \frac{\lambda}{2}}^{\ell_1 + \frac{\lambda}{2}} X_j dx \Big/ \lambda\rho \int\limits_0^{\ell} X_j^2 dx.$$

It is easily seen that the system of Eqs. (2.12) is the complete analogy of the differential equations of vibrations of a system with single degree of freedom. To avoid repetition, we shall refer below to the paper [1], where the problem of construction of a solution of the system of equations of the form (2.12) is considered in detail.

Thus, following the results of [1], we conclude that when the frequency ν of an electric circuit is near to ω_1, then the beam considered vibrates strongly with frequency ν (parametric resonance). This type of resonance takes place also when the frequency ν is near to $\omega_2, \omega_3, \ldots$ However, it must be emphasized that in the system with distributed parameters the vibrations with the lowest frequency (ω_1) play the main role.

Some experiments were performed with beams and systems of several degrees of freedom. The experimental results in the cases considered were in good agreement with the theoretical results. This fact testifies to the acceptability of the limitations used is the problem and shows that the approximate solutions found by using the assumption concerning the one-frequency regime of vibrations in the regions of resonance can be adopted for practical purpose.

For the cantilever beam with parameters $E = 2 \cdot 10^7$ N/cm^2, $J = 16 \cdot 10^{-3}$ cm^4, $\varphi = 10^{-4} N \cdot s^2/cm^2$, $\ell = 46$ cm; therefore, $\omega_1 = 14.8$, $\omega_2 = 93.7$, strong vibrations with frequency of electric circuit ν when ν is in the region 13.5-14.3 Hz ..., were very small. For the same beam, but when $\ell = 58$ cm and therefore $\omega_1 = 26.3$, substantial parametric resonance when ν is the region 27.1-29.4 Hz was observed.

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DAO ĐỘNG THAM SỐ CÁC HỆ CƠ HỌC NHIỀU BẬC TỰ DO GÂY NÊN BỞI TÁC DỤNG CỦA LỰC ĐIỆN TỪ

Công trình này là sự tiếp tục của công trình đã được công bố [1]. Trong công trình này kết quả nghiên cứu dao động của hệ cơ học với n bậc tự do và của dầm khi chúng chịu tác dụng với lực điện từ của dao động kích động tham số tương tự như trong [1] dao động tham số được khảo sát có tần số bằng tần số dao động trong khung điện. Xác định biên độ dao động và nghiên cứu ổn định của chúng.