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ASYMPTOTIC SOLUTION OF THE HIGH ORDER PARTIAL DIFFERENTIAL EQUATION

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Abstract. In the present paper, the authors have constructed an asymptotic solution of the high order equation with partial derivatives by means of the asymptotic method for the high order systems. The improved first approximation of the solution of the given boundary value problem is determined.

1. INTRODUCTION

The problem of the oscillation of the creepy elastic beam with linear boundary conditions in the autonomous case has been studied [3]. In this work, the authors inverstigate the oscillation of the creepy elastic beam, in the non-autonomous case, described by the third order equation as follows:

$$\frac{\partial^3 y}{\partial t^3} + \xi \frac{\partial^2 y}{\partial t^2} + \omega^2 \frac{\partial^5 y}{\partial t \partial x^4} + \xi \omega^2 \frac{\partial^4 y}{\partial x^4} = \varepsilon F(x, y, \dot{y}, \theta, ...), \tag{1}$$

where ξ, ω, Ω are real constants, ε is a small parameter, $\theta = \theta(t)$, $y = y(x,t)$. The relevant homogeneous boundary conditons are

$$y|_{x=0} = 0, \quad \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=0} = 0, \quad y|_{x=0} = 0, \quad \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=0} = 0. \tag{2}$$

It is supposed that the resonance relation takes the from.

$$\Omega_1 = \frac{p}{q}\gamma + \varepsilon\Delta, \tag{3}$$

$$\frac{d\theta}{dt} = \gamma, \tag{4}$$

p, q are integers.

2. CONTRUCTION OF THE NON-AUTONOMOUS SYSTEM

With the boundary conditions (2) we get the fundamental functions and the eigenvalues in the form

$$Z_k(x) = \sin\frac{k\pi x}{\ell}, \beta_k = \frac{k^2\pi^2}{l^2} \quad (k = 1, 2, \ldots). \tag{5}$$

In this case, the partial solution of the equation (1) is found in form of the following series [6]

$$y(x,t) = a\cos\varphi Z_1 + \varepsilon U_1(x,a,\varphi,\theta) + \varepsilon^2 U_2(x,a,\varphi,\theta) + ... \tag{6}$$

$$\varphi = \left(\frac{p}{q}\theta + \psi\right) \tag{7}$$

where a, ψ are the functions satisfy the following differential equations

$$\frac{da}{dt} = \varepsilon A_1(a,\psi) + \varepsilon^2 A_2(a,\psi) + ... \tag{8}$$

$$\frac{d\psi}{dt} = \left(\Omega_1 - \frac{p}{q\gamma}\right) + \varepsilon B_1(a,\psi) + \varepsilon^2 B_2(a,\psi) + ... \tag{9}$$

Now differentiating the function $y(x,t)$ in the form (6) with respect to argument t, after calculation we get

$$\frac{\partial y}{\partial t} = \varepsilon A_1 \cos\varphi Z_1 - a\Omega_1 \sin\varphi Z_1 - \varepsilon\beta_1 a\sin\varphi Z_1 + \varepsilon\left(\Omega_1\frac{\partial U_1}{\partial\varphi} + \gamma\frac{\partial U_1}{\partial\theta}\right)$$

$$\begin{aligned}\frac{\partial^2 y}{\partial t^2} = {}& \varepsilon\frac{\partial A_1}{\partial\psi}\frac{d\psi}{dt}\cos\varphi Z_1 - \varepsilon A_1\sin\varphi\frac{d\varphi}{dt}Z_1 - \frac{da}{dt}\Omega_1\sin\varphi Z_1 - a\Omega_1\frac{d\varphi}{dt}\cos\varphi Z_1 \\ & - \varepsilon\frac{\partial B_1}{\partial\psi}\frac{d\psi}{dt}a\sin\varphi Z_1 - \varepsilon B_1 a\cos\varphi\frac{d\varphi}{dt}Z_1 + \varepsilon\left(\Omega_1^2\frac{\partial^2 U_1}{\partial\varphi^2} + 2\Omega_1\gamma\frac{\partial^2 U_1}{\partial\varphi\partial\theta} + \gamma^2\frac{\partial^2 U_1}{\partial\theta^2}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 y}{dt^2} = {}& \varepsilon\frac{\partial A_1}{\partial\psi}\left(\Omega_1 - \frac{p}{q}\gamma\right)\cos\varphi Z_1 - \varepsilon A_1\Omega_1\sin\varphi Z_1 - \varepsilon A_1\Omega_1\sin\varphi Z_1 - \varepsilon\Omega_1^2\cos\varphi Z_1 \\ & - \varepsilon B_1 a\Omega_1\cos\varphi Z_1 - \varepsilon\frac{\partial B_1}{\partial\psi}\left(\Omega_1 - \frac{p}{q}\gamma\right)a\sin\varphi Z_1 - \varepsilon B_1 a\Omega_1\cos\varphi Z_1 \\ & + \varepsilon\left(\Omega_1^2\frac{\partial^2 U_1}{\partial\varphi^2} + 2\Omega_1\gamma\frac{\partial^2 U_1}{\partial\varphi\partial\theta} + \gamma^2\frac{\partial^2 U_1}{\partial\theta^2}\right)\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\partial^2 y}{\partial t^2} = {}& \varepsilon\frac{\partial A_1}{\partial\psi}\left(\Omega_1 - \frac{p}{q}\gamma\right)\cos\varphi Z_1 - 2\varepsilon A_1\Omega_1\sin\varphi Z_1 - 2\varepsilon B_1 a\Omega_1\cos\varphi Z_1 \\ & - \varepsilon\frac{\partial B_1}{\partial\psi}\left(\Omega_1 - \frac{p}{q}\gamma\right)a\sin\varphi Z_1 - a\Omega_1^2\cos\varphi Z_1 + \varepsilon\,(\Omega_1^2\,\frac{\partial^2 U_1}{\partial\varphi^2} + 2\Omega_1\gamma\frac{\partial^2 U_1}{\partial\varphi\partial\theta} + \gamma^2\frac{\partial^2 U_1}{\partial\theta^2}).\end{aligned}$$

$$\begin{aligned}\frac{\partial^3 y}{\partial t^3} = {}& \varepsilon\frac{\partial^2 A_1}{\partial\psi^2}\,(\Omega_1 - \frac{p}{q}\gamma)^2\cos\varphi Z_1 - \varepsilon\frac{\partial A_1}{\partial\psi}(\Omega_1 - \frac{p}{q}\gamma)\sin\varphi\Omega_1 Z_1 \\ & - 2\varepsilon\frac{\partial A_1}{\partial\psi^2}(\Omega_1 - \frac{p}{q}\gamma)\Omega_1\sin\varphi Z_1 - 2\varepsilon A_1\Omega_1^2\cos\varphi Z_1 - 2\varepsilon\frac{\partial B_1}{\partial\psi}(\Omega_1 - \frac{p}{q}\gamma)a\Omega_1\cos\varphi Z_1 \\ & + 2\varepsilon B_1 a\Omega_1^2\sin\varphi Z_1 - \varepsilon\frac{\partial^2 B_1}{\partial\psi^2}(\Omega_1 - \frac{p}{q}\gamma)^2 a\sin\varphi Z_1 - \varepsilon\frac{\partial B_1}{\partial\psi}(\Omega_1 - \frac{p}{q}\gamma)a\Omega_1\cos Z_1 \\ & - \varepsilon A_1\Omega_1^2\cos\varphi Z_1 + a\Omega_1^3\sin\varphi Z_1 + \varepsilon B_1 a\Omega_1^2\sin\varphi Z_1 + \varepsilon(\Omega_1^3\frac{\partial^3 U_1}{\partial\varphi^3} + 3\Omega_1\gamma\frac{\partial^3 U_1}{\partial\varphi^2\partial\theta}\end{aligned}$$

$$+3\Omega_1\gamma\frac{\partial^3 U_1}{\partial\varphi\partial\theta^2}+\gamma^3\frac{\partial^3 U_1}{\partial\theta^3}).$$

$$\frac{\partial^3 y}{\partial t^3}=\varepsilon\frac{\partial^2 A_1}{\partial\psi^2}\left(\Omega_1-\frac{p}{q}\gamma\right)^2\cos\varphi Z_1-3\varepsilon\frac{\partial A_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)\Omega_1\sin\varphi Z_1-3\varepsilon A_1\Omega_1^2\cos\varphi Z_1$$
$$-\varepsilon\frac{\partial^2 B_1}{\partial\psi^2}\left(\Omega_1-\frac{p}{q}\gamma\right)^2 a\sin\varphi Z_1-3\varepsilon\frac{\partial B_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)a\Omega_1\cos\varphi Z_1+3\varepsilon B_1 a\Omega_1^2\sin\varphi Z_1$$
$$+a\Omega_1^3\sin\varphi Z_1+\varepsilon(\Omega_1^3\frac{\partial^3 U_1}{\partial\varphi^3}+3\Omega_1^2\gamma\frac{\partial^3 U_1}{\partial\varphi^2\partial\theta}+3\Omega_1\gamma^2\frac{\partial^3 U_1}{\partial\varphi\partial\theta^2}+\gamma^3\frac{\partial^3 U_1}{\partial\theta^3}). \qquad (11)$$

Now we calculate some of quantities in the equation (1).

$$\begin{aligned}\xi\frac{\partial^2 y}{\partial t^2}&=\varepsilon\xi\frac{\partial A_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)\cos\varphi Z_1-2\varepsilon\xi A_1\Omega_1\sin\varphi Z_1-2\varepsilon\xi B_1\Omega_1 a\cos\varphi Z_1\\&-\varepsilon\xi\frac{\partial B_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)a\sin\varphi Z_1-\xi a B_1\Omega_1^2\cos\varphi Z_1+\varepsilon\xi L_2[U_1]\end{aligned} \qquad (12)$$

$$\begin{aligned}\omega^2\frac{\partial^5 y}{\partial t\partial x^4}&=\varepsilon A_1\omega^2\frac{\pi^4}{\ell^4}\left(\Omega_1-\frac{p}{q}\gamma\right)\cos\varphi Z_1-a\Omega_1\omega^2\frac{\pi^4}{\ell^4}\sin\varphi Z_1-\varepsilon B_1 a\omega^2\frac{\pi^4}{\ell^4}\sin\varphi Z_1\\&+\varepsilon\left(\omega^2\Omega_1\frac{\partial^5 U_1}{\partial\varphi\partial x^4}+\omega^2\gamma\frac{\partial^5 U_1}{\partial\theta\partial x^4}\right)\end{aligned}$$

$$\omega^2\frac{\partial^5 y}{\partial t\partial x^4}=\varepsilon A_1\Omega_1^2\cos\varphi Z_1-a\Omega_1^3\sin\varphi Z_1-\varepsilon B_1 a\Omega_1^2\sin\varphi Z_1+\varepsilon\omega^2\frac{\partial}{\partial x^4}L_1[U_1] \qquad (13)$$

$$\xi\omega^2\frac{\partial^4 y}{\partial x^4}=\xi\omega^2 a\cos\varphi\frac{\pi^4}{\ell^4}Z_1+\varepsilon\xi\omega^2\frac{\partial^4 U_1}{\partial x^4}=\xi\Omega_1^2 a\cos\varphi Z_1+\varepsilon\xi\omega^2\frac{\partial^4 U_1}{\partial x^4}. \qquad (14)$$

Substituting expressions (11), (12), (13) and (14) into the equation (1) we obtain

$$\begin{aligned}&\varepsilon\frac{\partial^2 A_1}{\partial\psi^2}\left(\Omega_1-\frac{p}{q}\gamma\right)^2\cos\varphi Z_1-3\varepsilon\frac{\partial A_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)\Omega_1\sin\varphi Z_1-3\varepsilon A_1\Omega_1^2\cos\varphi Z_1\\&-\varepsilon\frac{\partial^2 B_1}{\partial\psi^2}\left(\Omega_1-\frac{p}{q}\gamma\right)^2 a\sin\varphi Z_1-3\varepsilon\frac{\partial B_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)a\Omega_1\cos\varphi Z_1\\&+3\varepsilon B_1 a\Omega_1^2\sin\varphi Z_1+a\Omega_1^3\sin\varphi Z_1+\varepsilon L_3[U_1]+\varepsilon\xi\frac{\partial A_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)\cos\varphi Z_1\\&-2\varepsilon\xi A_1\Omega_1\sin\varphi Z_1-2\varepsilon\xi B_1\Omega_1 a\cos\varphi Z_1-\varepsilon\xi\frac{\partial B_1}{\partial\psi}\left(\Omega_1-\frac{p}{q}\gamma\right)a\sin\varphi Z_1]\\&-\xi a\Omega_1^2\cos\varphi Z_1+\varepsilon\xi L_2[U_1+\varepsilon A_1\Omega_1^2\cos\varphi Z_1-a\Omega_1^3\sin\varphi Z_1-\varepsilon B_1 a\Omega_1^2\sin\varphi Z_1\\&+\varepsilon\omega^2\frac{\partial}{\partial x^4}L_1[U_1]+\xi\Omega_1^2 a\cos\varphi Z_1+\varepsilon\xi\omega^2\frac{\partial^4 U_1}{\partial x^4}+\ldots=\varepsilon F(a\cos\varphi Z_1,-a\Omega_1\sin\varphi,\ldots)\end{aligned} \qquad (15)$$

Comparing the coefficients of like powers of ε we have

$$L_3[U_1] + \xi L_2[U_1] + \omega^2 L_1\left[\frac{\partial^4 U_1}{\partial x^4}\right] + \xi\omega^2\frac{U_1}{\partial x^4} + \left\{\left[\left(\Omega_1 - \frac{p}{q}\gamma\right)^2\frac{\partial^2 A_1}{\partial\psi^2} + \xi\left(\Omega_1 - \frac{p}{q}\gamma\right)\right.\right.$$

$$\frac{\partial A_1}{\partial\psi} - 3a\Omega_1\left(\Omega_1 - \frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\Psi} - 2\Omega_1^2 A_1 - 2\Omega_1\xi\alpha B_1]\cos\varphi +$$

$$\left[-a\left(\Omega_1 - \frac{p}{q}\gamma\right)^2\frac{\partial^2 B_1}{\partial\psi^2} - \xi a\left(\Omega_1 - \frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\Psi} - 3\Omega_1\left(\Omega_1 - \frac{p}{q}\gamma\right)\frac{\partial A_1}{\partial\psi}\right] - 2\xi\Omega_1 A_1$$

$$\left.+ 2a\Omega_1^2 B_1]\sin\varphi U\right\}Z_1 = F_1 \tag{16}$$

where the linear operators take the from

$$\begin{aligned} L_1[U_1] &= \left(\frac{\partial}{\partial\varphi}\Omega_1 - \gamma\frac{\partial}{\partial\theta}\right)U_1, \\ L_2[U_1] &= \left(\Omega_1\frac{\partial}{\partial\varphi} + \gamma\frac{\partial}{\partial\theta}\right)^2 U_1, \\ L_3[U_1] &= \left(\Omega_1\frac{\partial}{\partial\varphi} + \gamma\frac{\partial}{\partial\theta}\right)^3 U_1 \end{aligned} \tag{17}$$

The relevant homogeneous boundary conditions of the function U_1 are

$$U_1|_{x=0} = 0\,, \quad \frac{\partial^2 U_1}{\partial x^2}|_{x=0} = 0, \quad U_1|_{x=l} = 0\,, \quad \frac{\partial^2 U_1}{\partial x^2}|_{x=l} = 0 \tag{18}$$

Now we expand the functions U_1 and F_1 into the series of functions $\{Z_K(x)\}$

$$\begin{aligned} U_1 &= \sum_{k=1}^{\infty} U_{1k}(a,\varphi,\theta)Z_K(x), \\ F_1 &= \sum_{k=1}^{\infty} F_{1k}(a,\varphi,\theta)Z_K(x), \\ F_{1k} &= \int_0^t F_1 Z_k dx \Big/ \int_0^t Z_k^2(x)dx. \end{aligned} \tag{19}$$

where $F_{1K}(a,\varphi,\theta)$ are defined, still $U_{1K}(a,\varphi,\theta)$ need to be determined.

We calculate some quantities in the equation (16)

$$\begin{aligned} L_3[U_1] &= \sum_{k=1}^{\infty} L_3[U_{1k}]Z_k, \xi L_2[U_1] = \xi\sum_{k=1}^{\infty} L_2[U_{lk}]Z_k \\ \omega^2 L_1\left[\frac{\partial^4 U_1}{\partial x^4}\right] &= \sum_{k=1}^{\infty} L_1[U_{lk}]\Omega_k^2 Z_k, \xi\omega^2\frac{\partial^4 U_1}{\partial x^4} = \sum_{k=1}^{\infty} U_{lk}\xi\Omega_k^2 Z_k. \end{aligned} \tag{20}$$

Substituting (15) into (16) we have

$$\begin{aligned}
&\sum_{K=1}^{\infty}\left\{L_3\left[U_{1K}\right]+\xi L_2\left[U_{1K}\right]+\Omega_K^2 L_1\left[U_{1K}\right]+\xi\Omega_K^2 U_{1K}\right\}Z_K+\\
&+\left[\left(\Omega_1-\frac{p}{q}\gamma\right)^2\frac{\partial^2 A_1}{\partial\psi^2}+\xi\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial A_1}{\partial\psi}-2a\Omega_1\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}\right.\\
&-2\Omega_1^2A_1-2\xi\Omega_1 aB_1]\cos\varphi+\left[-a\left(\Omega_1-\frac{p}{q}\gamma\right)^2\frac{\partial^2 B_1}{\partial\psi^2}-\xi a\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}\right.\\
&\left.-3\Omega_1\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial A_1}{\partial\psi}-2\xi\Omega_1A_1+2a\Omega_1^2B_1\right]\sin\varphi\}Z_1=\sum_{K=1}^{\infty}F_{1K}Z_K
\end{aligned}\tag{21}$$

In the case of $k=1$ we obtain the following equations

$$\begin{aligned}
&L_3\left[U_{11}\right]+\xi L_2\left[U_{11}\right]+\Omega_1^2 L_2\left[U_{11}\right]+\xi\Omega_1^2 U_{11}\\
&=\left[\left(\Omega_1-\frac{p}{\gamma}\right)^2\frac{\partial^2 A_1}{\partial\psi^2}+\xi\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial A_1}{\partial\psi}-3a\Omega_1\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}\right.\\
&-2\Omega_1^2A_1-2\xi a\Omega_1\,B_1]\cos\varphi+\left[a\left(\Omega_1-\frac{p}{q}\gamma\right)^2\frac{\partial B_1}{\partial\psi^2}+\xi a\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}\right.\\
&+3\Omega_1\left(\Omega_1-\frac{p}{q}\lambda\right)\frac{\partial A_1}{\partial\psi}+2\xi\Omega_1A_1-2a\Omega_1^2\,B_1]\sin\varphi+F_{11},
\end{aligned}\tag{22}$$

$$L_3\left[U_{1k}\right]+\xi L_2\left[U_{1k}\right]+\Omega_1^2 L_2\left[U_{1k}\right]+\xi\Omega_1^2 U_{1k}=F_{1k}\quad (k=2,\,3...)\tag{23}$$

In order to find the functions $U_{1k}(a,\varphi,\theta)$ we again expand U_{1k} and $U_{1k}(a,\varphi,\theta)$ into the Fourier series

$$U_{1k}(a,\varphi,\theta)=\sum_{m,n=-\infty}^{+\infty}U_{nm}^{(k)}(a)e^{i(n\theta+m\varphi)},\tag{24}$$

where $U_{nm}^{(k)}(a)$ need to be determined

$$\begin{aligned}
&F_{1k}(a,\varphi,\theta)=\sum_{m,n=-\infty}^{+\infty}F_{nm}^{(k)}(a)e^{i(n\theta+m\varphi)},\\
&F_{mn}^{(k)}(a)=\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}F_{1k}e^{-i(n\theta+m\varphi)}d\theta d\varphi.
\end{aligned}\tag{25}$$

From (24) and noting (17) we get

$$\begin{aligned}
&L_1[U_{1k}]=L_1\left[\sum_{m,n}U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta)}\right]\sum_{mn}U_{mn}^{(k)}(a)L_1\left[e^{i(m\varphi+n\theta)}\right],\\
&L_1[U_{1k}]=\sum_{mn}i(m\Omega_1+n\gamma)U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta)}.
\end{aligned}\tag{26}$$

$$L_2[U_{1k}] = L_2\left[\sum_{m,n} U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta)}\right]\sum_{mn} U_{mn}^{(k)}(a)L_2\left[e^{i(m\varphi+n\theta)}\right],$$
$$L_2[U_{1k}] = \sum_{mn} i^2(m\Omega_1+n\gamma)^2U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta)}. \tag{27}$$

$$L_3[U_{lk}] = L_3\left[\sum_{m,n} U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta)}\right] = \sum_{mn} U_{mn}^{(k)}(a)L_3\left[e^{i(m\phi+n\theta)}\right],$$
$$L_3[U_{lk}] = \sum_{mn} i^3(m\Omega_1+n\gamma)^3U_{mn}^{(k)}(a)e^{i(m\varphi+n\gamma)} \tag{28}$$

Substituting (25) – (28) into (23) we obtain

$$\sum_{mn} i^3(m\Omega_1+n\gamma\gamma^3U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta\theta} + \sum_{mn}\xi i i^2(m\Omega_1+\gamma\gamma^2\mathrm{U}_{mn}^{(k)}(a)e^{i(m\varphi+n\theta\theta)}$$
$$+\sum_{mn}\Omega_k^2 i(m\Omega_1+n\theta\theta)_{mn}^{(k)}(a)e^{i(m\varphi+n\theta\theta}+\xi\Omega_k^2\sum_{mn}U_{mn}^{(k)}(a)e^{i(m\varphi+n\theta\theta}$$
$$=\sum_{mn}F_{mn}^{(k)}(a)e^{i(m\varphi+n\theta\theta}.$$

From the above equation, after calculating we get the following expresstion

$$\left[i^3(m\Omega_1^+n\gamma)^3+\xi i^2(m\Omega_1+n\gamma)^2+i\Omega_k^2(m\Omega_1+n\gamma)+\xi\Omega_k^2\right]U_{mn}^{(k)}(a) = F_{mn}^{(k)}(a),$$
$$\left\{i^2\,(m\Omega_1+n\gamma)^2\left[(\xi+i(m\Omega_1+n\gamma)\right]+\Omega_k^2\left[\xi+i(m\Omega_1+n\gamma)\right]U_{mn}^k(a) = F_{mn}^{(k)}(a), \tag{29}$$

$$\begin{cases}\left[\Omega_k^2-(m\Omega_1+n\gamma\right]\left[\xi+i(m\Omega_1^+n\gamma\right]U_{mn}^{(k)}(a) = F_{mn}^{(k)}(a),\\ U_{mn}^{(k)}(a) = \dfrac{F_{mn}^{(k)}(a)}{\left[\xi+i(m\Omega_1+n\gamma)\right]\left[\Omega_k^2-(m\Omega_1+n\gamma)^2\right]}\end{cases}$$

Substituting (29) into the expression (24) the funtions $U_{1k}(a,\varphi,\theta)$ are determined

$$U_{lk}(a,\varphi,\theta) = \sum_{mn}\frac{\int_0^{2\pi}\int_0^{2\pi}F_{lk}(a,\varphi,\theta)e^{-i(m\varphi+n\theta)}d\varphi d\theta}{4\pi^2\left[\xi+i(m\Omega_1+n\gamma)\right]\left[\Omega_k^2-(m\Omega_1+n\gamma)^2\right]} \tag{30}$$

It is seen that when k=1 we have

$$\left[\Omega_1^4-(m\Omega_1+n\gamma^2\right]\neq 0$$
$$\left[\Omega_1(m\pm 1)+n\gamma\right]\neq 0,$$

Substituting $\Omega_1 \approx \dfrac{p}{q}\gamma$ into the above inequalities yields

$$\left[p(m\pm 1)+nq\right]\neq 0. \tag{31}$$

Substituting (30) into (29) we obtain the expression for the function $U_1 = U_1(x, a, \varphi, \theta)$ as follows

$$U_1 = \sum_{mn}\sum_{k=1}^{\infty}\left\{\frac{\int\limits_0^{2\pi}\int\limits_0^{2\pi}\left[\int\limits_0^{l} F_1 Z_k(x)dx\right] e^{-i(m\varphi+n\theta)}d\varphi d\theta . Z_k(x)e^{i(m\varphi+n\theta)}}{4\pi^2\left[\xi + i(m\Omega_{1+n\gamma)}\right]\left[\Omega_k^2 - (m\Omega_1 + n\gamma)^2\right]\int\limits_0^{l} Z_k^2(x)dx}\right\} \tag{32}$$

When k=1 we have

$$[p(m \pm 1) + np] \neq 0$$

In order to determine A_1, B_1, now we substitute (26), (27) and (28) into (23), supposing that U_1 does not contain $\cos\varphi$, $\sin\varphi$. It means

$$\langle U_1 \cos\varphi\rangle = 0, \langle U_1 \sin\varphi\rangle = 0$$

Therefore the equation (22) becomes

$$\begin{aligned}&-\left[\left(\Omega_1 - \frac{p}{q}\gamma\right)^2 \frac{\partial^2 A_1}{\partial\psi^2} + \xi(\Omega_1 - \frac{p}{q}\gamma)\frac{\partial A_1}{\partial\Psi} - 3a\Omega_1\left(\Omega_1 - \frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}\right.\\ &\left.-2\Omega_1^2 A - 2\xi\Omega_1 a B_1\right]\cos\varphi\left[a\left(\Omega_1 - \frac{p}{q}\gamma\right)^2\frac{\partial^2 B_1}{\partial\psi^2} + \xi a\left(\Omega_1 - \frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}\right.\\ &\left.-3\Omega_1\left(\Omega_1 - \frac{p}{q}\gamma\right)\frac{\partial A_1}{\partial\psi} + 2\xi\Omega_1 A_a - 2a\Omega_1^2 B_1\right]\sin\varphi + F_{11} = 0\end{aligned} \tag{33}$$

where

$$\begin{aligned}&F_{11}(a, \varphi, \theta) = \sum_{mn} F_{mn}^{(1)}(a)e^{1(m\varphi+n\theta\theta},\\ &F_{mn}^{(1)}(a) = \frac{1}{4\pi^2}\int\limits_0^{2\pi}\int\limits_0^{2\pi} F_{11}e^{-i(m\varphi+n\theta\theta}d\varphi d\theta,\end{aligned} \tag{34}$$

In the expresstions (34), the terms contain $\cos\varphi$, $\sin\varphi$ corresponding to the values m, n = -$\infty \div +\infty$, for which

$$m\varphi + n\theta = \pm\varphi + \mu \tag{35}$$

where the quantity μ needs be determined. Noting (7) we have

$$(m \mp 1)\varphi + n\theta = \mu \rightarrow (m \mp 1)\frac{p}{q}\theta + (m \mp 1)\psi + n\theta = \mu,$$

Putting:

$$\frac{(m \pm 1)}{q} = r \rightarrow m = rq \pm 1 \tag{36}$$

We have

$$rp\theta + rq\psi + n\theta = \mu \rightarrow rp\theta + n\theta = 0 \rightarrow n = -rp \tag{37}$$

$$\mu = rq\psi\,(\,r = -\infty \div +\infty) \tag{38}$$

where r is a proportional coefficient.

Now we have to calculate the quantity in (34) in two cases where

$$\begin{aligned} m\varphi + n\theta &= +\varphi + \mu \\ m\varphi + n\theta &= -\varphi + \mu \end{aligned} \tag{39}$$

Substituting (39) into (34) and noting (38) we obtain

$$F_{11}(a,\varphi,\theta) = \sum_r e^{-irq\psi} e^{i\varphi} \frac{1}{4\pi^2} \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} e^{-i\varphi} d\varphi d\theta$$

$$+ \sum_r e^{irq\psi} e^{-ir} \frac{1}{4\pi^2} \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} e^{+i\varphi} d\varphi d\theta$$

$$\begin{aligned} F_{11} =& \frac{1}{4\pi^2} \sum_r e^{irq\psi} \left\{ (\cos\varphi + i\sin\varphi) \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} (\cos\varphi - i\sin\varphi) d\varphi d\theta \right\} \\ &+ (\cos\varphi - i\sin\varphi) \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} (\cos\varphi - i\sin\varphi) d\varphi d\theta \} \end{aligned} \tag{40}$$

The expression (40) can be written in the form

$$\begin{aligned} F_{11} =& \frac{1}{4\pi^2} \sum_r e^{irq\psi} \left\{ \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} [(\cos\varphi d\varphi d\theta) \cos\varphi - i(\sin\varphi d\varphi d\theta)\cos\varphi \right. \\ &+ (\sin\varphi d\varphi d\theta)\sin\varphi + i(\cos\varphi d\varphi d\theta)\sin\varphi + (\cos\varphi d\varphi d\theta)\sin\varphi \\ &\left. + i(\sin\varphi d\varphi d\theta)\cos\varphi - i(\cos\varphi d\varphi d\theta)\sin\varphi + (\sin\varphi d\varphi d\theta)\sin\varphi] \right\} \end{aligned}$$

It is seen that the function F_{11} is rewritten

$$\begin{aligned} F_{11} =& \frac{2}{4\pi^2} \sum_r e^{irq\psi} \left\{ \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} \cos\varphi d\varphi d\theta \cos\varphi + \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{irq\psi} + \sin\varphi d\varphi d\theta \sin\varphi \right\} \\ =& \left[\frac{1}{2\pi^2} \sum_r e^{irq\psi} \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} \cos\varphi d\varphi d\theta \right] \cos\varphi \\ &+ \left[\frac{1}{2\pi^2} \sum_r e^{irq\psi} \int_0^{2\pi}\int_0^{2\pi} F_{11} e^{-irq\psi} \sin\varphi d\varphi d\theta \right] \sin\varphi \end{aligned} \tag{41}$$

Substituting (41) into (33) and then comparing the coefficients of $\cos\varphi$, $\sin\varphi$ we get .

$$\left(\Omega_1-\frac{p}{q}\gamma\right)^2\frac{\partial^2 A_1}{\partial\psi^2}+\xi\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial^2 A_1}{\partial\psi}-3a\Omega_1\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi}-2\Omega^2A_1-2\xi\Omega_1 aB_1$$
$$=\frac{1}{2\pi^2}\sum_{\gamma}e^{irq\psi}\int_0^{2\pi}\int_0^{2\pi}F_{11}e^{-irq\psi}\cos\varphi d\varphi d\theta[a\left(\Omega_1-\frac{p}{q}\gamma\right)^2\frac{\partial^2 B_1}{\partial\psi^2}+\xi a\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial B_1}{\partial\psi} \quad (42)$$
$$+3\Omega_1\left(\Omega_1-\frac{p}{q}\gamma\right)\frac{\partial A_1}{\partial\psi}+2\xi\Omega_1A_1-2a\Omega_1^2B_1]=-\frac{1}{2\pi^2}\sum_{r}e^{irq\psi}\int_0^{2\pi}\int_0^{2\pi}F_{11}e^{-irq\psi}\sin\varphi d\varphi d\theta$$

It is supposed that there exists a resonance relation

$$\Omega_1\approx\frac{p}{q}\gamma+\varepsilon\Delta \quad (43)$$

Substituting (43) into (42), neglecting the small terms of ε^2 order we get

$$\Omega_1^2A_1+\xi\Omega_1aB_1=-\frac{1}{4\pi^2}\sum_{r}e^{eirq\psi}\int_0^{2\pi}\int_0^{2\pi}F_{11}e^{-irq\left(\varphi-\frac{p}{q}\theta\right)}\cos\varphi d\varphi d\theta=G$$
$$\xi\Omega_1A_1-\Omega_{11}^2aB_1=-\frac{1}{4\pi^2}\sum_{r}e^{eirq\psi}\int_0^{2\pi}\int_0^{2\pi}F_{11}e^{-irq\left(\varphi-\frac{p}{q}\theta\right)}\sin\varphi d\varphi d\theta=H \quad (44)$$

It is seen from (??) that A_1 and B_1 are determined

$$A_1=\frac{(\Omega_1G+\xi H)}{\Omega_1(\Omega_1^2+\xi^2};\qquad B_1=\frac{\xi G-\Omega_1H}{a\Omega_1(\Omega_1^2+\xi^2)} \quad (45)$$

Thus, in the improved first approximation the partial solution of equation (6) is determined

$$y=a\cos\left(\tfrac{p}{q}\theta+\psi\right)\sin\frac{\pi x}{\ell}$$
$$+\varepsilon\sum_{mn}\sum_{k=1}^{\infty}\left\{\frac{\int_0^{2\pi}\int_0^{2\pi}\left[\int_0^{2\pi}F_1(x,a,\varphi,\theta)\sin\frac{k\pi\pi}{\ell}dx\right]e^{-i(m\varphi+n\theta\theta}d\varphi d\theta\theta\sin\frac{k\pi\pi}{\ell}e^{i(m\varphi+n\theta\theta}}{4\pi^2\left[\xi+i(m\Omega_1+n\gamma\gamma\right]\left[\Omega_k^2-(m\Omega_1+n\gamma)^2\right]\int_0^{\ell}\sin^2\frac{k\pi\pi}{\ell}dx}\right\} \quad (46)$$

When k=1 we have {p(m±1) + nq] $\neq 0$
a and ψ are determined from the following equations

$$\frac{da}{dt}=\varepsilon A_1(a,\psi)+\varepsilon^2...,$$
$$\frac{d\psi}{dt}=\left(\Omega_1-\frac{p}{q}\gamma\right)+\varepsilon B_1(a,\psi)+\varepsilon^2... \quad (47)$$

However, practically one only needs to find the particular solution in the first approximation

$$y(x,t) = a\cos\left(\frac{p}{q}\theta + \eta\right)\sin\frac{\pi x}{l} \tag{48}$$

3. CONCLUSION

In this work, the authors have investigated the nonlinear oscillation of creepy elastic beam in the non-autonmous case with the homogeneous linear boundary condition. It is easy seen that the motion of the creepy elastic beam is described by the partial differential equation of the third order with respect to argument.

The solution of this equation has been constructed by the asymptotic method for the high order systems. In the improved approximation the solution of the boundary value problem has been determined.

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NGHIỆM TIỆM CẬN CỦA PHƯƠNG TRÌNH ĐẠO HÀM RIÊNG CẤP CAO

Trong bài báo này, các tác giả xây dựng mô hình nghiệm tiệm cận của phương trình đạo hàm riêng cấp cao mô tả dao động của dầm đàn hồi phi tuyến từ biến trong trường hợp hệ không ôtônôm. Trong xấp xỉ thứ nhất hoàn thiện nghiệm tiệm cận của bài toán biên đó được xác định.