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PARAMETER OPTIMIZATION OF TUNED MASS DAMPER FOR THREE-DEGREE-OF-FREEDOM VIBRATION SYSTEMS

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Abstract. There are problems in mechanical, structural and aerospace engineering that can be formulated as Nonlinear Programming. In this paper, the problem of parameters optimization of tuned mass damper for three-degree-of-freedom vibration systems is investigated using sequential quadratic programming method. The objective is to minimize the extreme vibration amplitude of vibration models. It is shown that the constrained formulation, that includes lower and upper bounds on the updating parameters in the form of inequality constraints, is important for obtaining a correct updated model.

Keywords: Vibration, tuned mass damper, optimal design, nonlinear programming.

1. INTRODUCTION

Optimal design of multibody systems is characterized by a specific kind of optimization problem. Generally, an optimization problem is formulated to determine the design variable values that will minimize an objective function subject to constraints. Additionally, for many engineering applications, multibody analysis routine are used to calculate the kinematic and dynamic behavior of the mechanical design. As a result, most objective function and constraint values follow from the numerical analysis.

Use of the tuned mass damper (TMD) as an independent means of vibration control is especially important, particularly in the case where it is almost the only or main means of vibration protection [1-6]. A tuned mass damper, also known as an active mass damper (AMD) or harmonic absorber, is a device mounted in structures to reduce the amplitude of vibrations. Its application can prevent discomfort, damage, or outright structural failure. It is frequently used in power transmission, automobiles, machine and buildings.

In this paper we consider a problem of parameter optimization of tuned mass damper for three-degree-of-freedom vibration systems using sequential quadratic programming method [7-12].

2. REVIEW OF SEQUENTIAL QUADRATIC PROGRAMMING METHOD

The sequential quadratic programming, or called SQP, is an efficient and powerful algorithm to solve nonlinear programming problems. The method has a theoretical basis that is related to (1) the solution of a set of nonlinear equations using Newton's method, and (2) the derivation of simultaneous nonlinear equations using Kuhn–Tücker conditions to the Lagrangian of the constrained optimization problem. In this section we review some basic concepts of SQP method [7-10] for understanding the parameter optimization of the TMD installed in vibration systems.

Consider a nonlinear optimization problem with equality constraints:

Find $\mathbf{x}$ which minimizes $f(\mathbf{x})$

subject to

$$h_k(\mathbf{x}) = 0, k = 1, 2, \ldots, p. \tag{1}$$

The Lagrange function $L(\mathbf{x}, \boldsymbol{\lambda})$, for this problem is

$$L(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) + \sum_{k=1}^{p} \lambda_k h_k(\mathbf{x}) = f(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x}), \tag{2}$$

where λ_k is the Lagrange multiplier for the equality constraint h_k. The Kuhn–Tücker necessary conditions can be stated as

$$\nabla_x L = \mathbf{0} \quad \Rightarrow \quad \nabla f(\mathbf{x}) + \sum_{k=1}^{p} \lambda_k \nabla h_k = \mathbf{0} \quad \text{or} \quad \nabla f(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x}) = \mathbf{0}, \tag{3}$$

$$\nabla_\lambda L = \mathbf{0} \quad \Rightarrow \quad h_k(\mathbf{x}) = 0, k = 1, 2, \ldots, p \quad \text{or} \quad \mathbf{h}(\mathbf{x}) = \mathbf{0}. \tag{4}$$

Eqs. (3) and (4) represent a set of $n + p$ nonlinear equations with $n + p$ unknowns ($\mathbf{x}_i, i = 1, 2, \ldots, n$ and $\lambda_k, k = 1, 2, \ldots, p$). These nonlinear equations can be solved using Newton's method. For convenience, we rewrite Eqs. (3) and (4) as

$$\mathbf{b}(\mathbf{y}) = \mathbf{0}, \tag{5}$$

where

$$\mathbf{b} = \begin{Bmatrix} \nabla L \\ \mathbf{h} \end{Bmatrix}_{(n+p)\times 1}, \quad \mathbf{y} = \begin{Bmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{Bmatrix}_{(n+p)\times 1}, \quad 0 = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix}_{(n+p)\times 1}. \tag{6}$$

According to Newton's method, the solution of Eqs. (5) can be found iteratively as

$$\begin{bmatrix} \nabla_x^2 L(\mathbf{y}_i) & \mathbf{J}_h^T(\mathbf{x}_i) \\ \mathbf{J}_h(\mathbf{x}_i) & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \Delta \mathbf{x}_i \\ \Delta \boldsymbol{\lambda}_i \end{Bmatrix} = - \begin{Bmatrix} \nabla_x L(\mathbf{y}_i) \\ \mathbf{h}(\mathbf{x}_i) \end{Bmatrix}, \tag{7}$$

and

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta \mathbf{x}_i, \quad \boldsymbol{\lambda}_{i+1} = \boldsymbol{\lambda}_i + \Delta \boldsymbol{\lambda}_i. \tag{8}$$

The first set of equations in (7) can be written separately as

$$\nabla_x^2 L(\mathbf{y}_i) \Delta \mathbf{x}_i + \mathbf{J}_h^T(\mathbf{x}_i) \Delta \boldsymbol{\lambda}_i = -\nabla_x L(\mathbf{y}_i) \tag{9}$$

Using Eq. (8) for $\Delta \boldsymbol{\lambda}_i$ and Eq. (3) for $\nabla_x L(\mathbf{y}_i)$, Eq. (9) can be expessed as

$$\nabla_x^2 L(\mathbf{y}_i) \Delta \mathbf{x}_i + \mathbf{J}_h^T(\boldsymbol{\lambda}_{i+1} - \boldsymbol{\lambda}_i) = -\nabla f(\mathbf{x}_i) - \mathbf{J}_h^T(\mathbf{x}_i) \boldsymbol{\lambda}_i, \tag{10}$$

which can be simplified to obtain

$$\nabla_x^2 L(\mathbf{y}_i)\Delta\mathbf{x}_i + \mathbf{J}_h^T(\mathbf{x}_i)\boldsymbol{\lambda}_{i+1} = -\nabla f(\mathbf{x}_i). \tag{11}$$

Eq. (11) and the second set of equations in (7) can now be combined as

$$\begin{bmatrix} \nabla_x^2 L(\mathbf{y}_i) & \mathbf{J}_h^T(\mathbf{x}_i) \\ \mathbf{J}_h(\mathbf{x}_i) & \mathbf{0} \end{bmatrix}_j \begin{Bmatrix} \Delta\mathbf{x}_i \\ \boldsymbol{\lambda}_{i+1} \end{Bmatrix} = -\begin{Bmatrix} \nabla f(\mathbf{x}_i) \\ \mathbf{h}(\mathbf{x}_i) \end{Bmatrix}. \tag{12}$$

Eqs. (12) can be solved to find the change in the design vector $\Delta\mathbf{x}_i$ and the new values of the Lagrange multipliers, $\boldsymbol{\lambda}_{i+1}$. The iterative process indicated by Eq. (12) can be continued until convergence is achieved.

Now consider the following quadratic programming problem:

Find $\mathbf{d} = \Delta\mathbf{x}$ that minimizes the quadratic objective function

$$Q(\mathbf{d}) = \nabla_x f(\mathbf{x}_i)^T\mathbf{d} + \frac{1}{2}\mathbf{d}^T\nabla_x^2 L(\mathbf{x}_i, \boldsymbol{\lambda}_i)\mathbf{d}, \tag{13}$$

subject to the linear equality constraints

$$h_k(\mathbf{x}_i) + \nabla h_k^T(\mathbf{x}_i)\mathbf{d} = 0, \quad k = 1, 2, \ldots, p \;\Rightarrow\; \mathbf{h}(\mathbf{x}_i) + \mathbf{J}_h(\mathbf{x}_i)\mathbf{d} = 0. \tag{14}$$

The Lagange function $\tilde{L}$, corresponding to the problem of Eqs. (13) and (14) is given by

$$\tilde{L}(\mathbf{d}, \boldsymbol{\lambda}) = \nabla_x f(\mathbf{x}_i)^T\mathbf{d} + \frac{1}{2}\mathbf{d}^T\nabla_x^2 L(\mathbf{x}_i, \boldsymbol{\lambda}_i)\mathbf{d} + \boldsymbol{\lambda}^T\left[\mathbf{h}\left(\mathbf{x}_i\right) + \mathbf{J}_h\left(\mathbf{x}_i\right)\mathbf{d}\right]. \tag{15}$$

The Kuhn – Tücker necessary conditions can be stated as

$$\nabla_x f(\mathbf{x}_i) + \nabla_x^2 L(\mathbf{x}_i, \boldsymbol{\lambda}_i)\mathbf{d} + \mathbf{J}_h^T\left(\mathbf{x}_i\right)\boldsymbol{\lambda} = \mathbf{0}, \tag{16}$$

$$\mathbf{h}(\mathbf{x}_i) + \mathbf{J}_h(\mathbf{x}_i)\mathbf{d} = \mathbf{0}. \tag{17}$$

The Eqs. (16) and (17) can be combined in the following matrix form as

$$\begin{bmatrix} \nabla_x^2 L(\mathbf{y}_i) & \mathbf{J}_h^T(\mathbf{x}_i) \\ \mathbf{J}_h(\mathbf{x}_i) & \mathbf{0} \end{bmatrix}_j \begin{Bmatrix} \mathbf{d}_i \\ \boldsymbol{\lambda}_i \end{Bmatrix} = -\begin{Bmatrix} \nabla f(\mathbf{x}_i) \\ \mathbf{h}(\mathbf{x}_i) \end{Bmatrix}. \tag{18}$$

Eq. (18) can be identified to be same as Eq. (12) in matrix form. This shows that the orginal problem of Eq. (1) can be solved iteratively by solving the quadratic programming problem defined by Eq. (13).

In fact, when inequality constraints are added to the original problem, the quadratic programming problem of Eqs. (13) and (14) becomes

Find $\mathbf{x}$ which minimizes

$$Q(\mathbf{d}) = (\nabla f(\mathbf{x}_i))^T\mathbf{d} + \frac{1}{2}\mathbf{d}^T\nabla_x^2 L(\mathbf{x}_i, \boldsymbol{\lambda}_i, \boldsymbol{\mu}_i)\mathbf{d}, \tag{19}$$

subject to

$$h_k(\mathbf{x}_i) + (\nabla h_k(\mathbf{x}_i))^T\mathbf{d} = 0, k = 1, 2, \ldots, p \tag{20}$$

$$g_j(\mathbf{x}_i) + (\nabla g_j(\mathbf{x}_i))^T\mathbf{d} \leq 0, j = 1, 2, \ldots, m \tag{21}$$

with the Lagrange function given by

$$L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f(\mathbf{x}) + \sum_{k=1}^{p}\lambda_k h_k(\mathbf{x}) + \sum_{j=1}^{m}\mu_j g_j(\mathbf{x}) = f(\mathbf{x}) + \boldsymbol{\lambda}^T\mathbf{h}(\mathbf{x}) + \boldsymbol{\mu}^T\mathbf{g}(\mathbf{x}) \tag{22}$$

Since the minimum of the augmented Lagrange function is involved, the sequential quadratic programming method is also known as the projected Lagrangian method.

3. CALCULATING OPTIMAL PARAMETERS OF TMD FOR THE THREE-DEGREE-OF-FREEDOM VIBRATION SYSTEMS

In this section we study the influence of installed position of TMD on the behaviour of three-degree-of-freedom vibration systems using the sequential quadratic programming algorithm.

3.1. Vibration equation of system with the excited harmonic force at the mass m_1

Consider a damped linear vibration system of three-degree-of-freedom as shown in Fig. 1a. The vibrating system has three masses m$_1$, m_2, m_3; stiffness coefficients, respectively, k_1, k_2, k_3 and viscous coefficients, respectively, c_1, c_2, c_3; the mass m_1 is excited by harmonic force $F(t) = F_0 \cos(\Omega t)$. The motion equations of the system have the following form

$$\begin{aligned}
&m_1\ddot{y}_1 + (c_1 + c_2)\dot{y}_1 - c_2\dot{y}_2 + (k_1 + k_2)y_1 - k_2y_2 = F_0\cos(\Omega t)\\
&m_2\ddot{y}_2 - c_2\dot{y}_1 + (c_2 + c_3)\dot{y}_2 - c_3\dot{y}_3 - k_2y_1 + (k_2 + k_3)y_2 - k_3y_3 = 0\ .\\
&m_3\ddot{y}_3 - c_3\dot{y}_2 + c_3\dot{y}_3 - k_3y_2 + k_3y_3 = 0
\end{aligned} \tag{23}$$

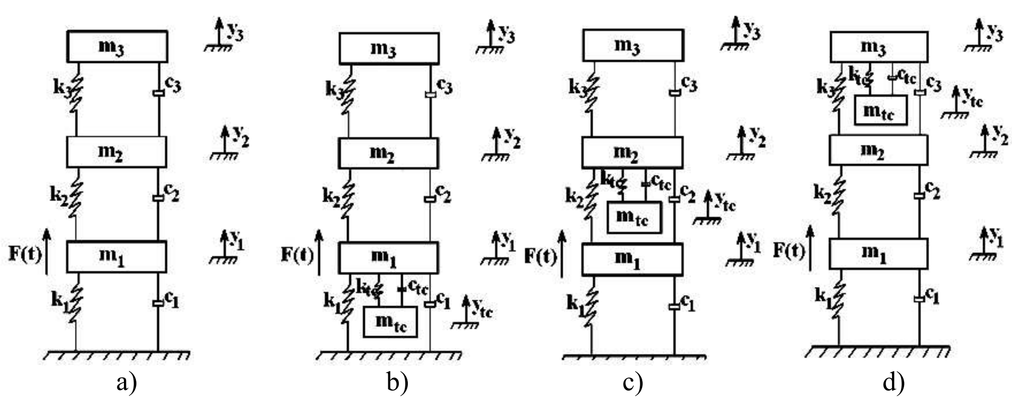

Fig. 1. The system of three-degree-of-freedom under excited force at m_1
a) Primary system without TMD; b) System with TMD at m_1
c) System with TMD at m_2; d) System with TMD at m_3

The steady-state response of the system has the form

$$\mathbf{y}(t) = \mathbf{a}\cos(\Omega t) + \mathbf{b}\sin(\Omega t) \tag{24}$$

with

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix};\mathbf{a}_0 = \begin{bmatrix} a_{01} \\ a_{02} \\ a_{03} \end{bmatrix};\mathbf{b}_0 = \begin{bmatrix} b_{01} \\ b_{02} \\ b_{03} \end{bmatrix}.$$

From Eq. (23) and Eq. (24), comparing coefficients of $\cos(\Omega t)$ and $\sin(\Omega t)$, we get the system of linear algebraic equations for unknown elements of vectors **a** and **b**

$$\begin{array}{l} (k_1+k_2-m_1\Omega^2)a_{01}+(c_1+c_2)\Omega b_{01}-k_2a_{02}-c_2\Omega b_{02}=F_0 \\ -(c_1+c_2)\Omega a_{01}+(k_1+k_2-m_1\Omega^2)b_{01}+c_2\Omega a_{02}-k_2b_{02}=0 \\ -k_2a_{01}-c_2\Omega b_{01}+(k_2+k_3-m_2\Omega^2)a_{02}+(c_2+c_3)\Omega b_{02}-k_3a_{03}-c_3\Omega b_{03}=0 \\ c_2\Omega a_{01}-k_2b_{01}-(c_2+c_3)\Omega a_{02}+(k_2+k_3-m_2\Omega^2)b_{02}+c_3\Omega a_{03}-k_3b_{03}=0 \\ -k_3a_{02}-c_3\Omega b_{02}+(k_3-m_3\Omega^2)a_{03}+c_3\Omega b_{03}=0 \\ c_3\Omega a_{02}-k_3b_{02}-c_3\Omega a_{03}+(k_3-m_3\Omega^2)b_{03}=0 \end{array}. \tag{25}$$

By solving the system of Eqs. (25), we receive the values of elements a_{0i}, b_{0i} $(i = 1, 2, 3)$ of vectors $\mathbf{a}_0$ and $\mathbf{b}_0$. For numeric calculation, the values of the coefficients are given as

$$m_1 = m_2 = m_3 = 100 \text{ kg}, k_1 = k_2 = k_3 = 10^5 \text{ N/m}, c_1 = c_2 = c_3 = 1000 \text{ Ns/m},$$
$$\Omega = 47 \text{ rad/s}, \quad F(t) = 10\cos(47t).$$

3.2. Installation positions of TMD

a) System installed TMD in m_1

As the first variant to quench vibrations of the system, we installed TMD with mass m_{tc}, spring stiffness k_{tc} and viscous resistance c_{tc} on mass m_1 (Fig. 1b). The equation of the system oscillations

$$\begin{array}{l} m_1\ddot{y}_1+(c_1+c_2+c_{tc})\dot{y}_1-c_2\dot{y}_2-c_{tc}\dot{y}_{tc}+(k_1+k_2+k_{tc})y_1-k_2y_2-k_{tc}y_{tc}=F_0\cos(\Omega t) \\ m_2\ddot{y}_2-c_2\dot{y}_1+(c_2+c_3)\dot{y}_2-c_3\dot{y}_3-k_2y_1+(k_2+k_3)y_2-k_3y_3=0 \\ m_3\ddot{y}_3-c_3\dot{y}_2+c_3\dot{y}_3-k_3y_2+k_3y_3=0 \\ m_{tc}\ddot{y}_{tc}-c_{tc}\dot{y}_1+c_{tc}\dot{y}_{tc}-k_{tc}y_1+k_{tc}y_{tc}=0 \end{array} \tag{26}$$

The steady-state response of the system has the form

$$\mathbf{y}(t) = \mathbf{a}\cos(\Omega t) + \mathbf{b}\sin(\Omega t) \tag{27}$$

where

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_{tc}(t) \end{bmatrix}; \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_{tc} \end{bmatrix}; \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_{tc} \end{bmatrix}$$

From Eqs. (26)-(27), comparing coefficients of $\cos(\Omega t)$ and $\sin(\Omega t)$ we get the system of linear algebraic equations for unknown elements of vectors **a** and **b**

$$\begin{array}{l} (k_1+k_2+k_{tc}-m_1\Omega^2)a_1+(c_1+c_2+c_{tc})\Omega b_1-k_2a_2-c_2\Omega b_2-k_{tc}a_{tc}-c_{tc}\Omega b_{tc}=F_0 \\ -(c_1+c_2+c_{tc})\Omega a_1+(k_1+k_2+k_{tc}-m_1\Omega^2)b_1+c_2\Omega a_2-k_2b_2+c_{tc}\Omega a_{tc}-k_{tc}b_{tc}=0 \\ -k_2a_1-c_2\Omega b_1+(k_2+k_3-m_2\Omega^2)a_2+(c_2+c_3)\Omega b_2-k_3a_3-c_3\Omega b_3=0 \\ c_2\Omega a_1-k_2b_1-(c_2+c_3)\Omega a_2+(k_2+k_3-m_2\Omega^2)b_2+c_3\Omega a_3-k_3b_3=0 \\ -k_3a_2-c_3\Omega b_2+(k_3-m_3\Omega^2)a_3+c_3\Omega b_3=0 \\ c_3\Omega a_2-k_3b_2-c_3\Omega a_3+(k_3-m_3\Omega^2)b_3=0 \\ -k_{tc}a_1-c_{tc}\Omega b_1+(k_{tc}-m_{tc}\Omega^2)a_{tc}+c_{tc}\Omega b_{tc}=0 \\ c_{tc}\Omega a_1-k_{tc}b_1-c_{tc}\Omega a_{tc}+(k_{tc}-m_{tc}\Omega^2)b_{tc}=0 \end{array}. \tag{28}$$

Solving the system of Eqs. (28), we receive the elements a_i, b_i $(i = 1, 2, 3)$ of vectors **a** and **b**.

For optimization problems, there is an optimization criterion (i.e. evaluation function) that has to be minimized or maximized. Here we must find the optimal values m_{tc}, k_{tc}, c_{tc} of TMD in order to minimize the expression of vibration amplitude of m_1

$$R_1 = \sqrt{a_1^2 + b_1^2},$$

with boundary constraints

$$5 \leq m_{tc}(\text{kg}) \leq 10; 1000 \leq k_{tc}(\text{N/m}) \leq 100000; 5 \leq c_{tc}(\text{Ns/m}) \leq 1000.$$

Using the sequential quadratic programming algorithm in MAPLE software, we can quickly and conveniently calculate the optimal parameters for TMD

$$R_1 = 0.00000451601155 \text{ m}; k_{tc} = 22099.62597299 \text{ N/m}; c_{tc} = 5 \text{ Ns/m}; m_{tc} = 10 \text{ kg}.$$

Some calculating results are provided in Tab. 1 and in Fig. 2.

Table 1. Effective vibration reduction system under excited force at m_1 before and after installing TMD at m_1

Location	Vibration amplitude (m)		Efficient vibration damping (%)	
	Without TMD	With TMD	increase	Reduced
m_1	0.0000653278	0.000004516		93.08
m_2	0.0000393333	0.000002719		93.08
m_3	0.0000335052	0.000002316		93.08

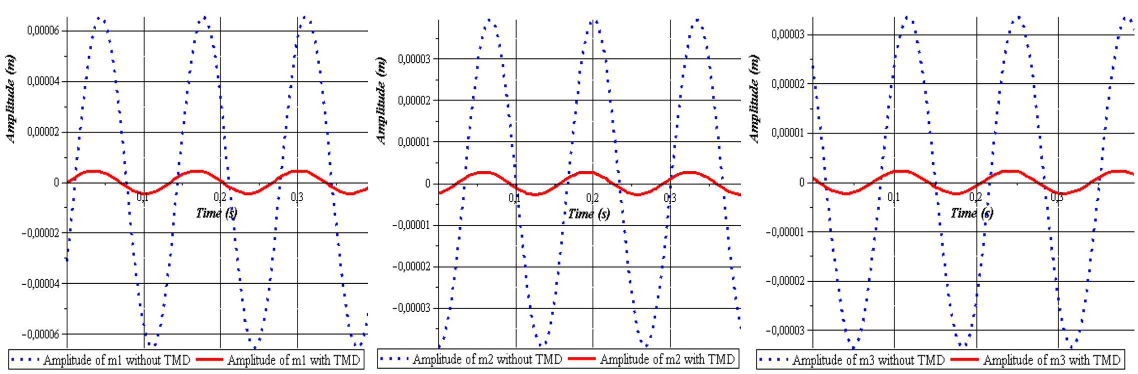

Fig. 2. Amplitude of three degrees of freedom system under excited force at m_1 before and after installing TMD at m_1

b) System installed TMD in m_2

As second variant to quench vibrations of the system, we installed TMD with mass m_{tc}, spring stiffness, k_{tc} and viscous resistance, c_{tc} on mass m_1 (see Fig. 1c). The vibration equations of the system have following form

$$\begin{aligned}
&m_1\ddot{y}_1 + (c_1+c_2)\dot{y}_1 - c_2\dot{y}_2 + (k_1+k_2)y_1 - k_2y_2 = F_0\cos(\Omega t)\\
&m_2\ddot{y}_2 - c_2\dot{y}_1 + (c_2+c_3+c_{tc})\dot{y}_2 - c_3\dot{y}_3 - c_{tc}\dot{y}_{tc} - k_2y_1 + (k_2+k_3+k_{tc})y_2 - k_3y_3 - k_{tc}y_{tc} = 0\\
&m_3\ddot{y}_3 - c_3\dot{y}_2 + c_3\dot{y}_3 - k_3y_2 + k_3y_3 = 0\\
&m_{tc}\ddot{y}_{tc} - c_{tc}\dot{y}_2 + c_{tc}\dot{y}_{tc} - k_{tc}y_2 + k_{tc}y_{tc} = 0
\end{aligned} \tag{29}$$

From Eq. (27) and Eq. (29), comparing coefficients of $\cos(\Omega t)$ and $\sin(\Omega t)$ we get the system of linear algebraic equations for unknown elements of vectors **a** and **b**

$$\begin{aligned}
&(k_1+k_2-m_1\Omega^2)a_1 + (c_1+c_2)\Omega b_1 - k_2a_2 - c_2\Omega b_2 = F_0\\
&-(c_1+c_2)\Omega a_1 + (k_1+k_2-m_1\Omega^2)b_1 + c_2\Omega a_2 - k_2b_2 = 0\\
&-k_2a_1 - c_2\Omega b_1 + (k_2+k_3+k_{tc}-m_2\Omega^2)a_2 + (c_2+c_3+c_{tc})\Omega b_2 - k_3a_3 - c_3\Omega b_3 - k_{tc}a_{tc} - c_{tc}\Omega b_{tc} = 0\\
&c_2\Omega a_1 - k_2b_1 - (c_2+c_3+c_{tc})\Omega a_2 + (k_2+k_3+k_{tc}-m_2\Omega^2)b_2 + c_3\Omega a_3 - k_3b_3 + c_{tc}\Omega a_{tc} - k_{tc}b_{tc} = 0\\
&-k_3a_2 - c_3\Omega b_2 + (k_3-m_3\Omega^2)a_3 + c_3\Omega b_3 = 0\\
&c_3\Omega a_2 - k_3b_2 - c_3\Omega a_3 + (k_3-m_3\Omega^2)b_3 = 0\\
&-k_{tc}a_2 - c_{tc}\Omega b_2 + (k_{tc}-m_{tc}\Omega^2)a_{tc} + c_{tc}\Omega b_{tc} = 0\\
&c_{tc}\Omega a_2 - k_{tc}b_2 - c_{tc}\Omega a_{tc} + (k_{tc}-m_{tc}\Omega^2)b_{tc} = 0
\end{aligned} \tag{30}$$

Solving the system of Eqs. (30), we receive the elements a_i, b_i $(i = 1, 2, 3)$ of vectors **a** and **b**. Thus, to minimize the vibration amplitude of m_2 we must find optimal values m_{tc}, k_{tc}, c_{tc} of TMD to minimize the expression $R_2 = \sqrt{a_2^2 + b_2^2}$ with boundary constraints

$$5 \le m_{tc}\ (\text{kg}) \le 10;\ 1000 \le k_{tc}\ (\text{N/m}) \le 100000;\ 5 \le c_{tc}\ (\text{Ns/m}) \le 1000.$$

Using SQP, we find the optimal parameters for TMD

$$R_2 = 0.00000485578798\ \text{m};\ k_{tc} = 22099.07992772\ \text{N/m};\ c_{tc} = 5\ \text{Ns/m};\ m_{tc} = 10\ \text{kg}.$$

Some calculating results are shown in Tab. 2 and in Fig. 3.

Table 2. Effective vibration reduction system under excited force at m_1 before and after installing TMD at m_2

Location	Vibration amplitude (m)		Efficient vibration damping (%)	
	Without TMD	With TMD	increase	reduced
m_1	0.0000653278	0.0000992695	51.95	
m_2	0.0000393333	0.0000048558		87.65
m_3	0.0000335052	0.0000041363		87.65

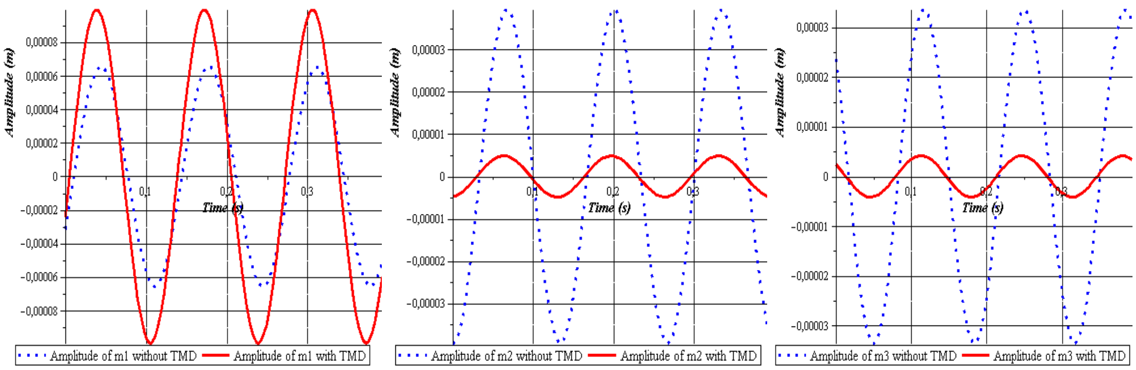

Fig. 3. Vibration amplitude of system under excited force at m_1 before and after installing TMD at m_2

c) System installed TMD in m_3

As third variant to quench vibrations of the system, we installed TMD with mass m_{tc}, spring stiffness, k_{tc} and viscous resistance, c_{tc} on mass m_3 (see Fig. 1d).

The equation of the system oscillations

$$\begin{array}{l} m_1\ddot{y}_1 + (c_1+c_2)\dot{y}_1 - c_2\dot{y}_2 + (k_1+k_2)y_1 - k_2y_2 = F_0\cos\Omega t \\ m_2\ddot{y}_2 - c_2\dot{y}_1 + (c_2+c_3)\dot{y}_2 - c_3\dot{y}_3 - k_2y_1 + (k_2+k_3)y_2 - k_3y_3 = 0 \\ m_3\ddot{y}_3 - c_3\dot{y}_2 + (c_3+c_{tc})\dot{y}_3 - c_{tc}\dot{y}_{tc} - k_3y_2 + (k_3+k_{tc})y_3 - k_{tc}y_{tc} = 0 \\ m_{tc}\ddot{y}_{tc} - c_{tc}\dot{y}_3 + c_{tc}\dot{y}_{tc} - k_{tc}y_3 + k_{tc}y_{tc} = 0 \end{array}. \tag{31}$$

From Eq. (27) and Eq. (31), comparing coefficients of $\cos(\Omega t)$ and $\sin(\Omega t)$ we get the system of linear algebraic equations for unknown elements of vectors **a** and **b**

$$\begin{array}{l} (k_1+k_2-m_1\Omega^2)a_1 + (c_1+c_2)\Omega b_1 - k_2a_2 - c_2\Omega b_2 = F_0 \\ -(c_1+c_2)\Omega a_1 + (k_1+k_2-m_1\Omega^2)b_1 + c_2\Omega a_2 - k_2b_2 = 0 \\ -k_2a_1 - c_2\Omega b_1 + (k_2+k_3-m_2\Omega^2)a_2 + (c_2+c_3)\Omega b_2 - k_3a_3 - c_3\Omega b_3 = 0 \\ c_2\Omega a_1 - k_2b_1 - (c_2+c_3)\Omega a_2 + (k_2+k_3-m_2\Omega^2)b_2 + c_3\Omega a_3 - k_3b_3 = 0 \\ -k_3a_2 - c_3\Omega b_2 + (k_3+k_{tc}-m_3\Omega^2)a_3 + (c_3+c_{tc})\Omega b_3 - k_{tc}a_{tc} - c_{tc}\Omega b_{tc} = 0 \\ c_3\Omega a_2 - k_3b_2 - (c_3+c_{tc})\Omega a_3 + (k_3+k_{tc}-m_3\Omega^2)b_3 + c_{tc}\Omega a_{tc} - k_{tc}b_{tc} = 0 \\ -k_{tc}a_3 - c_{tc}\Omega b_3 + (k_{tc}-m_{tc}\Omega^2)a_{tc} + c_{tc}\Omega b_{tc} = 0 \\ c_{tc}\Omega a_3 - k_{tc}b_3 - c_{tc}\Omega a_{tc} + (k_{tc}-m_{tc}\Omega^2)b_{tc} = 0 \end{array}. \tag{32}$$

Solving the system of Eqs. (32), and identify the elements a_i, b_i $(i = 1, 2, 3)$ of vectors **a** and **b.** Thus, to minimize the vibration amplitude of m_3 we must find optimal values m_{tc}, k_{tc}, c_{tc} of TMD to minimize the expression $R_3 = \sqrt{a_3^2 + b_3^2}$ with boundary constraints

$$5 \le m_{tc}\ (\text{kg}) \le 10;\ 1000 \le k_{tc}\ (\text{N/m}) \le 100000;\ 5 \le c_{tc}\ (\text{Ns/m}) \le 1000.$$

Using SQP, we find the optimal parameters for TMD

$$R_3 = 0.00000266217877\ \text{m};\ k_{tc} = 22106.994965140063\ \text{N/ m};\ c_{tc} = 5\ \text{Ns/ m};\ m_{tc} = 10\ \text{kg}.$$

Some calculating results are shown in Tab. 3 and in Fig. 4.

Table 3. Effective vibration reduction system under excited force at m_1 before and after installing TMD at m_3

Location	Vibration amplitude (m)		Efficient vibration damping (%)	
	Without TMD	With TMD	increase	reduced
m_1	0.0000653278	0.0000471		27.88
m_2	0.0000393333	0.0000514	30.64	
m_3	0.0000335052	0.00000266		92.05

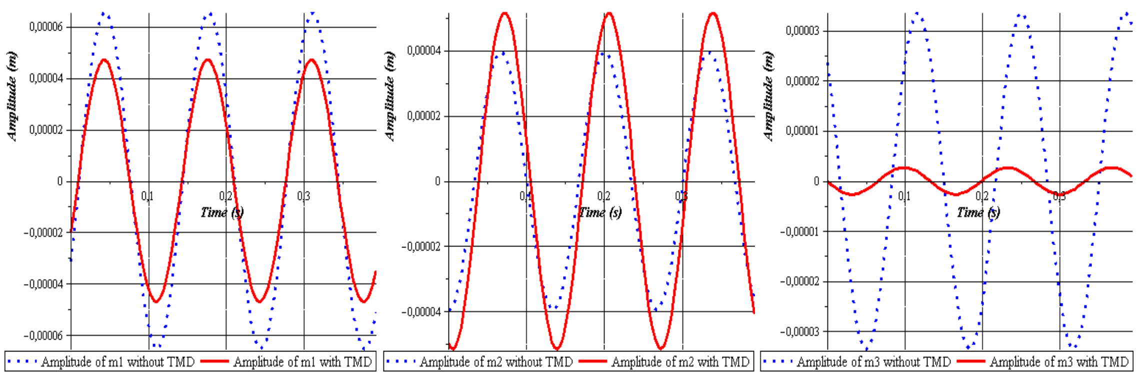

Fig. 4. Vibration amplitude of system under excited force at m_1 before and after installing TMD at m_3

From the simulation results in Figs 1-4 we have the following observations: When the TMD is installed on mass m_1, the vibration amplitudes of masses m_1, m_2, m_3 are significantly reduced. When the TMD is installed on mass m_2, the vibration amplitude of masses m_2 and m_3 are significantly reduced, and the vibration amplitudes of mass m_1 decreased very little. When the TMD is installed on the mass m_3, the vibration amplitude of mass m_3 significantly reduced, and the vibration amplitudes of masses m_1, m_2 decreased very little.

4. CONCLUSION

In this paper, the sequential quadratic programming (SQP) method is used to calculating parameter optimization of the tuned mass damper (TMD) for three-degree-of-freedom vibration systems. The following concluding remarks have been reached:

- If the TMD is attached to the vibration source (excited force or kinematical excitement), the effect of vibration reduction will be achieved globally.
- If the TMD is attached to the place far away from the vibration source, the effect of vibration reduction will be achieved in the upper masses from the position of TMD.

- The SQP method can be used in solving complex constrained optimization problems for multibody systems.

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