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DYNAMIC REDUCTION METHOD FOR FRAME STRUCTURES

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Abstract. This paper proposes a new method for reducing MDOF (Multi Degree of Freedom) system of frame structures into SDOF (Single Degree of Freedom) system and recovering back the results of dynamic analysis from the SDOF to the MDOF system. In most of dynamic condensation methods, the reduction of MDOF into an SDOF system is usually done by means of modal analysis. As a result, simplification against the MDOF is made forward-wise and not reversible. In other words, after the responses of SDOF system evaluated by the modal and dynamic analyses, then it is not possible to recover back the responses of the other condensed degree of freedoms accurately. In the new Strain Reduction Method, displacement and rotational DOFs of all nodal points in the frame structures except an arbitrarily selected representative node are transformed into their strains field before the reduction process. By adopting previous authors' work in improving the dynamic reduction method, small order in magnitude of strains field can be separated as secondary DOFs, hence leaving the representative node as an SDOF system. After conducting dynamic analysis to the SDOF system, the time historical responses of the SDOF system can be used to recover back the time historical responses in the MDOF system of frame structures with negligible error.

Keywords: SDOF, MDOF, structural dynamics, strain reduction method, reduction, recovering.

1. INTRODUCTION

To guarantee the safety and performance of building against damages during the occurrences of earthquake, it is necessary to have an understanding of the response behavior of buildings. Therefore, accuracy in design calculation has to be given consideration to achieve a safe and reliable design of buildings.

In the calculation design works, a designer has to select a design tool which uses a simple design procedure referred to the seismic design codes or conducts more complex MDOF dynamic analysis to improve the accuracy of calculation. However, conducting a fully complex MDOF dynamic analysis is still considered time consuming for practical design works. Although the present development in computing technology and hardware is rapidly increased, it seems like for design calculation and practical analysis purposes, there is still a need in seeking for a simple and easy yet accurate design procedure.

Most of seismic design codes including charts, tables and graphs were developed based on an equivalent SDOF model for evaluating responses of structures against the ground motion seismic excitations. Such as response spectral of accelerations, velocities and displacements are still a valuable and practical tool for design practice due to its simplicity.

In this paper, a new method for reducing MDOF system to SDOF system for design purpose which is simple and easy is proposed. In the proposed method, the way of dynamic equation of motion is reduced is different with the ones of Guyan [1]; Dynamic Condensation [2]; IRS [3] and SEREP [4]. In most of the dynamic condensation methods proposed, the reduction of MDOF system into an SDOF system is merely done by using modal analysis to get natural frequencies and modal shapes of the structures. Therefore, simplification against the MDOF is made forward-wise and not reversible. In other words, after the responses of the SDOF system evaluated by the modal and dynamic analyses, it is not possible to get the responses of the other condensed DOFs accurately, because the higher frequency modes are usually neglected in the calculation.

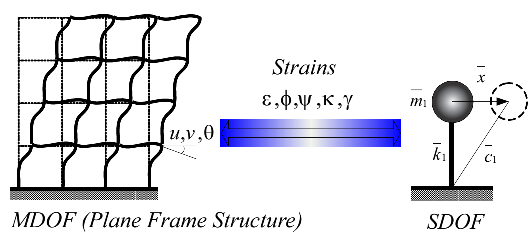

Fig. 1. Reduction and recovering method using strains transformation illustration

In the present Strain Reduction Method, before using the reduction method [5-9] to the dynamic equation of motion matrices, all the DOFs in the MDOF system are transformed into their corresponding strains field. Hence, the secondary DOFs are scaled down to their strain fields which have lower order in magnitude compared with the main displacements DOF. After conducting dynamic analysis to the SDOF system, the time historical responses of the SDOF system can be used to recover back the time historical responses in the MDOF system of frame structures. Fig. 1 shows the illustration of the present proposed method where u, v, θ denote the nodal displacement fields and $\varepsilon, \phi, \psi, \kappa, \gamma$ denote the transformed strain fields of element member of frame structures.

2. GENERAL STRAIN TRANSFORMATION METHOD

Let $\bar{x}, \bar{y}, \bar{z}$ are the local coordinate system of a beam, column or other element member of a space frame structure in the x, y, z global coordinate system as depicted in Fig. 2.

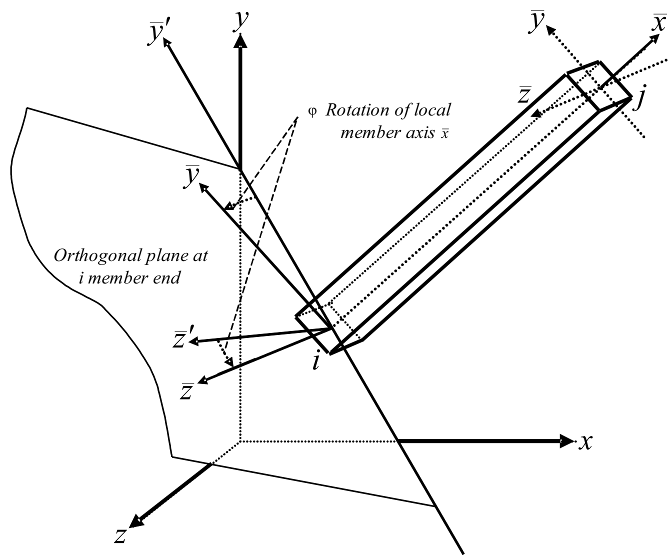

Fig. 2. Element member of space frame structure

Along the member element $\bar{x}$ local axis, the displacements and rotational DOFs are given by $\bar{u}, \bar{v}, \bar{w}, \phi$ in the $\bar{x}, \bar{y}, \bar{z}$ coordinate systems, respectively. By assuming the displacement modes at one end of the element member which are: linear in terms of axial displacement and rotational DOFs $\bar{u}, \phi$; and cubic in terms of transversal displacements DOFs $\bar{v}, \bar{w}$, the strains field along the element member can be given by

$$\begin{gathered}\varepsilon = \frac{d\bar{u}}{d\bar{x}},\ \phi_y = -\frac{d\bar{w}}{d\bar{x}},\ \phi_z = \frac{d\bar{v}}{d\bar{x}},\ \psi_x = \frac{d\bar{\phi}_x}{d\bar{x}},\\ \kappa_y = \frac{d\bar{\phi}_y}{d\bar{x}}, \kappa_z = -\frac{d\bar{\phi}_z}{d\bar{x}}, \gamma_y = \frac{d\kappa_y}{d\bar{x}}, \gamma_z = \frac{d\kappa_z}{d\bar{x}},\end{gathered} \tag{1}$$

with the element member length ℓ and both member ends DOFs at $i, j\ \ (j > i)$ are given as $\bar{u}_i, \bar{u}_{\bar{j}}, \bar{v}_i, \bar{v}_j, \bar{w}_i, \bar{w}_j, \bar{\varphi}_{xi}, \bar{\varphi}_{xj}, \bar{\varphi}_{yi}, \bar{\varphi}_{yj}, \bar{\varphi}_{zi}, \bar{\varphi}_{zj}, \psi_x, \bar{\kappa}_y, \bar{\kappa}_z, \gamma_y, \gamma_z$.

By integrating each equation in Eq. (1) and substituting the DOFs at both member ends as the integration constants, the displacement and rotational DOFs which related to the strains field along the element member can be expressed in a matrix form as follow

$$\left\{ \begin{array}{c} \bar{u}_j \\ \bar{\varphi}_j \end{array} \right\} = \left[\begin{array}{cc} [I] & [\bar{\ell}] \\ [0] & [I] \end{array} \right] \left\{ \begin{array}{c} \bar{u}_i \\ \bar{\varphi}_i \end{array} \right\} + [b]\{\varepsilon\}, \tag{2}$$

where,

$[I]$ is the unit matrix,

$\left\{ \begin{array}{c} \bar{u}_i \\ \bar{\phi}_i \end{array} \right\} = \left\{\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{\phi}_{xi}, \bar{\phi}_{yi}, \bar{\phi}_{zi}\right\}$ is the displacements, slopes and twist at end i,

$\left\{ \begin{array}{c} \bar{u}_j \\ \bar{\phi}_j \end{array} \right\} = \left\{\bar{u}_j, \bar{v}_j, \bar{w}_j, \bar{\phi}_{xj}, \bar{\phi}_{yj}, \bar{\phi}_{zj}\right\}$ is the displacements, slopes and twist at end j,

$\{\varepsilon\}^T = \{\varepsilon, \psi_x, \bar{\kappa}_y, \bar{\kappa}_z, \gamma_y, \gamma_z\}$ is the axial, twist rate, bending and shear strains, $\bar{\kappa}_y, \bar{\kappa}_z$ are the average bending strains,

$$[\bar{\ell}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \ell \\ 0 & \ell & 0 \end{bmatrix},$$

$$[b] = \begin{bmatrix} \ell & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\ell^2 & 0 & \frac{1}{12}\ell^3 \\ 0 & 0 & -\frac{1}{2}\ell^2 & 0 & \frac{1}{12}\ell^3 & 0 \\ 0 & \ell & 0 & 0 & 0 & 0 \\ 0 & 0 & -\ell & 0 & 0 & 0 \\ 0 & 0 & 0 & -\ell & 0 & 0 \end{bmatrix}.$$

Suppose, the coordinates of both i, j nodes are given as (x_i, y_i, z_i) and (x_j, y_j, z_j). The length ℓ and the cosines directions of axis $\bar{x}$ of the structural element member can be calculated from

$$\begin{aligned} &\ell = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}, \\ &\ell_{\bar{x}} = (x_j - x_i)/\ell, \\ &m_{\bar{x}} = (y_j - y_i)/\ell, \\ &n_{\bar{x}} = (z_j - z_i)/\ell. \end{aligned}$$

The global coordinate system x, y, z and local coordinate system $\bar{x}, \bar{y}, \bar{z}$ of the structural element member are related by

$$\{\bar{\mathbf{x}}\} = [T_e]\{\mathbf{x}\} \tag{3}$$

where

$$\begin{aligned} &\{\bar{\mathbf{x}}\}^T = \{\bar{x}, \bar{y}, \bar{z}\}, \\ &\{\mathbf{x}\}^T = \{x, y, z\}, \\ &[T_e] = [\phi]\,[\ell_{\bar{x}}], \\ &[\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix}, \\ &[\ell_{\bar{x}}] = \begin{bmatrix} \ell_{\bar{x}} & m_{\bar{x}} & n_{\bar{x}} \\ -m_{\bar{x}}/a & \ell_{\bar{x}}/a & 0 \\ -n_{\bar{x}}\ell_{\bar{x}}/a & -m_{\bar{x}}n_{\bar{x}}/a & a \end{bmatrix}, \\ &c = \cos\varphi, s = \sin\varphi, \text{ and } a = \sqrt{\ell_{\bar{x}}^2 + m_{\bar{x}}^2}. \end{aligned}$$

In the case that the structural element member axis $\bar{x}$ is parallel with the global coordinate axis z, the $[T_e]$ matrix in Eq. (3) becomes

$$[T_e] = \begin{bmatrix} 0 & 0 & 1 \\ c & s & 0 \\ -s & c & 0 \end{bmatrix}, \tag{4}$$

which holds

$$\begin{aligned} \left\{ \begin{array}{c} \bar{u}_i \\ \bar{\varphi}_i \end{array} \right\} &= \begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix} \left\{ \begin{array}{c} u_i \\ \varphi_i \end{array} \right\}, \\ \left\{ \begin{array}{c} \bar{u}_j \\ \bar{\varphi}_j \end{array} \right\} &= \begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix} \left\{ \begin{array}{c} u_j \\ \varphi_j \end{array} \right\}, \end{aligned} \tag{5}$$

and substituting both relationships from Eq. (5) into Eq. (2) gives

$$\begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix} \left\{ \begin{array}{c} u_j \\ \varphi_j \end{array} \right\} = \begin{bmatrix} [I] & [\bar{\ell}] \\ [0] & [I] \end{bmatrix} \begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix} \left\{ \begin{array}{c} u_i \\ \varphi_i \end{array} \right\} + [b]\{\varepsilon\}, \tag{6}$$

which can be rewritten as

$$\left\{ \begin{array}{c} u_j \\ \phi_j \end{array} \right\} = \begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix}^{-1} \begin{bmatrix} [I] & [\bar{\ell}] \\ [0] & [I] \end{bmatrix} \begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix} \left\{ \begin{array}{c} u_i \\ \phi_i \end{array} \right\} + \begin{bmatrix} [T_e] & 0 \\ 0 & [T_e] \end{bmatrix}^{-1} [b]\{\varepsilon\}. \tag{7}$$

Here, Eq. (7) is an equation for relating strains field with their corresponding displacement DOFs.

3. STRAIN REDUCTION METHOD FOR PLANE FRAME

To give a clear understanding of the present Strain Reduction Method, a plane frame problem is described herein. By using the same formulations, spatial related DOFs are eliminated from the equations, hence yield to equations for plane formulations.

Components of frame structure such as beam and column are based on the $\bar{x}, \bar{y}$ coordinates as their local coordinate system. Fig. 3 illustrates the displacement vector from local coordinate system $\bar{x}, \bar{y}$ which is projected onto the global coordinate systems x, y, respectively.

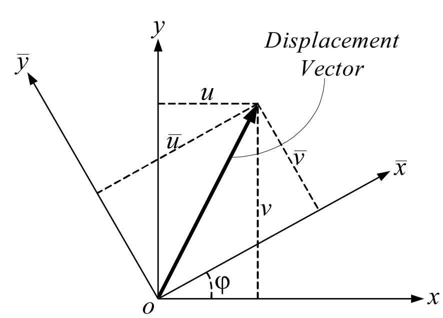

Fig. 3. Local and global coordinates of a displacement vector

Suppose, the coordinates of both i, j nodes are given as (x_i, y_i) and (x_j, y_j). The length ℓ and cosines directions of axis $\bar{x}$ of the structural element member can be calculated as

$$\begin{aligned}
&\ell = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2},\\
&\ell_{\bar{x}} = (x_j - x_i)/\ell,\\
&m_{\bar{x}} = (y_j - y_i)/\ell.
\end{aligned}$$

By assuming the displacement shapes at one end of the element member are: linear in terms of axial displacement DOF $\bar{u}$; and cubic in terms of transversal displacement DOF $\bar{v}$, the strain field along the element member can expressed as

$$\varepsilon = \frac{d\bar{u}}{d\bar{x}}, \quad \theta = \frac{d\bar{v}}{d\bar{x}}, \quad \kappa = \frac{d\bar{\theta}}{d\bar{x}}, \quad \gamma = \frac{d\kappa}{d\bar{x}}, \tag{8}$$

with the element member length ℓ and both member ends DOFs $i, j\,(j > i)$ are given as $\bar{u}_i, \bar{u}_j, \bar{v}_i, \bar{v}_j, \bar{\theta}_i, \bar{\theta}_j, \kappa_i, \kappa_j$.

By integrating each equation in Eq. (8) and substituting the DOFs at both ends as the integration constants, the displacement and rotational DOFs which related to the strains field along the element member can be expressed in a matrix form as follow

$$\begin{Bmatrix} \bar{u}_j \\ \bar{v}_j \\ \bar{\theta}_j \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \ell \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{\theta}_i \end{Bmatrix} + \begin{bmatrix} \ell & 0 & 0 \\ 0 & -\frac{1}{2}\ell^2 & \frac{1}{12}\ell^3 \\ 0 & -\ell & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \bar{\kappa} \\ \gamma \end{Bmatrix} \tag{9}$$

where

$\bar{\kappa} = \dfrac{1}{2}(\kappa_i + \kappa_j)$ is the average bending strain,

$\gamma = \dfrac{1}{\ell}(\kappa_j - \kappa_i)$ is the shear strain.

The displacements and rotational DOFs u, v, θ of beam and column element can be transformed to the global coordinate system by the following relationship

$$\begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{\theta}_i \end{Bmatrix} = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \theta_i \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \bar{u}_j \\ \bar{v}_j \\ \bar{\theta}_j \end{Bmatrix} = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_j \\ v_j \\ \theta_j \end{Bmatrix} \tag{10}$$

where $c = \cos\varphi$ and $s = \sin\varphi$.

By substituting Eq. (10) into Eq. (9), an equation for converting strains field to their corresponding displacement fields yields to

$$\begin{Bmatrix} u_j \\ v_j \\ \theta_j \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\ell s \\ 0 & 1 & \ell c \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \theta_i \end{Bmatrix} + \begin{bmatrix} \ell c & \frac{1}{2}\ell^2 s & -\frac{1}{12}\ell^3 s \\ \ell s & -\frac{1}{2}\ell^2 c & \frac{1}{12}\ell^3 c \\ 0 & -\ell & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \bar{\kappa} \\ \gamma \end{Bmatrix} \tag{11}$$

where, $u_i, v_i, u_j, v_j, \theta_i, \theta_j$ are the displacements and rotations in global coordinate system x, y and $\varepsilon, \bar{\kappa}, \gamma$ are the axial strain, average bending strain and shear strain, respectively.

4. PLANE FRAME EXAMPLE

Fig. 4 shows a symmetric and simple 5-story steel plane frame structure and nodal numbering scheme.

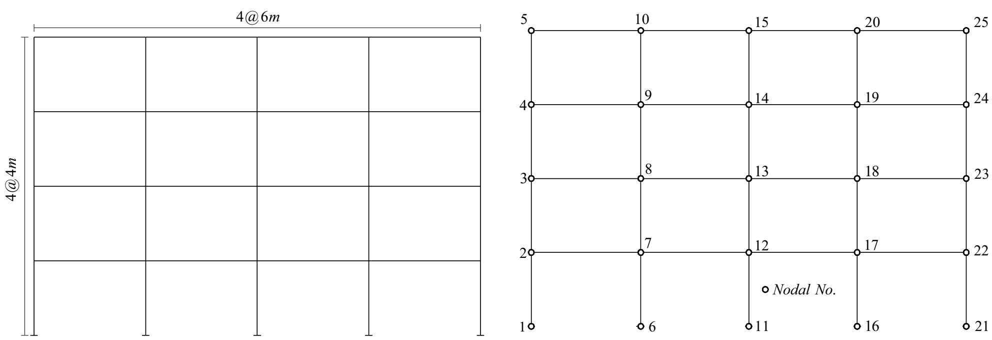

Fig. 4. Plane frame dimension and nodal numbering scheme

Table 1. Sectional Properties

Member	Floor	Sectional Property
Beam	1 to 5	H-522X470X20X35
Column	1 to 5	H-600X300X12X25

The structural member properties for the plane frame structural members are listed in Tab. 1.

4.1. Determining representative nodes

In order to introduce the computation procedure of the Strain Reduction Method, a global representative node needs to be selected from the structure. From where, this node will be the target of condensation from others DOFs which are reduced into their strains field beforehand. Fig. 5 shows the frame model with their type of representative nodes to demonstrate the applicability and validity of the current introduced Strain Reduction Method.

The way to reduce all the DOFs in the plane frame into their corresponding strains field will be given in several stages transformation as will be given from the following sections.

4.2. Transforming Horizontal Displacements

Firstly, nodes at the middle columns with horizontal displacement DOFs u_{12}, u_{13}, u_{14}, u_{15} are selected as representative nodes. Then, nodes with horizontal displacement DOFs at each floor such as beams are transformed into strains field and then condensed into their representative node at its floor.

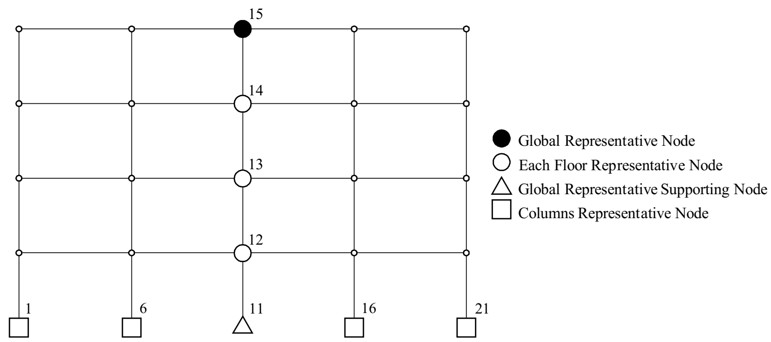

Fig. 5. Representative nodal scheme for plane frame example

The algorithms of transformation of horizontal displacement DOFs into their strains are given as follow

At 2$^{\text{nd}}$ Floor, $u_2, u_7, u_{17}, u_{22} \rightarrow u_{12}, \varepsilon_{2,7}, \varepsilon_{7,12}, \varepsilon_{12,17}, \varepsilon_{17,22}$

At 3$^{\text{rd}}$ Floor, $u_3, u_4, u_{18}, u_{23} \rightarrow u_{13}, \varepsilon_{3,8}, \varepsilon_{8,13}, \varepsilon_{13,18}, \varepsilon_{18,23}$

At 4$^{\text{th}}$ Floor, $u_4, u_9, u_{19}, u_{24} \rightarrow u_{14}, \varepsilon_{4,9}, \varepsilon_{9,14}, \varepsilon_{14,19}, \varepsilon_{19,24}$

At 5$^{\text{th}}$ Floor, $u_5, u_{10}, u_{20}, u_{25} \rightarrow u_{15}, \varepsilon_{5,10}, \varepsilon_{10,15}, \varepsilon_{15,20}, \varepsilon_{20,25}$

By using Eq. (11) with $\varphi = 0$, transformation of horizontal displacement DOF between i node and j node can be defined as

$$u_j = u_i + \ell_{ij}\varepsilon_{ij} \quad \text{or} \quad u_i = u_j - \ell_{ij}\varepsilon_{ij}$$

Accordingly, the transformation of horizontal displacements DOFs of all floors are given as follow

$$\begin{aligned}
&u_2 = u_7 - \ell_{2,7}\varepsilon_{2,7} = u_{12} - \ell_{2,7}\varepsilon_{2,7} - \ell_{7,12}\varepsilon_{7,12}\\
&u_7 = u_{12} - \ell_{7,12}\varepsilon_{7,12}\\
&u_{17} = u_{12} + \ell_{12,17}\varepsilon_{12,17}\\
&u_{22} = u_{17} + \ell_{17,22}\varepsilon_{17,22} = u_{12} + \ell_{12,17}\varepsilon_{12,17} + \ell_{17,22}\varepsilon_{17,22}\\
&u_3 = u_8 - \ell_{3,8}\varepsilon_{3,8} = u_{13} - \ell_{3,8}\varepsilon_{3,8} - \ell_{8,13}\varepsilon_{8,13}\\
&u_8 = u_{13} - \ell_{8,13}\varepsilon_{8,13}\\
&u_{18} = u_{13} + \ell_{13,18}\varepsilon_{13,18}\\
&u_{23} = u_{18} + \ell_{18,23}\varepsilon_{18,23} = u_{13} + \ell_{13,18}\varepsilon_{13,18} + \ell_{18,23}\varepsilon_{18,23}\\
&u_4 = u_9 - \ell_{4,9}\varepsilon_{4,9} = u_{14} - \ell_{4,9}\varepsilon_{4,9} - \ell_{9,14}\varepsilon_{9,14}\\
&u_9 = u_{14} - \ell_{9,14}\varepsilon_{9,14}\\
&u_{19} = u_{14} + \ell_{14,19}\varepsilon_{14,19}\\
&u_{24} = u_{19} + \ell_{19,24}\varepsilon_{19,24} = u_{14} + \ell_{14,19}\varepsilon_{14,19} + \ell_{19,24}\varepsilon_{19,24}\\
&u_5 = u_{10} - \ell_{5,10}\varepsilon_{5,10} = u_{15} - \ell_{5,10}\varepsilon_{5,10} - \ell_{10,15}\varepsilon_{10,15}\\
&u_{10} = u_{15} - \ell_{10,15}\varepsilon_{10,15}\\
&u_{20} = u_{15} + \ell_{15,20}\varepsilon_{15,20}\\
&u_{25} = u_{20} + \ell_{20,25}\varepsilon_{20,25} = u_{15} + \ell_{15,20}\varepsilon_{15,20} + \ell_{20,25}\varepsilon_{20,25}
\end{aligned}$$

which can be expressed in the following matrix form equation

$$\{U\} = \left[{}^{1}T\right]\{E_{u\varepsilon}\}. \tag{12}$$

4.3. Transforming Vertical Displacements

Next, nodes at the bottom of columns with vertical displacement DOFs $v_1, v_6, v_{11}, v_{16}, v_{21}$ are selected as representative nodes. Then, nodes with vertical displacement DOFs at each column are transformed into strains field and then condensed into their representative node at the bottom of the columns.

The algorithms of transformation of vertical displacement DOFs into strains are given as

At 1^{st} Columns, $v_2, v_3, v_4, v_5 \to v_1, \varepsilon_{1,2}, \varepsilon_{2,3}, \varepsilon_{3,4}, \varepsilon_{4,5}$

At 2^{nd} Columns, $v_7, v_8, v_9, v_{10} \to v_6, \varepsilon_{6,7}, \varepsilon_{7,8}, \varepsilon_{8,9}, \varepsilon_{9,10}$

At 3^{rd} Columns, $v_{12}, v_{13}, v_{14}, v_{15} \to v_{11}, \varepsilon_{11,12}, \varepsilon_{12,13}, \varepsilon_{13,14}, \varepsilon_{14,15}$

At 4^{th} Columns, $v_{17}, v_{18}, v_{19}, v_{20} \to v_{16}, \varepsilon_{16,17}, \varepsilon_{17,183}, \varepsilon_{18,19}, \varepsilon_{19,20}$

At 5^{th} Columns, $v_{22}, v_{23}, v_{24}, v_{25} \to v_{21}, \varepsilon_{21,22}, \varepsilon_{22,23}, \varepsilon_{23,24}, \varepsilon_{24,25}$

By using Eq. (11) with $\varphi = \pi/2$, transformation of vertical displacement DOF between i node and j node can be defined as

$$v_j = v_i + \ell_{ij}\varepsilon_{ij} \quad or \quad v_i = v_j - \ell_{ij}\varepsilon_{ij}.$$

Accordingly, the transformation of vertical displacement DOFs of columns are given as

$$
\begin{aligned}
&v_2 = v_1 + \ell_{1,2}\varepsilon_{1,2}\\
&v_3 = v_2 + \ell_{2,3}\varepsilon_{2,3} = v_1 + \ell_{1,2}\varepsilon_{1,2} + \ell_{2,3}\varepsilon_{2,3}\\
&v_4 = v_3 + \ell_{3,4}\varepsilon_{3,4} = v_1 + \ell_{1,2}\varepsilon_{1,2} + \ell_{2,3}\varepsilon_{2,3} + \ell_{3,4}\varepsilon_{3,4}\\
&v_5 = v_4 + \ell_{4,5}\varepsilon_{4,5} = v_1 + \ell_{1,2}\varepsilon_{1,2} + \ell_{2,3}\varepsilon_{2,3} + \ell_{3,4}\varepsilon_{3,4} + \ell_{4,5}\varepsilon_{4,5}\\
&v_7 = v_6 + \ell_{6,7}\varepsilon_{6,7}\\
&v_8 = v_7 + \ell_{7,8}\varepsilon_{7,8} = v_6 + \ell_{6,7}\varepsilon_{6,7} + \ell_{7,8}\varepsilon_{7,8}\\
&v_9 = v_8 + \ell_{8,9}\varepsilon_{8,9} = v_6 + \ell_{6,7}\varepsilon_{6,7} + \ell_{7,8}\varepsilon_{7,8} + \ell_{8,9}\varepsilon_{8,9}\\
&v_{10} = v_9 + \ell_{9,10}\varepsilon_{9,10} = v_6 + \ell_{6,7}\varepsilon_{6,7} + \ell_{7,8}\varepsilon_{7,8} + \ell_{8,9}\varepsilon_{8,9} + \ell_{9,10}\varepsilon_{9,10}\\
&v_{12} = v_{11} + \ell_{11,12}\varepsilon_{11,12}\\
&v_{13} = v_{12} + \ell_{12,13}\varepsilon_{12,13} = v_{11} + \ell_{11,12}\varepsilon_{11,12} + \ell_{12,13}\varepsilon_{12,13}\\
&v_{14} = v_{13} + \ell_{13,14}\varepsilon_{13,14} = v_{11} + \ell_{11,12}\varepsilon_{11,12} + \ell_{12,13}\varepsilon_{12,13} + \ell_{13,14}\varepsilon_{13,14}\\
&v_{15} = v_{14} + \ell_{14,15}\varepsilon_{14,15} = v_{11} + \ell_{11,12}\varepsilon_{11,12} + \ell_{12,13}\varepsilon_{12,13} + \ell_{13,14}\varepsilon_{13,14} + \ell_{14,15}\varepsilon_{14,15}\\
&v_{17} = v_{16} + \ell_{16,17}\varepsilon_{16,17}\\
&v_{18} = v_{17} + \ell_{17,18}\varepsilon_{17,18} = v_{16} + \ell_{16,17}\varepsilon_{16,17} + \ell_{17,18}\varepsilon_{17,18}\\
&v_{19} = v_{18} + \ell_{18,19}\varepsilon_{18,19} = v_{16} + \ell_{16,17}\varepsilon_{16,17} + \ell_{17,18}\varepsilon_{17,18} + \ell_{18,19}\varepsilon_{18,19}\\
&v_{20} = v_{19} + \ell_{19,20}\varepsilon_{19,20} = v_{16} + \ell_{16,17}\varepsilon_{16,17} + \ell_{17,18}\varepsilon_{17,18} + \ell_{18,19}\varepsilon_{18,19} + \ell_{19,20}\varepsilon_{19,20}\\
&v_{22} = v_{21} + \ell_{21,22}\varepsilon_{21,22}\\
&v_{23} = v_{22} + \ell_{22,23}\varepsilon_{22,23} = v_{21} + \ell_{21,22}\varepsilon_{21,22} + \ell_{22,23}\varepsilon_{22,23}\\
&v_{24} = v_{23} + \ell_{23,24}\varepsilon_{23,24} = v_{21} + \ell_{21,22}\varepsilon_{21,22} + \ell_{22,23}\varepsilon_{22,23} + \ell_{23,24}\varepsilon_{23,24}\\
&v_{25} = v_{24} + \ell_{24,25}\varepsilon_{24,25} = v_{21} + \ell_{21,22}\varepsilon_{21,22} + \ell_{22,23}\varepsilon_{22,23} + \ell_{23,24}\varepsilon_{23,24} + \ell_{24,25}\varepsilon_{24,25}
\end{aligned}
$$

which can be expressed in the following matrix form equation

$$\{V\} = \left[{}^{2}T\right]\{E_{v\varepsilon}\}. \tag{13}$$

4.4. Horizontal Displacement to Rotations and Shear Strain Transformation

Next, the horizontal displacement DOFs of 2^{nd}, 3^{rd} and 4^{th} floors u_{12}, u_{13}, u_{14} are transformed into a horizontal displacement u_{11}, rotations $\theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}$ and shear strain

$\gamma_{11,12}, \gamma_{12,13}, \gamma_{13,14}$ by using Eq. (11) with $\varphi = \pi/2$. The transformation from different type of displacements between node i and node j takes the forms

$$u_j = u_i - \ell_{ij}\theta_i + \tfrac{1}{2}\ell_{ij}^2\bar{\kappa}_{ij} - \tfrac{1}{12}\ell_{ij}^3\gamma_{ij} \quad \text{and} \quad \theta_j = \theta_i - \ell_{ij}\bar{\kappa}_{ij} \quad \text{or} \quad \bar{\kappa}_{ij} = \frac{(\theta_i - \theta_j)}{\ell_{ij}}$$

which yields to, $u_j = u_i - \frac{1}{2}\ell_{ij}(\theta_i + \theta_j) - \frac{1}{12}\ell_{ij}^3\gamma_{ij}$ then, gives the horizontal displacement DOFs of u_{12}, u_{13}, u_{14} as,

$$\begin{aligned}
&u_{12} = u_{11} - \tfrac{1}{2}\ell_{11,12}(\theta_{11} + \theta_{12}) - \tfrac{1}{12}\ell_{11,12}^3\gamma_{11,12} \\
&u_{13} = u_{11} - \tfrac{1}{2}\ell_{11,12}\theta_{11} - \tfrac{1}{2}(\ell_{11,12} + \ell_{12,13})\theta_{12} \\
&-\tfrac{1}{2}\ell_{12,13}\theta_{13} - \tfrac{1}{12}\ell_{11,12}^3\gamma_{11,12} - \tfrac{1}{12}\ell_{12,13}^3\gamma_{12,13} \\
&u_{14} = u_{11} - \tfrac{1}{2}\ell_{11,12}\theta_{11} - \tfrac{1}{2}(\ell_{11,12} + \ell_{12,13})\theta_{12} \\
&-\tfrac{1}{2}\ell_{12,13}\theta_{13} - \tfrac{1}{2}\ell_{13,14}(\theta_{13} + \theta_{14}) \\
&-\tfrac{1}{12}\ell_{11,12}^3\gamma_{11,12} - \tfrac{1}{12}\ell_{12,13}^3\gamma_{12,13} - \tfrac{1}{12}\ell_{13,14}^3\gamma_{13,14}
\end{aligned}$$

which can be expressed in a following matrix form equation.

$$\{\Phi_u\} = \left[{}^3T\right]\{E_{u\gamma\theta}\} \tag{14}$$

4.5. Transforming Rotational Displacements

Finally, rotational and displacement DOFs of all nodes at columns are transformed into their bending strains that will be represented by the nodes at bottom of the columns.

The algorithms of transformation of rotational displacements into its bending strains are given as follow

At 1st Columns, $\theta_2, \theta_3, \theta_4, \theta_5 \rightarrow \theta_1, \bar{\kappa}_{1,2}, \bar{\kappa}_{2,3}, \bar{\kappa}_{3,4}, \bar{\kappa}_{4,5}$

At 2nd Columns, $\theta_7, \theta_8, \theta_9, \theta_{10} \rightarrow \theta_6, \bar{\kappa}_{6,7}, \bar{\kappa}_{7,8}, \bar{\kappa}_{8,9}, \bar{\kappa}_{9,10}$

At 3rd Columns, $\theta_{12}, \theta_{13}, \theta_{14}, \theta_{15} \rightarrow \theta_{11}, \bar{\kappa}_{11,12}, \bar{\kappa}_{12,13}, \bar{\kappa}_{13,14}, \bar{\kappa}_{14,15}$

At 4th Columns, $\theta_{17}, \theta_{18}, \theta_{19}, \theta_{20} \rightarrow \theta_{16}, \bar{\kappa}_{16,17}, \bar{\kappa}_{17,18}, \bar{\kappa}_{18,19}, \bar{\kappa}_{19,20}$

At 5th Columns, $\theta_{22}, \theta_{23}, \theta_{24}, \theta_{25} \rightarrow \theta_{21}, \bar{\kappa}_{21,22}, \bar{\kappa}_{22,23}, \bar{\kappa}_{23,24}, \bar{\kappa}_{24,25}$

By using Eq. (11), transformation of rotational displacement DOF between i node and j node can be defined as

$$\theta_j = \theta_i - \ell_{ij}\bar{\kappa}_{ij}.$$

Accordingly, the transformation of rotational displacement DOFs of all columns are given as follow

$$\begin{aligned}
&\theta_2 = \theta_1 + \ell_{1,2}\bar{\kappa}_{1,2} \\
&\theta_3 = \theta_2 + \ell_{2,3}\bar{\kappa}_{2,3} = \theta_1 + \ell_{1,2}\bar{\kappa}_{1,2} + \ell_{2,3}\bar{\kappa}_{2,3} \\
&\theta_4 = \theta_3 + \ell_{3,4}\varepsilon_{3,4} = \theta_1 + \ell_{1,2}\bar{\kappa}_{1,2} + \ell_{2,3}\bar{\kappa}_{2,3} + \ell_{3,4}\bar{\kappa}_{3,4} \\
&\theta_5 = \theta_4 + \ell_{4,5}\varepsilon_{4,5} = \theta_1 + \ell_{1,2}\bar{\kappa}_{1,2} + \ell_{2,3}\bar{\kappa}_{2,3} + \ell_{3,4}\bar{\kappa}_{3,4} + \ell_{4,5}\bar{\kappa}_{4,5} \\
&\theta_7 = \theta_6 + \ell_{6,7}\bar{\kappa}_{6,7} \\
&\theta_8 = \theta_7 + \ell_{7,8}\bar{\kappa}_{7,8} = \theta_6 + \ell_{6,7}\bar{\kappa}_{6,7} + \ell_{7,8}\bar{\kappa}_{7,8} \\
&\theta_9 = \theta_8 + \ell_{8,9}\bar{\kappa}_{8,9} = \theta_6 + \ell_{6,7}\bar{\kappa}_{6,7} + \ell_{7,8}\bar{\kappa}_{7,8} + \ell_{8,9}\bar{\kappa}_{8,9}
\end{aligned}$$

$$
\begin{aligned}
&\theta_{10} = \theta_9 + \ell_{9,10}\bar{\kappa}_{9,10} = \theta_6 + \ell_{6,7}\bar{\kappa}_{6,7} + \ell_{7,8}\bar{\kappa}_{7,8} + \ell_{8,9}\bar{\kappa}_{8,9} + \ell_{9,10}\bar{\kappa}_{9,10}\\
&\theta_{12} = \theta_{11} + \ell_{11,12}\bar{\kappa}_{11,12}\\
&\theta_{13} = \theta_{12} + \ell_{12,13}\bar{\kappa}_{12,13} = \theta_{11} + \ell_{11,12}\bar{\kappa}_{11,12} + \ell_{12,13}\bar{\kappa}_{12,13}\\
&\theta_{14} = \theta_{13} + \ell_{13,14}\bar{\kappa}_{13,14} = \theta_{11} + \ell_{11,12}\bar{\kappa}_{11,12} + \ell_{12,13}\bar{\kappa}_{12,13} + \ell_{13,14}\bar{\kappa}_{13,14}\\
&\theta_{15} = \theta_{14} + \ell_{14,15}\bar{\kappa}_{14,15} = \theta_{11} + \ell_{11,12}\bar{\kappa}_{11,12} + \ell_{12,13}\bar{\kappa}_{12,13} + \ell_{13,14}\bar{\kappa}_{13,14} + \ell_{14,15}\bar{\kappa}_{14,15}\\
&\theta_{17} = \theta_{16} + \ell_{16,17}\bar{\kappa}_{16,17}\\
&\theta_{18} = \theta_{17} + \ell_{17,18}\bar{\kappa}_{17,18} = \theta_{16} + \ell_{16,17}\bar{\kappa}_{16,17} + \ell_{17,18}\bar{\kappa}_{17,18}\\
&\theta_{19} = \theta_{18} + \ell_{18,19}\bar{\kappa}_{18,19} = \theta_{16} + \ell_{16,17}\bar{\kappa}_{16,17} + \ell_{17,18}\bar{\kappa}_{17,18} + \ell_{18,19}\bar{\kappa}_{18,19}\\
&\theta_{20} = \theta_{19} + \ell_{19,20}\bar{\kappa}_{19,20} = \theta_{16} + \ell_{16,17}\bar{\kappa}_{16,17} + \ell_{17,18}\bar{\kappa}_{17,18} + \ell_{18,19}\bar{\kappa}_{18,19} + \ell_{19,20}\bar{\kappa}_{19,20}\\
&\theta_{22} = \theta_{21} + \ell_{21,22}\bar{\kappa}_{21,22}\\
&\theta_{23} = \theta_{22} + \ell_{22,23}\bar{\kappa}_{22,23} = \theta_{21} + \ell_{21,22}\bar{\kappa}_{21,22} + \ell_{22,23}\bar{\kappa}_{22,23}\\
&\theta_{24} = \theta_{23} + \ell_{23,24}\bar{\kappa}_{23,24} = \theta_{21} + \ell_{21,22}\bar{\kappa}_{21,22} + \ell_{22,23}\bar{\kappa}_{22,23} + \ell_{23,24}\bar{\kappa}_{23,24}\\
&\theta_{25} = \theta_{24} + \ell_{24,25}\bar{\kappa}_{24,25} = \theta_{21} + \ell_{21,22}\bar{\kappa}_{21,22} + \ell_{22,23}\bar{\kappa}_{22,23} + \ell_{23,24}\bar{\kappa}_{23,24} + \ell_{24,25}\bar{\kappa}_{24,25}
\end{aligned}
$$

which can be expressed in the following matrix form equation

$$\{\Phi\} = \left[{}^4T\right]\{E_{\theta\bar{\kappa}}\}. \tag{15}$$

4.6. Global Transformation Matrix

The transformation matrices $\left[{}^1T\right], \left[{}^2T\right], \left[{}^3T\right], \left[{}^4T\right]$ for strain reduction method in Eq. (12) to Eq. (15) are shown in the matrix forms for clarity purpose only, the matrices must not be used directly, instead the contents inside the matrices have to be assembled corresponding to the DOFs in the global MDOF system, then multiplied as follow

$$[T_G] = \left[{}^1T_G\right] \times \left[{}^2T_G\right] \times \left[{}^3T_G\right] \times \left[{}^4T_G\right]$$

where,

$[T_G]$ is transformation matrix for the whole DOFs of MDOF system of the frame,

$\left[{}^1T_G\right], \left[{}^2T_G\right], \left[{}^3T_G\right], \left[{}^4T_G\right]$ are the transformation matrices $\left[{}^1T\right], \left[{}^2T\right], \left[{}^3T\right], \left[{}^4T\right]$ in Eq. (12) to Eq. (15) which have been assembled into their corresponding global MDOF system. The transformation from the displacement and rotational DOFs to their strains fields is then, given by the following relationship

$$\{\mathbf{u}\} = [T_G]\{\mathbf{e}\} \tag{16}$$

where,

$\{\mathbf{u}\}$ is the displacements and rotational DOFs vector consists of u, v, θ DOFs,

$\{\mathbf{e}\}$ is the strains field and rotational DOFs vector consists of $\varepsilon, \gamma, \theta, \bar{\kappa}$ DOFs.

From the principle virtual work, the dynamic equation of motion in terms of displacement field $\{\mathbf{u}\}$ can be expressed in terms of strains field $\{\mathbf{e}\}$ by using the transformation relationship in Eq. (16) as

$$[M]\{\ddot{\mathbf{e}}\} + [C]\{\dot{\mathbf{e}}\} + [K]\{\mathbf{e}\} = [P] \tag{17}$$

where,

$$\begin{aligned} [M] &= [T_G]^T [m] [T_G], \\ [C] &= [T_G]^T [c] [T_G], \\ [K] &= [T_G]^T [k] [T_G], \\ [P] &= [T_G]^T \{p\} \end{aligned}$$

$\{\ddot{\mathbf{e}}\}, \{\dot{\mathbf{e}}\}$ are twice and once derivation of strains field $\{\mathbf{e}\}$ in time,

$[m], [c], [k]$ and $\{p\}$ are mass matrix, damping matrix, stiffness matrix and external force vector of the original $\{\mathbf{u}\}$ DOFs in global coordinate system before being transformed into their strains field DOFs.

4.7. Reduction of the Dynamic Equation of Motion

The reduction procedure to Eq. (17) is made by separating it into the main and secondary DOFs equations. After separating the dynamic equation of motion, the secondary equation is then, differentiated by time to the 3$^{\text{rd}}$ and 4$^{\text{th}}$ order and substituted back to the main equation which yields to a SDOF system of dynamic equation of motion.

Consider a dynamic equation of motion which can be separated into main, secondary DOFs and their corresponding external force DOFs. The dynamic equation of motion [5-9] can be written as follow

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} \tag{18}$$

where, $\dot{x} = dx/dt$, $\ddot{x} = d^2x/dt^2$, $t =$ time. Expanding the above equation resulted in

$$m_{11}\ddot{e}_1 + m_{12}\ddot{e}_2 + c_{11}\dot{e}_1 + c_{12}\dot{e}_2 + k_{11}e_1 + k_{12}e_2 = p_1 \tag{19}$$

$$m_{21}\ddot{e}_1 + m_{22}\ddot{e}_2 + c_{21}\dot{e}_1 + c_{22}\dot{e}_2 + k_{21}e_1 + k_{22}e_2 = p_2 \tag{20}$$

Rewriting and taking twice differentiations respect to time against the Eq. (20), and neglecting higher orders terms, results in the following equations

$$m_{22}\ddot{e}_2 + c_{22}\dot{e}_2 + k_{22}e_2 = p_2 - m_{21}\ddot{e}_1 - c_{21}\dot{e}_1 - k_{21}e_1 \tag{21}$$

$$c_{22}\ddot{e}_2 + k_{22}\dot{e}_2 = -c_{21}\ddot{e}_1 - k_{21}\dot{e}_1 \tag{22}$$

$$k_{22}\ddot{e}_2 = -k_{21}\ddot{e}_1 \tag{23}$$

The purpose transforming the displacement fields into strain fields is to reduce errors due to neglecting the higher order terms in Eqs. (21-23), since the strain fields are in lower order compared to the displacement fields.

Substitution of Eqs. (21-23) into Eq. (19), eliminating the $\ddot{e}_2, \dot{e}_2, e_2$ terms, results in the following SDOF dynamic equation of the main displacement as

$$\begin{aligned} &(m_{11} - m_{12}k_{22}^{-1}k_{21} - c_{12}k_{22}^{-1}c_{21} - k_{12}k_{22}^{-1}m_{21} + c_{12}k_{22}^{-1}c_{22}k_{22}^{-1}k_{21} \\ &+k_{12}k_{22}^{-1}m_{22}k_{22}^{-1}k_{21} + k_{12}k_{22}^{-1}c_{22}k_{22}^{-1}c_{21} - k_{12}k_{22}^{-1}c_{22}k_{22}^{-1}c_{22}k_{22}^{-1}k_{21})\,\ddot{e}_1 \\ &\quad + \left(c_{11} - c_{12}k_{22}^{-1}k_{21} - k_{12}k_{22}^{-1}c_{21} + k_{12}k_{22}^{-1}c_{22}k_{22}^{-1}k_{21}\right)\dot{e}_1 \\ &\quad + \left(k_{11} - k_{12}k_{22}^{-1}k_{21}\right)e_1 = p_1 - k_{12}k_{22}^{-1}p_2 \end{aligned} \tag{24}$$

After the SDOF equation is solved for dynamic problems, the secondary equation can be obtained easily by substituting back the solutions. Reversely, if required the solutions to the original equation of motion can be obtained by using Eq. (16).

5. NUMERICAL STUDY

5.1. Model Analysis and Dynamic Properties

After all the DOFs are transformed into their strains field, reduction from MDOF system of plane frame into a SDOF system as shown in Fig. 6, resulted in an SDOF system with equation of motion parameters and dynamical properties which are given in Tab. 2 and Tab. 3.

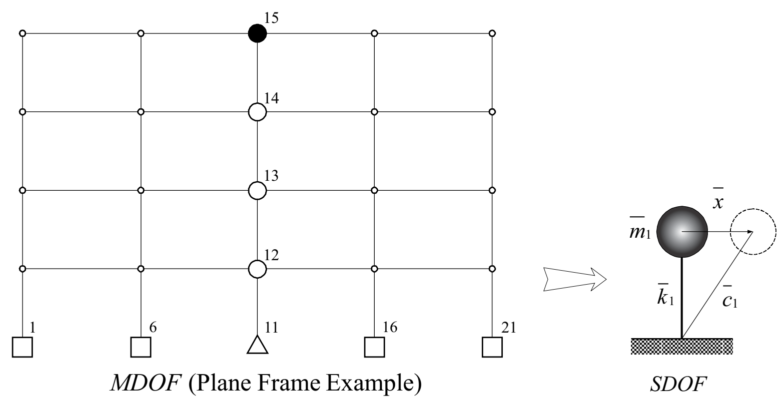

Fig. 6. Illustration of Reduction Method from MDOF to SDOF systems

Table 2. Equation of Motion Parameters

SDOF System	
Mass, M	2,355.7 kg
Damping, C	58,894.2 N.s/mm
Stiffness, K	635,848.3 N/mm
Initial Mass, M_0	3,531.6 kg

Table 3. Dynamic Properties

SDOF System	
Natural Frequency, ω	3.15 rad
Period, T	2.00 sec
Frequency, f	0.50 Hz

5.2. Time History Analysis

An initial 5% critical ratio of viscous damping is used in the dynamic analysis. This viscous damping is assumed linearly depend on the stiffness matrices. The structures are subjected to the seismic ground acceleration of El-Centro Earthquake 1940 North-South components with time interval of 0.02 sec. The time history analysis responses of the structures are evaluated by using the linear acceleration step-by-step method [10] with the time integration of 0.001 sec.

5.3. Time History Analysis Responses

The results of time history analysis both the MDOF (plane frame example) and its reduced SDOF models are compared. The displacements, velocities and accelerations of the SDOF mass and the representative node $u_{15}, \dot{u}_{15}, \ddot{u}_{15}$ in the MDOF system are depicted in Fig. 7, Fig. 8 and Fig. 9, respectively.

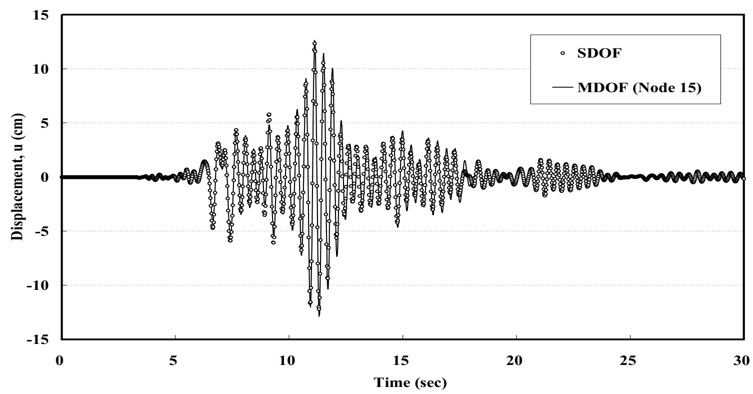

Fig. 7. Response Displacements

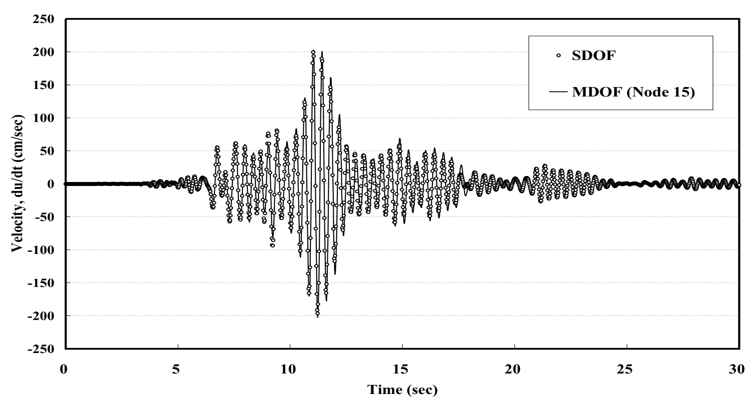

Fig. 8. Response Velocities

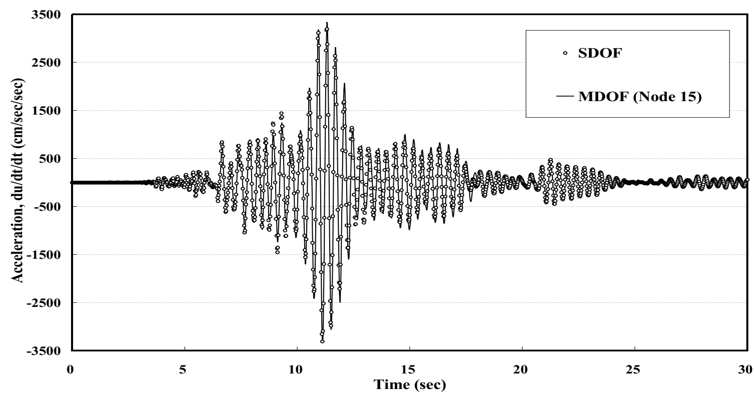

Fig. 9. Response Accelerations

5.4. Comparison of Results

Comparison of responses between the reduced SDOF system and the MDOF system are given in Tab. 4. From Tab. 4, the maximum error of 4.3%, 1.7% and 2.3% are observed from the results of predictions to the MDOF behavior at Node 15 by SDOF behavior for the horizontal displacement, velocity and acceleration, respectively.

Table 4. Comparison of Time History Analysis Results

Maximum Value	SDOF	MDOF at Node 15	% Error
Displacement, u_{15}, (cm)	12.59	12.88	2.3
Velocity, $\dot{u}_{15}$, (cm/sec)	199.66	203.02	1.7
Acceleration, $\ddot{u}_{15}$, (cm/sec^2)	3,196.7	3,341.9	4.3

From the example illustrated, the maximum error of 2.3% for horizontal displacement was observed. This error is negligible and relatively small, thus acceptable for design works. These errors were the results of neglecting the small order of strains field during the reduction process as discussed in [11-13]. Therefore, the present Strain Reduction Method shows satisfactory results in predicting the behavior of the MDOF system by the reduced SDOF system.

From the results of the SDOF system, by following the procedure discussed in [5-9], the other DOFs can easily be recovered to obtain the time history for each DOF in the MDOF system.

6. CONCLUSIONS

A dynamic reduction method for reducing MDOF system to SDOF system of frame structures has been presented. The method is straightforward and easy to be implemented into a computer code. The numerical example show the efficiency and accuracy of the propose method. The main conclusion of the paper can be summarized as follows.

The Strain Reduction Method is simple and easy to be used in practical dynamic analysis design of frame structures.

The main feature in the present method is that its capability in predicting the time history results of dynamic responses by using SDOF system and recoverable to its MDOF system by using matrices derived, not only predicting the maximum values like the other practical design method does.

The proposed method also showed its soundness and accuracy in predicting the behavior of MDOF plane frame structure within tolerable error for design works.

Automation in searching for representative nodes for the present method is considered as a crucial task, thus future research will be focused on solving this issue.

Verifications and validations by using other structural models and conditions are required for future enhancement of the proposed method.

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