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Short Communication

WEIGHTED DUAL APPROACH TO THE PROBLEM OF EQUIVALENT REPLACEMENT

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Abstract. The problem of equivalent replacement plays a basic role in many fields of science and technology. In the classical approach the original object A is to be replaced by an equivalent object kB using the mean square minimum criterion. When the difference between A and kB is significant the replacement leads to unaccepted errors. In order to reduce the errors one may apply the dual approach. One of significant advantages of the dual conception is its consideration of two different aspects of a problem in question allowing the investigation to be more appropriate. The main idea of this short communication is to propose a special form of weighted dual mean square criterion. Numerical results are carried out for linear and nonlinear systems to show an improved accuracy of the proposed criterion.

Keywords: Weighted, dual approach, mean square, equivalent replacement.

1. INTRODUCTION

The problem of equivalent replacement plays a basic role in many fields of science and technology. The solution of this problem allows transforming a difficult problem to one that is much easier to be investigated. In the classical approach the original object A is to be replaced by an equivalent object kB using the mean square criterion. When the difference between A and kB is significant the replacement leads to unaccepted errors, see for example [1]. In order to reduce the errors one may apply the dual approach recently proposed and developed in [2-5]. One of significant advantages of the dual conception is its consideration of two different aspects of a problem in question allowing the investigation to be more appropriate. The main idea of this short communication is to extend the result of [2] and propose a special form of weighted dual mean square criterion.

2. WEIGHTED DUAL MEAN SQUARE CRITERION

Let first reconsider briefly the basic idea of the classical mean square criterion. Let given functions $A(x)$ and $B(x)$. Using the classical mean square criterion one can replace the function $A(x)$ by the equivalent function $k_cB(x)$ with the equivalent coefficient k_c found from the optimal condition

$$J_c = < (A(x) - k_cB(x))^2 > \to \min_{k_c} \tag{1}$$

The symbol $< \cdot >$ is a corresponding averaging operator. In the case of deterministic functions defined on the interval (a, b) one gets

$$< \cdot > = \frac{1}{a-b}\int_a^b (\cdot)dx \tag{2}$$

In the case of random functions $< \cdot >$ is the mathematical expectation operator. Using the dual approach to the problem of equivalent replacement one may suggest a consideration respect two aspects, namely, forward and back replacements. Thus, a weighted dual mean square criterion can be introduced as follows [2]

$$J_d = (1-p) < (A(x) - k_dB(x))^2 > + p < (k_dB(x) - \mu_dA(x))^2 > \to \min_{k_c, \mu_d} \tag{3}$$

where p is a weighting parameter. It is seen that (3) leads to the classical and dual mean square criteria [4] when $p = 0$ and $p = 1/2$, respectively. Taking partial derivatives of J_d respect to k_d, μ_d one gets

$$\frac{1}{2}\frac{\partial}{\partial k_d}J_d = (1-p) < (A(x) - k_dB(x))(-B(x)) > + p < (k_dB(x) - \mu_dA(x))B(x)) > = 0, \tag{4}$$

$$\frac{1}{2}\frac{\partial}{\partial \mu_d}J_d = p < (k_dB(x) - \mu_dA(x))(-A(x)) > = 0. \tag{5}$$

Solving this system yields

$$k_d = \frac{1-p}{1-p\eta}\frac{< A(x)B(x) >}{< B^2(x) >}, \tag{6}$$

$$\mu_d = \frac{(1-p)\eta}{1-p\eta}. \tag{7}$$

where it is denoted

$$\eta = \frac{< A(x)B(x) >^2}{< A^2(x) >< B^2(x) >}. \tag{8}$$

It is easy to show

$$0 < \eta \leq 1, \tag{9}$$

and the equality takes the place when $A = \lambda B, \lambda = const$. Hence, the value

$$d = 1 - \eta \tag{10}$$

can be considered as a measurement for the difference between $A(x)$ and $B(x)$. The weighting parameter p depends on d and can be expanded in a polynomial form of d as follows

$$p = p_1d + p_1d^2 + \ldots + p_nd^n. \tag{11}$$

The coefficients p_i $(i = 1, 2, ..., n)$ are chosen in such a way based on the analysis of exact solutions of linear and nonlinear systems. An expression of p is introduced as follows

$$p = \frac{5}{12}(1-\eta) + \frac{1}{2}(1-\eta)^2. \tag{12}$$

Substituting (12) into (3) yields the following form of the weighted dual mean square criterion

$$\begin{aligned} J_d = {} & (1 - \frac{5}{12}(1-\eta) - \frac{1}{2}(1-\eta)^2) < (A(x) - k_d B(x))^2 > + \\ & + (\frac{5}{12}(1-\eta) + \frac{1}{2}(1-\eta)^2) <(k_d B(x) - \mu_d A(x))^2 > \to \min_{k_c, \mu_d}. \end{aligned} \tag{13}$$

where η is given by (8). Thus, the corresponding equivalent coefficient k_d is defined by (6) where p is determined by (12). In the next section the accuracy of the weighted dual mean square criterion (13) is examined for some typical linear and nonlinear systems.

3. NUMERICAL EXAMPLES

3.1. Critical load for a beam subjected to axial compression

For illustration of possible uses (13) for linear system consider a beam subjected to axial compression and fixed at $x = 0, x = L$

$$\frac{d^2 y(x)}{dx^2} + P y(x) = 0, \tag{14}$$

where $y(x)$ is the deflection defined in the domain $D\colon x \in [0, L]$ and satisfying boundary conditions

$$y(0) = y(L) = 0. \tag{15}$$

An approximate solution of (14) satisfying (5) is taken in the form

$$y(x) = x(x - L). \tag{16}$$

Here one has

$$\begin{aligned} & A(x) = \frac{d^2 y(x)}{dx^2} = 2,\ B(x) = y(x),\ < A(x)B(x) > = -\frac{L^2}{3}, < A^2(x) > = 4, \\ & < B^2(x) > = 4,\ \eta = \frac{< A(x)B(x) >^2}{< A^2(x) >< B^2(x) >} = \frac{5}{6},\ p = \frac{1}{12}. \end{aligned} \tag{17}$$

Substituting (17) into (6) and using (14) give

$$P_d = -k_d = 9.851/L^2. \tag{18}$$

The exact and Galerkin' solutions are, respectively,

$$P_e = \pi^2/L^2 = 9.870/L^2,\ P_G = -k_G = 10/L^2. \tag{19}$$

It is seen that the solution (18) is much closer to the exact solution than Galerkin' one.

3.2. Duffing oscillator excited to dynamic load

We consider the Duffing oscillator

$$\ddot{u} + 2h\dot{u} + \gamma u^3 = \sigma\varsigma(t), \tag{20}$$

where $\varsigma(t)$ is white noise process with unit intensity. First, consider free vibration of Duffing system

$$\ddot{u} + \gamma u^3 = 0, \quad \dot{u}(0) = 0,\ u(0) = a = const. \tag{21}$$

The nonlinear term γu^3 is replaced by an equivalent linear one $k_d u$. One gets

$$\begin{gathered} A(\varphi) = \gamma u^3 = \gamma a^3 \cos^3\varphi,\ B(x) = u = a\cos\varphi, \\ < A(\varphi)B(\varphi) > = < \gamma a^4 \cos^4\varphi > = \frac{3}{8}a^4, < A^2(\varphi) > = < \gamma^2 a^6 \cos^6\varphi > = \frac{5}{16}a^6, \\ < B^2(\varphi) > = < a^2\cos^2\varphi > = \frac{1}{2}a^2,\ \eta = \frac{< A(\varphi)B(\varphi)>^2}{< A^2(\varphi) >< B^2(\varphi) >} = \frac{9}{10},\ p = \frac{7}{150}. \end{gathered} \tag{22}$$

Substituting (22) into (6) yields the following frequency of free vibration of Duffing system (21)

$$\Omega_d = \sqrt{k_d} = \sqrt{\frac{1-p}{1-p\eta}\frac{< A(\varphi)B(\varphi) >}{< B^2(\varphi) >}} = \sqrt{0.7464a^2} = 0.864a. \tag{23}$$

The exact and Galerkin' solutions are, respectively,

$$\Omega_e = 0.847a,\ \Omega_G = 0.866a. \tag{24}$$

It is seen solution (23) is a litle slightly closer to the exact solution than Galerkin' one.

Consider now random vibration of Duffing oscillator (20). The nonlinear term γu^3 is replaced by an equivalent linear one $k_d u$. One gets

$$\begin{gathered} A(\varphi) = \gamma u^3,\ B(x) = u\,,\ < A(u)B(u) > = < \gamma u^4 > = 3 < u^2 >, \\ < A^2(u) > = < \gamma^2 u^6 > = 15 < u^2 >^3, \\ < B^2(u) > = < u^2 >,\ \eta = \frac{< A(u)B(u) >^2}{< A^2(u) >< B^2(u) >} = \frac{3}{5},\ p = \frac{37}{150}. \end{gathered} \tag{25}$$

Substituting (25) into (6) yields the second moment of random vibration of Duffing system (20) with standard parameters $\sigma^2/(4h) = 1$,

$$< u^2 > = \sqrt{\frac{1-p\eta}{3(1-p)}} = 0.614. \tag{26}$$

The exact and Galerkin' solutions are, respectively,

$$< u^2 >_e = 0.66,\ < u^2 >_G = 0.57. \tag{27}$$

It is seen that solution (26) is much closer to the exact solution than Galerkin' one.

4. CONCLUSION

In this short communication the main idea of the dual conception is further extended to suggest a special form of weighted dual mean square criterion. Numerical results are carried out for a linear system and Duffing oscillator. It is observed that the approximate solutions of weighted dual mean square minimum criterion obtained for thse systems are more accurate than Galerkin' solutions. Further detailed investigations will be presented in a full publication.

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