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GENERALIZED PSEUDO INVERSE KINEMATICS AT SINGULARITIES FOR DEVELOPING FIVE-AXES CNC MACHINE TOOL POSTPROCESSOR

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Abstract. This paper presents an analytical scheme for analyzing the singular configuration problem of general five-axes CNC machine tool. A computational technique based on the generalized pseudo inverse kinematics for finding out feasible solution of the problem is proposed. The technique is then used for developing postprocessor algorithm of five-axes CNC machine tool. Typical examples of singularity analysis are carried out, and real cutting parts are implemented for verifying the research result.

Keywords: Five-axis CNC machine, Kinematics singularity, Five-axis CNC postprocessor.

1. INTRODUCTION

Five-axis milling CNC machine (five-axis milling Robot) has been widely utilized as an efficient tool for fabricating products of complex geometry which may include numerous freeform surfaces. The products are widely used in several high technology industries such as the aerospace industry, automotive industry, shipbuilding industry, etc. Using 5-axis CNC systems integrated with CAMs, end-to-end component design and manufacturing is highly automated. Today's commercial CAMs have been shown their advantages in preparing complex G-codes program file which is used for controlling the machines.

The main function of any CAM system is to compute the cutter trajectory (tool path) in task space (so-called workpiece space) defined inside the system by programmer, basing on the input of the part surface modeling, surface quality requirement (surface error), cutter definition, tool path pattern, etc. The tool path is piecewise curves passing through CL points (cutter location points). In task space, each CL point is defined as a set of 6 components, (x, y, z, I, J, K) where (x, y, z) is coordinates of the tool tip and (I, J, K) are direction cosines of the tool axis vector, respectively. All the CL points outputted by CAM are recorded one by one in a text file that is so called the CL data file.

As introduced in [1, 2, 3], the 5-axis CNC mechanisms usually consists of three translational axes (x, y, z) and two rotation axes $(A, B), (B, C)$ or (A, C). In machining process, the controller must compile G-code commands for controlling the axes of the machine in such a way that the cutter removes material on the workpiece as required. As for free form surface milling, most of the G-codes are the straight line interpolation syntaxes which control simultaneously these 5 independent axes (joints) of the machine. Therefore, a postprocessor is needed for transforming CL data (output of CAMs) into G-codes running the CNC machine The main task of the transformation is to compute the inverse kinematics of the machine mechanisms.

Consider the generalized 5-axes mechanism, if we denote $\mathbf{r} = \begin{bmatrix} x & y & z & I & J & K \end{bmatrix}^T \in W^6 \subset \mathbb{R}^6$ as a CL point in the workpiece space W^6, and $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix}^T \in Q^5 \subset \mathbb{R}^5$ as a joint variable vector in the joint space Q^5 where (q_1, q_2, q_3) are the notations of three translational joint (axis) variables, and (q_4, q_5) are the notations of the rotational joint (axis) variables, the forward kinematics equation can be described as follows

$$\mathbf{r} = \mathbf{f}(\mathbf{q}), \tag{1}$$

where $\mathbf{f}$ is a nonlinear mapping: $\mathbf{f} : Q^5 \to W^6$. The differential relationships of the kinematics is written as

$$\delta\mathbf{r} = \mathbf{J}(\mathbf{q})\,\delta\mathbf{q}, \tag{2}$$

where $\mathbf{J}(\mathbf{q}) = \dfrac{\partial\mathbf{f}}{\partial\mathbf{q}} \in \mathbb{R}^{6\times5}$ is the Jacobian matrix. To calculate the inverse kinematics for the mentioned above postprocessor, we can solve either Eq. (1) for $\mathbf{q}$ or Eq. (2) for $\delta\mathbf{q}$. Calculating the inverse kinematics of any 5-axis CNC machine is to find out the inverse function of either the nonlinear function of Eq. (1) or the linear function of Eq. (2). For both cases, the condition for existence of the inverse function is that the determinant of Jacobian matrix is not equal to zero [4]

$$Det\ \mathbf{J}(\mathbf{q}) \neq 0 \tag{3}$$

Theoretically, if $Det\ \mathbf{J}(\mathbf{q}) = 0$ the configuration of the machine will degenerate, and it is called the singular configuration of the kinematics. The singular configuration limits the tool tip trajectories of the machine in solving inverse kinematics. Moreover, the most serious problem of the singularity is not at singular point which has zero volume in W^6, but rather in the neighborhood of singular point which has a finite volume. In the considering neighborhood, even for a small change of $\mathbf{r}$, an enormous change of $\mathbf{q}$ is often required. Thus, it causes a large error in real cutting path. This problem increases the under cuts and over cuts, reduces the effectiveness and efficiency of the surface machining and increases the production cost.

In the literature, the forward, inverse kinematics and postprocessor for specific kinds of 5-axes CNC machines have been presented and discussed in several papers, such as by She [5], Lee and She [6], She and Huang [7], Jung et al [8] and Sorby [9]. For the non-orthogonal five-axis configuration, an approximation scheme is proposed by Sorby [9] for positioning the tool tip near singularities. The problem of singular point is also dealt with in the research of Affouard et al [10] who developed a method for avoiding the machine singularity through a tool path planning algorithm in the CAD/CAM system. The method proposed by Munlin et al [11] is for reducing the machining error near singularities of an orthogonal axes machine type, by optimizing the sequence of machine rotations. For the non-orthogonal five-axes configuration, another approximation scheme is proposed by

Sorby [11] for positioning the tool tip near singularities. However, most of the Literature to be found on the inverse kinematics and singularity for five-axes machines focuses on specific CNC machines. Instead of finding out the inverse kinematics solution at singularities, most of the papers cope with the singularity problem by modifying the tooltip positioning strategy near the points [10, 11]. This paper discusses the singularity in terms of the differential kinematics for the general mechanism of five-axes CNC machine. An analytical formulation is considered for checking singularities, and a computational technique based on the generalized pseudo inverse kinematics is proposed for finding out feasible solution in the neighborhood of singularities. Typical examples of singularity analysis are carried out, and real cutting parts are implemented for verifying the research result.

2. SINGULAR CONFIGURATION ANALYSIS

Generally, each specific five-axes kinematics model could compose singular points; some of them locate on the boundary, and the others locate in the middle of the domain Q^5. In controlling practice, all motions of the axes are not allowed to reach to the joint limits. This is to avoid unwanted collisions and damages of the machine. Hence, in the case of five-axes machining, singularities locating within the domain Q^5 play a more serious role than which locates on the exact boundary.

Given a five-axes CNC machine and its forwards kinematics equation, the singular configuration of the machine can be analyzed by solving the following equation

$$Det\,\mathbf{J}\left(\mathbf{q}\right) = 0 \tag{4}$$

Eq. (4) is an analytical formulation for singularities analysis. The solution for $\mathbf{q}$ of Eq. (4) is called the singular points. For any kind of five-axes kinematics chain, the forward kinematics equation can be formulated as Eq. (1), and the Jacobian matrix, $\mathbf{J}\left(\mathbf{q}\right)$, is then derived. Consequently, the determinant of the Jacobian matrix is obtained as a function of $\mathbf{q}$. Solving the determinant function yields singular values of joint variables $\mathbf{q}$ at which the configuration degenerates.

In general, Eq. (4) formulated for the generalized 5-DOF machinery mechanism is complex and could have no analytical solution. In that cases, the numerical recipes (evaluation of function) [12] and softwares (like Matlab) are utilized for finding out the numerical solution as desired. Fortunately, (I, J, K) are the direction cosines of the tool axis vector, therefore $I^2 + J^2 + K^2 = 1$. Owing to this relation, the number of linearly independent equations of the forward kinematics modeling is reduced to 5; hence, for some common 5-axis machines, it is not so difficult to find out the analytical solutions of Eq. (4) with the support of Maple software. To illustrate the analysis sequences, examples below are considered in details with the support of Maple Software.

Example 2.1: Consider a typical orthogonal five-axes configuration, MAHO 600e. Now we analyze the singularity of the kinematics model with the forward kinematics equation described as follows

$$\begin{array}{l} x = q_1 Sq_5 Cq_4 + q_2 Sq_4 + \left(Z_{0f} + q_3\right) Cq_4 Cq_5 + X_{01} \\ y = q_1 Sq_5 Sq_4 - q_2 Cq_4 + \left(Z_{0f} + q_3\right) Sq_4 Cq_5 + Y_{01} \\ z = q_1 Cq_5 - \left(Z_{0f} + q_3\right) Sq_5 + Z_{03} + Z_{01} \\ I = Cq_4 Cq_5 \\ J = Sq_4 Cq_5 \end{array} . \tag{5}$$

C^* and S^* are denoted for cosine and since functions. $X_{01}, Y_{01}, Z_{01}, Z_{0f}$ are constants.

Based on Eq. (5), the determinant of Jacobian matrix is calculated as

$$Det\,\mathbf{J}\left(\mathbf{q}\right) = Sq_5 Cq_5. \tag{6}$$

The solution of $Det\,\mathbf{J}\left(\mathbf{q}\right) = 0$ is $q_5 = k\dfrac{\pi}{2}$, where $k \in$ Z (integer number Set). Since $0 > q_5 > -\pi$ (B axis of the machine varies in the range (-π ,0)), the singular configuration is found at $q_5 = -\dfrac{\pi}{2}$.

Example 2.2: As for another typical non-orthogonal five-axes configuration, Deckel MAHO DMU 50e, we can use the same procedure for finding out the singularity. The forward kinematics equation can be found as follows

$$\begin{array}{l} x = q_1 Cq_5 \left[\Delta_z^2 \left(1 - Cq_4\right) + Cq_4\right] + q_1 Sq_4 \left[\Delta_y \Delta_x \left(1 - Cq_4\right) - \Delta_z Sq_4\right] \\ \quad + q_2 Cq_5 \left[\Delta_y \Delta_x \left(1 - Cq_4\right) + \Delta_z Sq_4\right] + + q_2 Sq_4 \left[\Delta_y^2 \left(1 - Cq_4\right) - Sq_4\right] \\ \quad + q_3 Cq_5 \left[\Delta_z \Delta_x \left(1 - Cq_4\right) + \Delta_y Sq_4\right] + q_3 Sq_5 \left[\Delta_z \Delta_x \left(1 - Cq_4\right) - \Delta_x Sq_4\right] + G_x \\ y = -q_1 Sq_5 \left[\Delta_x^2 \left(1 - Cq_4\right) + Cq_4\right] + q_1 Cq_4 \left[\Delta_y \Delta_x \left(1 - Cq_4\right) - \Delta_z Sq_4\right] \\ \quad - q_2 Sq_5 \left[\Delta_y \Delta_x \left(1 - Cq_4\right) + \Delta_z Sq_4\right] + + q_2 Cq_4 \left[\Delta_y^2 \left(1 - Cq_4\right) - Sq_4\right] \\ \quad - q_3 Sq_5 \left[\Delta_z \Delta_x \left(1 - Cq_4\right) + \Delta_y Sq_4\right] + q_3 Cq_5 \left[\Delta_z \Delta_x \left(1 - Cq_4\right) - \Delta_x Sq_4\right] + G_y \\ z = q_1 \left[\Delta_z \Delta_x \left(1 - Cq_4\right) + \Delta_y Sq_4\right] + q_3 \left[\Delta_z \Delta_y \left(1 - Cq_4\right) - \Delta_x Sq_4\right] \\ \quad + q_3 \left[\Delta_z^2 \left(1 - Cq_4\right) + Cq_4\right] + G_z \\ i = Cq_5 \left[\Delta_x \Delta_z \left(1 - Cq_4\right) - \Delta_y Sq_4\right] + Sq_5 \left[\Delta_y \Delta_z \left(1 - Cq_4\right) - \Delta_x Sq_4\right] \\ j = -Sq_5 \left[\Delta_x \Delta_z \left(1 - Cq_4\right) - \Delta_y Sq_4\right] + Cq_5 \left[\Delta_y \Delta_z \left(1 - Cq_4\right) - \Delta_x Sq_4\right] \end{array} . \tag{7}$$

In Eq. (7), $G_x, G_y, G_z, \Delta_x, \Delta_y, \Delta_z$ are constants. Calculating the determinant of the Jacobian matrix yields

$$Det\,\mathbf{J}\left(\mathbf{q}\right) = \frac{1}{4} Sq_4 \left(1 + Cq_4\right). \tag{8}$$

Finally, the singular configuration is found at $q_4 = 0$ since $q_4 \in [0, \pi)$ as shown in the machine Catalog.

3. INVERSE KINEMATICS AT SINGULARITIES

At singularities, $\mathbf{J}(\mathbf{q})^{-1}$ is not defined, hence the inverse solution of Eq. (2) is not existed. In this case, the generalized pseudo inverse of $\mathbf{J}\left(\mathbf{q}\right)$, denoted by $\mathbf{J}^*\left(\mathbf{q}\right)$, has been proposed as a remedy for this problem since it is defined even at singular points [13]. Using $\mathbf{J}^*\left(\mathbf{q}\right)$ the inverse solution found is continuous and feasible even at or in the neighborhood of singularities. In definition, the generalized pseudo inverse matrix, $\mathbf{J}^*\left(\mathbf{q}\right)$ satisfies

Eq. (8) as follows

$$\begin{aligned} &\mathbf{J}\mathbf{J}^*\mathbf{J} = \mathbf{J} \\ &\mathbf{J}^*\mathbf{J}\mathbf{J}^* = \mathbf{J}^* \\ &(\mathbf{J}\mathbf{J}^*)^T = \mathbf{J}\mathbf{J}^* \\ &(\mathbf{J}^*\mathbf{J})^T = \mathbf{J}^*\mathbf{J} \end{aligned}. \tag{9}$$

In particular linear algebra, a pseudo inverse $\mathbf{J}^*$ of a matrix $\mathbf{J}$ is a generalization of the inverse matrix, and it is so called the generalized pseudo inverse matrix (Moore-Penrose pseudo inverse matrix). Depending on the number of row, m, and the number of column, n, of the matrix $\mathbf{J}$, either the left inverse or right inverse formula is used. If $m \geq n$ the left inverse will be used, and if $m \leq n$ the right inverse will be used.

For the 5-axis CNC kinematics chain, $\mathbf{J}(\mathbf{q}) = \dfrac{\partial \mathbf{f}}{\partial \mathbf{q}} \in \mathbb{R}^{6\times 5}$ $(m \geq n)$, so we use the left inverse of $\mathbf{J}$. Using $\mathbf{J}^*(\mathbf{q})$. Eq. (2) is now transformed as

$$\delta\mathbf{q} = \mathbf{J}^*(\mathbf{q})\,\delta\mathbf{r}. \tag{10}$$

In essence, Eq. (10) offers a least-square solution with a minimum norm for Eq. (2) at singular point. In the other words, $\delta\mathbf{q}$ calculated by Eq. (10) satisfying

$$\min \|\delta\mathbf{q}\|, \tag{11}$$

among all $\delta\mathbf{q}$ that fulfill

$$\min \|\delta\mathbf{r} - \mathbf{J}(\mathbf{q})\,\delta\mathbf{q}\|. \tag{12}$$

Now we need to find out $\delta\mathbf{q}^*$ that satisfies Eqs. (11, 12). It is proposed that the following equation as an evaluation criterion for the solution of Eq. (2)

$$\min \|\delta\mathbf{e}\|_W, \tag{13}$$

where the error vector, $\delta\mathbf{e}$ is defined as

$$\delta\mathbf{e} = \begin{bmatrix} \delta\mathbf{r} - \mathbf{J}(\mathbf{q})\,\delta\mathbf{q} \\ \delta\mathbf{q} \end{bmatrix}. \tag{14}$$

We can find out the optimal solution $\delta\mathbf{q}^*$ in such a way that the norm $\|\delta\mathbf{e}\|_W \to 0$. The real deviation of the real cutting passes and the desired one (the machining tolerance) is controlled by the linearization scheme [13].

With the consideration of the weight matrix, $\|\delta\mathbf{e}\|_W$ is written as follows

$$\|\delta\mathbf{e}\|_W^2 = \delta\mathbf{e}^T\mathbf{W}\delta\mathbf{e}. \tag{15}$$

Eq. (15) implies the weighted norm of the error vector, $\delta\mathbf{e}$. The weight is chosen as

$$\mathbf{W} = \begin{bmatrix} \mathbf{E}_{5\times 5} & 0 \\ 0 & k\mathbf{E}_{5\times 5} \end{bmatrix}, \tag{16}$$

where k is a scalar constant. k implies the weight between the exactness, $\delta\mathbf{r} - \mathbf{J}(\mathbf{q})\,\delta\mathbf{q}$, and the feasibility, $\delta\mathbf{q}$, of the solution. The weight matrix reflects physically the proportional factor between the input and the output.

From Eq. (13), $\delta\mathbf{e}$ must satisfy

$$\|\delta\mathbf{e}\|_W^2 = \|\delta\mathbf{r} - \mathbf{J}(\mathbf{q})\,\delta\mathbf{q}\|_W^2 + \|\delta\mathbf{q}\|_W^2. \tag{17}$$

Eqs. (12) and (17) imply that we expect a solution by evaluating simultaneously the exactness and the feasibility. Eq. (17) is now rewritten as the following quadratic function.

$$\|\delta\mathbf{e}\|_W^2 = (\delta\mathbf{q} - \delta\mathbf{q}^*)^T \left(\mathbf{J}^T\mathbf{J} + k\mathbf{E}\right)(\delta\mathbf{q} - \delta\mathbf{q}^*) + \delta\mathbf{r}^T\mathbf{W}^*\delta\mathbf{r}, \tag{18}$$

where

$$\delta\mathbf{q}^* = \left(\mathbf{J}^T\mathbf{J} + k\mathbf{E}\right)^{-1}\mathbf{J}^T\delta\mathbf{r}, \tag{19}$$

$$\mathbf{W}^* = \mathbf{E} - \mathbf{J}\left(\mathbf{J}^T\mathbf{J} + k\mathbf{E}\right)\mathbf{J}^T. \tag{20}$$

Notice that $\left(\mathbf{J}^T\mathbf{J} + k\mathbf{E}\right)$ is always positive definite matrix and it is therefore non-singular.

Eq. (18) is the quadratics function of $\delta\mathbf{q}$. Thus, $\delta\mathbf{q}^*$ is the unique solution for the evaluation criterion of Eq. (13). Finally, the inverse solution of the kinematics satisfying the criterion Eq. (13) becomes

$$\delta\mathbf{q}^* = \mathbf{J}^*\delta\mathbf{r}, \tag{21}$$

where

$$\mathbf{J}^* = \left(\mathbf{J}^T\mathbf{J} + k\mathbf{E}\right)^{-1}\mathbf{J}^T. \tag{22}$$

Eq. (21) is the solution for the five-axes inverse kinematics at singularities which will be used in the next section for improving the postprocessor algorithm.

4. FIVE-AXES POSTPROCESSOR ALGORITHM IMPROVEMENT

For any five-axes machine, the singularity of the configuration should be searched in the configuration space following the procedure presented in section 1. The singular CL points, $\mathbf{r}^s$, in CL file produced by CAMs is then determined by using the forward kinematics relationship. Take a look back Example 2.1. When the machine configuration degenerates, $q_5 = -\dfrac{\pi}{2}$, the component k of CL record is computed as $k = -\sin q_5$. So $k = 1$ for all q_1, q_2, q_3, q_4. Hence, any CL point in the data set, of which the component k equals to 1 is exactly the singular CL point, $\mathbf{r}^s$.

In practice, when the configuration degenerates in machining process, the corresponding CL data composes either exact singular CL points or CL points in the neighborhood of singular point. In the first case, it implies that the tool tip positions exactly at the singular points when machining. The second case means the cutting trajectories pass across the neighborhood of the singular CL points.

Suppose that we consider a singular CL record $\mathbf{r}^s$ locating in the range $[\mathbf{r}_i, \mathbf{r}_{i+1}]$. The increment of the tool tip position and orientation can be computed as $\delta\mathbf{r} = \mathbf{r}^s - \mathbf{r}_i$. Using Eq. (21) we can determine $\delta\mathbf{q}^*$ for the inverse solution. Finally, the vector of machine joint variables is determined as $\mathbf{q}_s = \mathbf{q}_i + \delta\mathbf{q}^*$.

It should be remarked that the actual tool path in the task space is not linear [14]. It is piecewise curve passing through CL points since the linear and rotary axes move simultaneously. The cutting curve deviates from the linearly interpolated straight line path between successive CL points, and this problem has been proceeded as the linearization technique. The linearization scheme presented in [14] is employed for modifying the tool path passing singular point. Consider three CL points, $\mathbf{r}_i, \mathbf{r}_s$ and $\mathbf{r}_{i+1}$, in the neighborhood of the singularity. The actual values of $\mathbf{q}$ can be expressed as

$$\mathbf{q}_{l,t} = \mathbf{q}_i + (\mathbf{q}_s - \mathbf{q}_i)\, t, \tag{23}$$

$$\mathbf{q}_{m,t} = \mathbf{q}_s + (\mathbf{q}_{i+1} - \mathbf{q}_s)\, t. \tag{24}$$

In Eqs. (22, 23), $t \in [0, 1]$ is a parameter for the linearization. Based on the values of $\mathbf{q}_i, \mathbf{q}_s$ and $\mathbf{q}_{i+1}$, the insertion points $\mathbf{q}_{l,t}$ and $\mathbf{q}_{m,t}$ are computed, and the corresponding $\mathbf{r}_{l,t}$ and $\mathbf{r}_{m,t}$ are then determined. If the error between the computed curve and the desired toolpath is still greater than the limitation, the new insertions points will be computed basing the input on the points having been determined. The time parameter t varies from 0 to 1 to generate $\mathbf{q}_{l,t}$ and $\mathbf{q}_{m,t}$. The corresponding $\mathbf{r}_{l,t}$ and $\mathbf{r}_{m,t}$ are computed by using the forward kinematics equation. The chordal deviation checking procedure is then used to decide how many inserted positions in between $[\mathbf{r}_i, \mathbf{r}_s]$ and $[\mathbf{r}_s, \mathbf{r}_{i+1}]$.

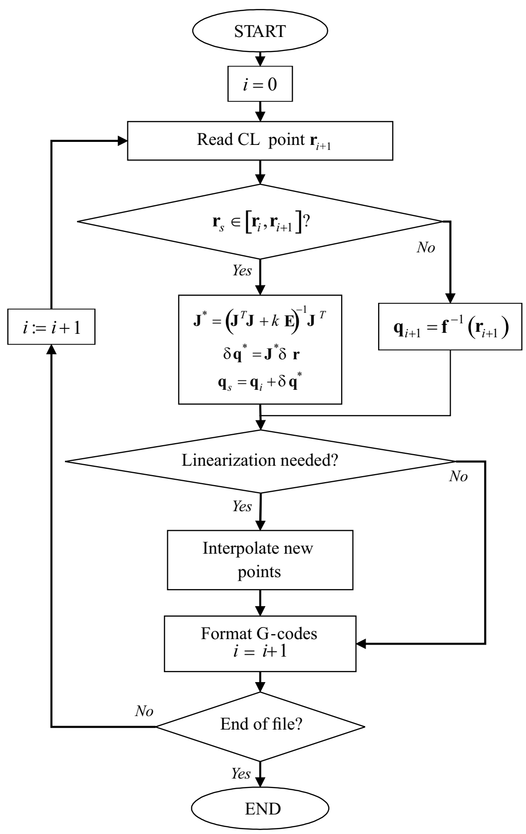

Fig. 1. Postprocessor algorithm

In brief, the algorithm for improving five-axes postprocessor which incorporates the generalized pseudo inverse kinematics at singularities can be described as in Fig. 1.

5. MACHINING TEST

The proposed algorithm is implemented for improving the postprocessor for Deckel Maho DMU 50e CNC machine. Experiments have been carried out for verifying the inverse kinematics solution at singularities. In particular, a given concave surface with singular points in the middle is taken into consideration for preparing CL data in Pro Engineer software. The tool path generated by the software and its simulation is shown in Fig. 2.

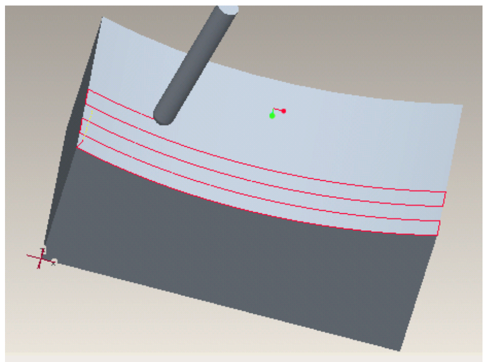

Fig. 2. Tool path generated by ProEngineer

Fig. 3. Real cutting path with dangerous error

Fig. 4. Smooth cutting path

To demonstrate the dangerous error of the real cutting passes when the tool cuts through the singularities, the old postprocessor is employed to generate the G-codes, based on the CL data yielded. As a result, the machined surface is damaged as shown in Fig. 3 since the tool always turn back approximately 1800 when the cutter cuts near the middle of each cutting curve (the singular point). In case of using the improved postprocessor, the cutter moves smoothly on the planned passes. The real cutting path is shown in Fig. 4.

6. CONCLUSIONS

The paper presents an analytical scheme for analyzing the singular configuration of general five-axes CNC machine This is useful for checking the singularities of any specific machine. The singular configuration problem causing big machining error is effectively solved and the proposed algorithm shows its advantage for producing smooth cutter trajectories crossing neighborhood of singular points. Remark that the proposed technique could be integrated directly in CAMs for developing generalized postprocessor of five-axes CNC machine tool.

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