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NON - LINEAR VIBRATION OF FUNCTIONALLY GRADED SHALLOW SPHERICAL SHELLS

Dao Huy Bich[1] **and Le Kha Hoa**[2]
[1] *Vietnam National University, Hanoi*
[2] *Military Logistic Academy*

Abstract. The present paper deals with the non-linear vibration of functionally graded shallow spherical shells. The properties of shell material are graded in the thickness direction according to the power law distribution in terms of volume fractions of the material constituents. In the derived governing equations geometric non-linearity in all strain-displacement relations of the shell is considered. From the deformation compatibility equation and the motion equation a system of partial differential equations for stress function and deflection of shell is obtained. The Galerkin method and Runge-Kutta method are used for dynamical analysis of shells to give expressions of natural frequencies and non-linear dynamic responses. Numerical results show the essential influence of characteristics of functionally graded materials and dimension ratios on the dynamical behaviors of shells.

1. INTRODUCTION

In recent years, functionally graded materials (FGMs) have gained considerable attention in the engineering applications. FGMs are microscopically inhomogeneous, in which the material properties vary smoothly and continuously from one surface to the other. Studies on the stability and vibration of functionally graded plates, cylindrical and shallow shells have been carried out. About vibration of FGM plates Vel S.S. and Batra R.C. [1] gave three dimensional exact solution for the vibration of FGM rectangular plates; Ferreira A.J. et al. [2] received natural frequencies of FGM plates by meshless method. Natural frequencies and buckling stresses of FGM plates and shallow shells were analyzed by Hiroyuki Matsunaga [3,4] using 2-D higher-order deformation theory. Pradyumna S. and Bandyspadhyay J.N. received natural frequencies of FGM curved panels using high-order finite element formulation [5]. Pradhan S.C et al. [6] and Loy C.T et al. [7] studied vibration of FGM cylindrical shells and the effects of boundary conditions and power law indices on the natural frequencies of shells; Yang Y. and Shen H.S. [8] studied free vibration and parametric response of shear deformable FGM cylindrical panels; Reddy J.N and Cheng Z.Q [9] gave frequency correspondence between membranes and functionally graded spherical shallow shell of polygonal plan form; Dao H.B. and Vu D. L. [10] studied dynamical behaviors of FGM shallow shells with geometrical imperfections and obtained dynamic responses and dynamic critical buckling loads of cylindrical and doubly-curved shallow shells subjected to dynamic loading Research on the stability of FGM cylindrical shells under aperiodic axial impulse and dynamic torsional loading can be seen in the works of Sofiyev A.H. et al. [11, 12]. Non-linear buckling analysis of FGM shallow spherical shells under pressure loads was presented by Ganapathi M. [13] by using finite element method, geometric non-linearity is assumed only on the meridional direction in strain-displacement relations. Dao H.B. [14] studied this problem using the approximated analytical method, geometric non-linearity is assumed in all strain-displacement relations.

The present paper deals with the non-linear vibration of functionally graded material shallow spherical shells. Derivations of governing equations of shells considering geometric non-linearity are based on the shell theory according to the von Karman theory for moderately large deflection and small strain with the assumption of power law composition for the constituent materials. The natural frequencies and non-linear dynamic responses of FGM spherical shells subjected to pressure loading are considered. The effects of characteristics of functionally graded materials and dimension ratios of shells on their dynamical behaviors are investigated.

2. GOVERNING EQUATIONS

Consider an axisymmetric functionally graded shallow spherical shell of thickness h, base radius r_0, shell radius R with the coordinates φ, θ and z along the meridional, circumferential and radial-thickness directions respectively as shown in Fig. 1. The shell is made of a mixture of ceramic and metal with continuously varying volume fractions. The materials in outer and inner surfaces of the shell are ceramic and metal respectively. Assume that, the modulus of elasticity E and the mass density ρ of the mixture change in the thickness direction by the power law of distribution; while Poisson's ratio ν is assumed to be constant [15]

$$\begin{aligned} E(z) &= E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k, \\ \rho(z) &= \rho_m + (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^k, \\ \nu(z) &= \nu = const \end{aligned} \tag{1}$$

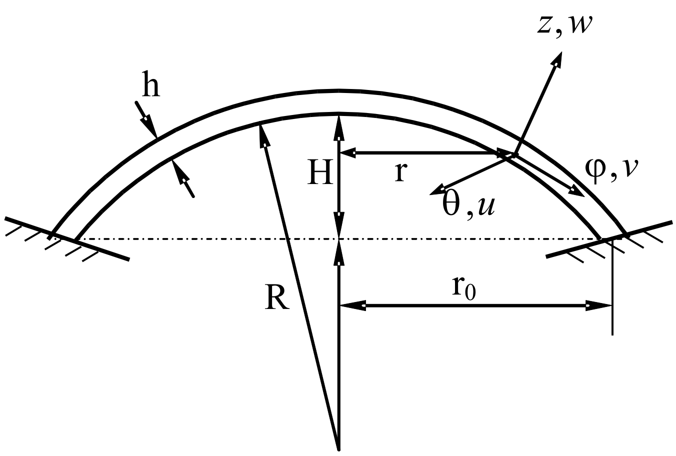

Fig. 1. Geometry of spherical cap

It is convenient to introduce an additional variable r defined by the relation $r = R\sin\varphi$, where r is the radius of the parallel circle. If the rise H of the shell is much smaller than the base radius r_0 we can take $\cos\varphi \approx 1$ and $Rd\varphi = dr$, such that points of the middle surface may be referred to coordinates r and θ.

The non-linear strain-displacement relationships based upon the von Karman theory for moderately large deflection and small strain are

$$\begin{aligned}
\varepsilon_r^0 &= \frac{\partial v}{\partial r} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2,\\
\varepsilon_\theta^0 &= \frac{1}{r}\left(\frac{\partial u}{\partial \theta} + v\right) - \frac{w}{R} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right)^2,\\
\gamma_{r\theta}^0 &= r\frac{\partial}{\partial r}\left(\frac{u}{r}\right) + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{r}\frac{\partial w}{\partial \theta}\frac{\partial w}{\partial r},
\end{aligned} \tag{2}$$

$$\begin{aligned}
\chi_r &= \frac{\partial^2 w}{\partial r^2},\\
\chi_\theta &= \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r}\frac{\partial w}{\partial r},\\
\chi_{r\theta} &= \frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta},
\end{aligned} \tag{3}$$

where u, v, w - displacements of the middle surface points along circumferential, meridional and radial directions respectively;

$\varepsilon_r^0, \varepsilon_\theta^0, \gamma_{r\theta}^0$ - strains in the middle surface;

$\chi_r, \chi_\theta, \chi_{r\theta}$ - changes of curvatures and twist.

They must be relative in the deformation compatibility equation

$$\frac{1}{r^2}\frac{\partial^2 \varepsilon_r^0}{\partial \theta^2} - \frac{1}{r}\frac{\partial \varepsilon_r^0}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \varepsilon_\theta^0}{\partial r}\right) - \frac{1}{r^2}\frac{\partial^2}{\partial r \partial \theta}(r\gamma_{r\theta}^0) = -\frac{\Delta w}{R} + \chi_{r\theta}^2 - \chi_r\chi_\theta, \tag{4}$$

where $\Delta = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \theta^2}$.

Integrating the stress-strain equations and their moments through the thickness of the shell ($-\frac{h}{2} \le z \le \frac{h}{2}$), taking into account the Young modulus $E(z)$ and the mass density $\rho(z)$ being power functions of z described by Eq.(1), we obtain the expressions of internal forces and moments resultants

$$\begin{aligned}
N_r &= \frac{E_1}{1-\nu^2}(\varepsilon_r^0 + \nu\varepsilon_\theta^0) - \frac{E_2}{1-\nu^2}(\chi_r + \nu\,\chi_\theta),\\
N_\theta &= \frac{E_1}{1-\nu^2}(\varepsilon_\theta^0 + \nu\varepsilon_r^0) - \frac{E_2}{1-\nu^2}(\chi_\theta + \nu\,\chi_r),\\
N_{r\theta} &= \frac{E_1}{2(1+\nu)}\gamma_{r\theta}^0 - \frac{E_2}{1+\nu}\chi_{r\theta}
\end{aligned} \tag{5}$$

or inversely

$$\begin{aligned}\varepsilon_r^0 &= \frac{1}{E_1}(N_r - \nu N_\theta) + \frac{E_2}{E_1}\chi_r,\\ \varepsilon_\theta^0 &= \frac{1}{E_1}(N_\theta - \nu N_r) + \frac{E_2}{E_1}\chi_\theta,\\ \gamma_{r\theta}^0 &= \frac{2(1+\nu)}{E_1}N_{r\theta} + \frac{2E_2}{E_1}\chi_{r\theta}\end{aligned} \tag{6}$$

and

$$\begin{aligned}M_r &= \frac{E_2}{1-\nu^2}(\varepsilon_r^0 + \nu\varepsilon_\theta^0) - \frac{E_3}{1-\nu^2}(\chi_r + \nu\chi_\theta) = \frac{E_2}{E_1}N_r - \frac{E_1E_3 - E_2^2}{E_1(1-\nu^2)}(\chi_r + \nu\chi_\theta),\\ M_\theta &= \frac{E_2}{1-\nu^2}(\varepsilon_\theta^0 + \nu\varepsilon_r^0) - \frac{E_3}{1-\nu^2}(\chi_\theta + \nu\chi_r) = \frac{E_2}{E_1}N_\theta - \frac{E_1E_3 - E_2^2}{E_1(1-\nu^2)}(\chi_\theta + \nu\chi_r),\\ M_{r\theta} &= \frac{E_2}{2(1+\nu)}\gamma_{r\theta}^0 - \frac{E_3}{1+\nu}\chi_{r\theta} = \frac{E_2}{E_1}N_{r\theta} - \frac{E_1E_3 - E_2^2}{E_1(1+\nu)}\chi_{r\theta},\end{aligned} \tag{7}$$

where

$$\begin{aligned}E_1 &= \int_{-h/2}^{h/2} E(z)dz = E_m h + \frac{(E_c - E_m)h}{k+1} = E_1^* h,\\ E_2 &= \int_{-h/2}^{h/2} E(z)zdz = \frac{(E_c - E_m)kh^2}{2(k+1)(k+2)} = E_2^* h^2,\\ E_3 &= \int_{-h/2}^{h/2} E(z)z^2dz = \frac{E_m h^3}{12} + (E_c - E_m)h^3\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4k+4}\right) = E_3^* h^3,\end{aligned} \tag{8}$$

with

$$\begin{aligned}E_1^* &= E_m + \frac{E_c - E_m}{k+1},\\ E_2^* &= \frac{(E_c - E_m)k}{2(k+1)(k+2)},\\ E_3^* &= \frac{E_m}{12} + (E_c - E_m)\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4k+4}\right).\end{aligned} \tag{9}$$

Suppose that the shallow spherical shell is acted on by external uniform pressure q. According to Love's theory the equations of motion are

$$\frac{1}{r}\frac{\partial}{\partial r}(rN_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(N_{r\theta}) - \frac{N_\theta}{r} = \rho_1\frac{\partial^2 v}{\partial t^2}, \tag{10}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rN_{r\theta}) + \frac{1}{r}\frac{\partial N_\theta}{\partial\theta} + \frac{N_{r\theta}}{r} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$\begin{aligned}&\frac{1}{r}\left[\frac{\partial^2}{\partial r^2}(rM_r) + 2\left(\frac{\partial^2 M_{r\theta}}{\partial r\partial\theta} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial\theta}\right) + \frac{1}{r}\frac{\partial^2 M_\theta}{\partial\theta^2} - \frac{\partial M_\theta}{\partial r}\right] + \frac{1}{r}(N_r + N_\theta) +\\ &+ \frac{1}{r}\frac{\partial}{\partial r}\left(rN_r\frac{\partial w}{\partial r} + N_{r\theta}\frac{\partial w}{\partial\theta}\right) + \frac{1}{r}\frac{\partial}{\partial\theta}\left(N_{r\theta}\frac{\partial w}{\partial r} + N_\theta\frac{\partial w}{r\partial\theta}\right) + q = \rho_1\frac{\partial^2 w}{\partial t^2},\end{aligned} \tag{12}$$

where

$$\rho_1 = \int\limits_{-h/2}^{h/2} \rho(z)dz = \rho_m h + \frac{(\rho_c - \rho_m)h}{k+1} = \rho^* h, \tag{13}$$

with $\rho^* = \rho_m + \dfrac{\rho_c - \rho_m}{k+1}$.

By taking the inertia forces $\rho\dfrac{\partial^2 u}{\partial t^2} \to 0$ and $\rho\dfrac{\partial^2 v}{\partial t^2} \to 0$ into consideration because of $u \ll w,\ v \ll w$ [16] equations (10) and (11) are satisfied by introducing the stress function F

$$N_r = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2},\ N_\theta = \frac{\partial^2 F}{\partial r^2},\ N_{r\theta} = \frac{1}{r^2}\frac{\partial F}{\partial \theta} - \frac{1}{r}\frac{\partial^2 F}{\partial r \partial \theta} \tag{14}$$

and the equation (12) can be rewritten as

$$\begin{aligned}&\frac{1}{r}\left[\frac{\partial^2}{\partial r^2}(rM_r) + 2\left(\frac{\partial^2 M_{r\theta}}{\partial r\partial\theta} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial\theta}\right) + \frac{1}{r}\frac{\partial^2 M_\theta}{\partial\theta^2} - \frac{\partial M_\theta}{\partial r}\right] + \frac{1}{r}(N_r + N_\theta) + \\ &\quad + N_r\frac{\partial^2 w}{\partial r^2} + 2\frac{N_{r\theta}}{r}\frac{\partial^2 w}{\partial r\partial\theta} + \frac{N_\theta}{r^2}\frac{\partial^2 w}{\partial\theta^2} + \frac{N_\theta}{r}\frac{\partial w}{\partial r} - \frac{2}{r^2}N_{r\theta}\frac{\partial w}{\partial\theta} + q = \rho_1\frac{\partial^2 w}{\partial t^2}.\end{aligned} \tag{15}$$

The substitution of eqs. (6) into the compatibility equation (4) and eqs. (7) into equation (15), taking into account relations (2), (3) and (14) yields a system of equations in terms of the stress function F and the deflection w

$$\frac{1}{E_1}\Delta\Delta F = -\frac{\Delta w}{R} + \left(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w}{\partial\theta}\right)^2 - \frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right), \tag{16}$$

$$\begin{aligned}&\rho_1\frac{\partial^2 w}{\partial t^2} + \frac{E_1E_3 - E_2^2}{E_1(1-\nu^2)}\Delta\Delta w - \frac{1}{R}\Delta F - \left(\frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial\theta^2}\right)\frac{\partial^2 w}{\partial r^2} - \frac{\partial^2 F}{\partial r^2}\frac{\partial^2 w}{r^2\partial\theta^2} + \\ &\quad + 2\left(\frac{1}{r}\frac{\partial^2 F}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial F}{\partial\theta}\right)\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{1}{r}\frac{\partial^2 F}{\partial r^2}\frac{\partial w}{\partial r} + \frac{2}{r^2}\left(\frac{1}{r^2}\frac{\partial F}{\partial\theta} - \frac{1}{r}\frac{\partial^2 F}{\partial r\partial\theta}\right)\frac{\partial w}{\partial\theta} = q\end{aligned} \tag{17}$$

3. NON-LINEAR DYNAMICAL ANALYSIS

The mentioned equations (16) and (17) combining with boundary conditions and initial conditions can be used in non-linear dynamical analysis of functionally graded spherical shells.

Suppose that the FGM spherical shell is clamped at its base edge $r = r_0$, the boundary conditions are

$$w = 0,\ \frac{\partial w}{\partial r} = 0 \text{ and } N_r = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial\theta^2} = 0 \text{ at } r = r_0.$$

The boundary conditions can be satisfied, if the deflection w and the stress function F are represented by

$$\begin{aligned}w(r,\theta,t) &= f(t)\frac{r^2(r_0 - r)^2}{r_0^4}\sin n\theta \\ F(r,\theta,t) &= \eta(t)\frac{r^2(r_0 - r)^2}{r_0^4}\sin n\theta\end{aligned} \tag{18}$$

Applying Galerkin method to Eqs. (16) and (17) in the range $0 \le \theta \le 2\pi,\ 0 \le r \le r_0$ yields a set of two equations with respect to $f(t)$ and $\eta(t)$

$$\eta = \frac{2E_1}{38-20n^2+12n^4}\left[\frac{f\,r_0^2}{7R}(3+2n^2) - \frac{2nf^2}{7\pi}\right],$$

$$\rho_1\frac{d^2f}{dt^2} + \frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}.\frac{7(38-20n^2+12n^4)}{4r_0^4}f + \frac{(3+2n^2)}{2Rr_0^2}\eta - \frac{2nf\eta}{\pi r_0^4} = \frac{14q}{\pi n}.$$

Eliminating η from two these equations leads to a non-linear second-order ordinary differential equation for f

$$\begin{aligned}\rho_1\frac{d^2f}{dt^2} + \left[\frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}\frac{7(38-20n^2+12n^4)}{4r_0^4} + \frac{E_1(3+2n^2)^2}{(38-20n^2+12n^4)7R^2}\right]f - \\ -\frac{6E_1(3+2n^2)n}{7\pi Rr_0^2(38-20n^2+12n^4)}f^2 + \frac{8E_1n^2}{7\pi^2r_0^4(38-20n^2+12n^4)}f^3 = \frac{14q(t)}{\pi n}.\end{aligned} \tag{19}$$

where n - odd number.

By use of notations (8), (9) and (13) Eq. (19) can be rewritten in the form

$$\begin{aligned}\frac{d^2f}{dt^2} + \frac{E_1^*}{\rho^*R^2}\left[\frac{7(E_1^*E_3^*-E_2^{*2})}{4\alpha^4E_1^{*2}(1-\nu^2)}\left(\frac{h}{R}\right)^2(38-20n^2+12n^4) + \frac{(3+2n^2)^2}{7(38-20n^2+12n^4)}\right]f - \\ -\frac{6E_1^*(3+2n^2)n}{\rho^*7\pi\alpha^2R^3(38-20n^2+12n^4)}f^2 + \frac{8E_1^*n^2}{\rho^*7\pi^2\alpha^4R^4(38-20n^2+12n^4)}f^3 = \frac{14q(t)}{\pi n\rho^*h}\end{aligned} \tag{20}$$

where putting $r_0 = \alpha R$, α is the ratio of the base radius and the shell radius.

The obtained equation (20) is a governing equation for dynamical analysis of functionally graded shallow spherical shells. Based on this equation the non-linear vibration of FGM spherical shells can be investigated and the post-buckling analysis of shells can be performed.

4. NON-LINEAR VIBRATION OF FGM SHALLOW SPHERICAL SHELLS

Consider a FGM shallow spherical shell acted on by an uniformly distributed excited pressure load $q(t) = Q\sin\Omega t$, the equation of motion has of the form

$$\begin{aligned}\frac{d^2f}{dt^2} + \frac{E_1^*}{\rho^*R^2}\left[\frac{7(E_1^*E_3^*-E_2^{*2})}{4\alpha^4E_1^{*2}(1-\nu^2)}\left(\frac{h}{R}\right)^2(38-20n^2+12n^4) + \frac{(3+2n^2)^2}{7(38-20n^2+12n^4)}\right]f - \\ -\frac{6E_1^*(3+2n^2)n}{\rho^*7\pi\alpha^2R^3(38-20n^2+12n^4)}f^2 + \frac{8E_1^*n^2}{\rho^*7\pi^2\alpha^4R^4(38-20n^2+12n^4)}f^3 = \frac{14Q\sin\Omega t}{\pi n\rho^*h}\end{aligned} \tag{21}$$

From Eq. (21) the fundamental frequencies of natural vibration of the shell ω_n can be determined by the relation

$$\omega_n^2 = \frac{E_1^*}{\rho^*R^2}\left[\frac{7(E_1^*E_3^*-E_2^{*2})}{4\alpha^4E_1^{*2}(1-\nu^2)}\left(\frac{h}{R}\right)^2(38-20n^2+12n^4) + \frac{(3+2n^2)^2}{7(38-20n^2+12n^4)}\right] \tag{22}$$

The FGM shallow shell considered here is a spherical cap of radius $R = 5m$, base radius $r_0 = 0,6R$, i.e. $\alpha = 0,6$, thickness $h = 0,01m$. The materials mixture consists of Silicon nitride Si_3N_4 ($E_c = 348,43.10^9\ N/m^2,\ \rho_c = 2370\,kg/m^3$) and Stainless steel SUS 304 ($E_m = 201,04.10^9\ N/m^2,\ \rho_m = 8166\,kg/m^3$). The Poisson ratio is chosen to be 0,3 for simplicity.

Five first natural frequencies of FGM spherical ω_n are shown in the Table 1.

Table 1. Effect of power law index k on frequencies

	ω				
k	ω_1	ω_2	ω_3	ω_4	ω_5
$k = 0$ (Ceramic)	670.8294	685.9349	734.8552	825.8576	837.2293
$k = 1$	398.6771	408.2874	435.6813	491.8383	498.6416
$k = 2$	350.8065	358.4667	384.6829	431.4895	437.4192
$k = \infty$ (Metal)	274.5143	280.6957	300.7148	337.9545	342.6079

Obviously the natural frequencies of FGM shallow spherical shells are observed to be dependent on the constituent volume fraction, they decrease when increasing the power law index k. This is attributed due to the stiffness reduction because of the increase in the metallic volumetric fraction. When k =0, representing a full ceramic shell the natural frequencies are considerably greater than frequencies of other FGM shells, when $k = \infty$ representing a full metal shell the frequencies are smaller than those of others. The reason is the higher value of assumed modulus of elasticity of the ceramic constituent and the lower value of elasticity modulus of the metal constituent.

The non-linear dynamic responses of FGM spherical shells acted on by the harmonic uniformly external pressure load $q(t) = Q \sin \Omega t$ are obtained by solving Eq. (21) combined with the initial conditions and by use of the Runge - Kutta method. Fig. 2 shows non-linear responses of FGM spherical shells with various power law indices subjected to the excited load of magnitude $Q = 3000N/m^2$ and frequency $\Omega = 300(s^{-1})$ different far from the natural frequencies of FGM shells with $k = 0, 1, 2$.

From the obtained results we can see that amplitudes of non-linear vibration of FGM shells increase when increasing the power law index k but frequencies decrease when increasing k. The non-linear dynamic responses perform the phenomenon of periodic cycles.

The pictures of non-linear responses of FGM shells are quite different when the frequencies of excited load are near to the natural frequencies of shells. Indeed, in the following examples we consider the non-linear dynamic responses of functionally graded spherical caps with various power law indices under external pressure loads $Q \sin \Omega t$, where the external frequencies Ω are chosen near to the natural frequencies and the external amplitudes are taken in two cases: (a) $Q = 1500N/m^2$ and (b) $Q = 500N/m^2$ respectively. Fig. 3a and Fig. 3b show the non-linear responses of a full ceramic shell (with k=0) , the natural frequency of which is $\omega\ = 837,2293s^{-1}$ and the frequency of external load is taken as $\Omega = 812,0760s^{-1}$.

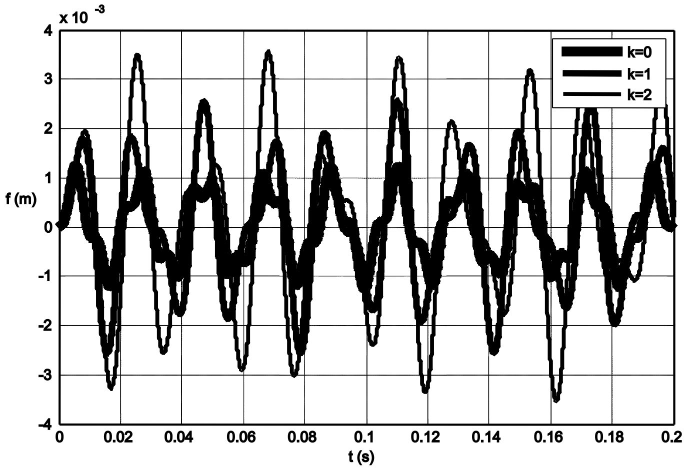

Fig. 2. Non-linear dynamic responses of FGM spherical shells with various k

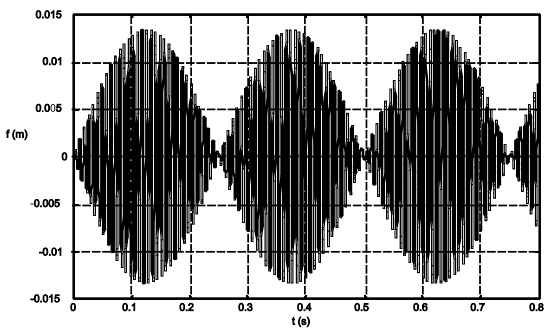

Fig. 3a. Non-linear responses of a full ceramic shell with $k = 0$ in case (a)

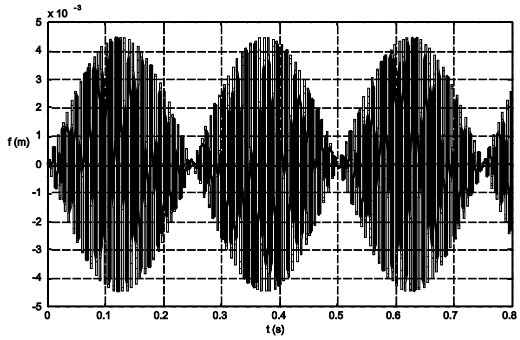

Fig. 3b. Non-linear responses in case (b)

Fig. 4a and Fig. 4b show the non-linear responses of a FGM shell with power law index k=1, the natural frequency of the shell $\omega_=498.6416s^{-1}$ and external frequency $\Omega =$ 466,2267s^{-1}

The non-linear responses of the FGM shell with power law index k=2 are indicated in the Fig. 5a and Fig. 5b, where the natural frequency of the shell $\omega = 437,4192s^{-1}$ and external frequency $\Omega = 402,4303s^{-1}$

And the non-linear responses of a full metal shell ($k = \infty$) are presented in the Fig. 6a and Fig. 6b with $\omega_=342,6079s^{-1}$ and $\Omega = 303,2078s^{-1}$

From obtained results we observe the interesting phenomenon like harmonic beat of a linear vibration, where the number of the beats is greater when increasing the power law index k, i.e. the period of beat of non-linear responses is shorter when increasing the index k. Furthemore frequencies of non-linear vibration of FGM shells also decrease when increasing the index k. Decreasing the amplitude of external load leads to decreasing the

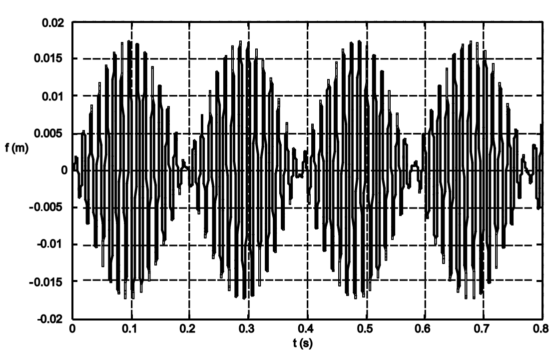

Fig. 4a. Non-linear responses of the FGM shell with $k = 1$ in case (a)

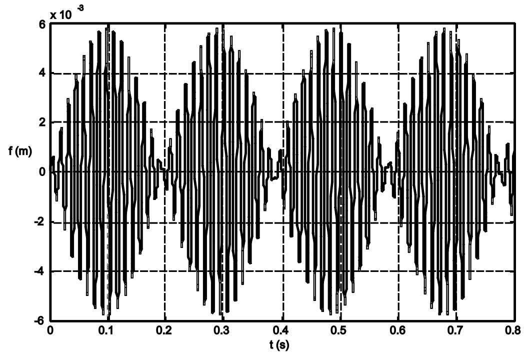

Fig. 4b. Non-linear responses in case (b)

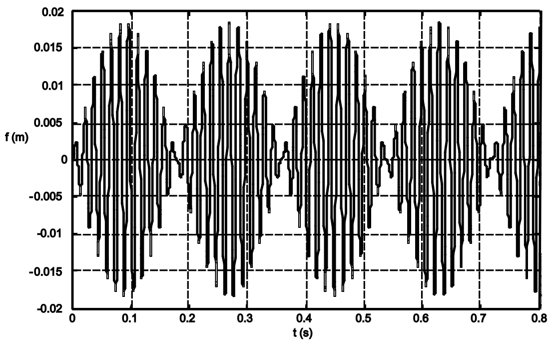

Fig. 5a. Non-linear responses of the FGM shell with $k = 2$ in case (a)

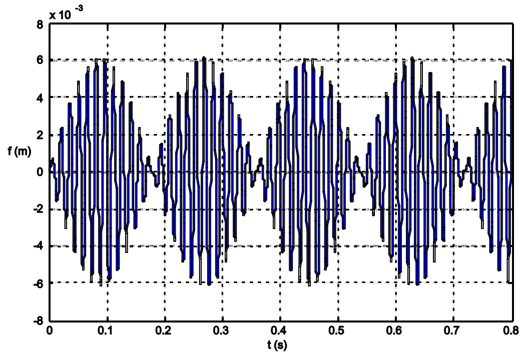

Fig. 5b. Non-linear responses in case (b)

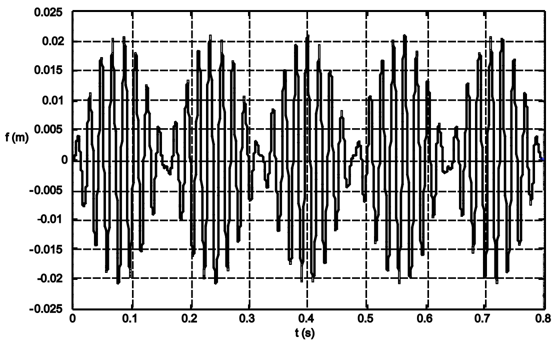

Fig. 6a. Non-linear responses of a full metal shell with $k = \infty$ in case (a)

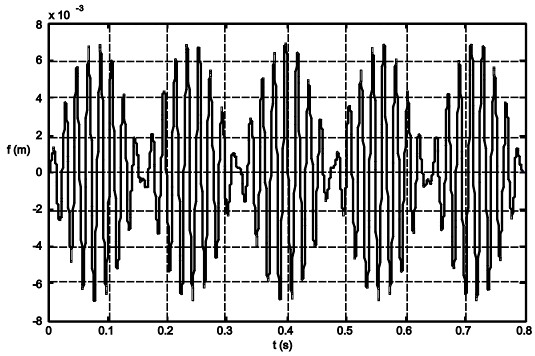

Fig. 6b. Non-linear responses in case (b)

vibration amplitude, of course, but the period of beat, the frequency of non-linear vibration of FGM shells are unchanged.

Next, the detailed investigation for dynamical characteristics of functionally graded spherical caps is carried out for different geometrical parameters. According to the relation

(22) calculated values of natural frequencies of the FGM spherical caps with k=1 corresponding to different dimension ratios R/h and H/r_0 , where $H = R\left(1 - \sqrt{1 - \left(\frac{r_0}{R}\right)^2}\right)$ is the rise of the shell, are given in the Table 2 and Table 3 respectively.

Table 2. Natural frequencies of FGM spherical cap with $k = 1$, $\alpha = 0,6$ versus ratios R/h

	ω				
R/h	ω_1	ω_2	ω_3	ω_4	ω_5
600	323.1338	337.6230	341.7290	409.4741	415.4575
500	398.6771	408.2874	435.6813	491.8383	498.6416
400	522.5359	517.5065	599.0702	615.8767	623.5149
300	761.7481	710.1611	936.6334	824.2681	831.9664
200	1.3840e+003	1.1472e+003	1.8740e+003	1.2496e+003	1.2506e+003

Table 3. Natural frequencies of FGM spherical cap with $k = 1$ versus ratios H/r_0

	ω				
H/r_0	ω_1	ω_2	ω_3	ω_4	ω_5
0,268	1.2556e+003	1.2747e+003	1.2910e+003	1.7589e+003	2.5465e+003
0,1716	845.1677	888.8869	1.0568e+003	1.6246e+003	2.4640e+003
0,127	642.2654	705.6727	961.4847	1.5749e+003	2.4345e+003
0,101	522.3149	602.2891	913.9948	1.5514e+003	2.4207e+003

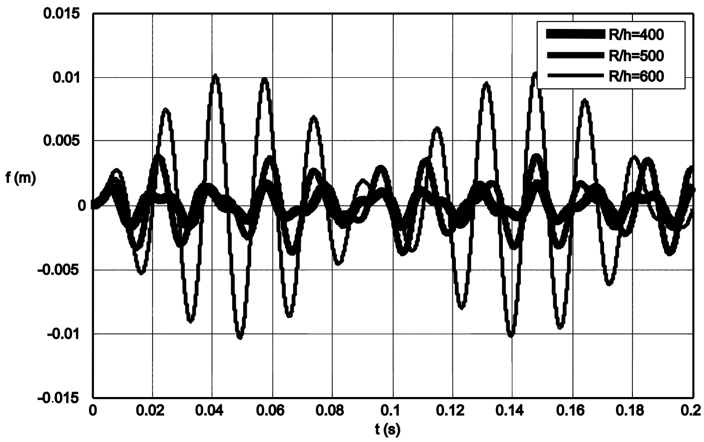

Fig. 7. Effect of dimension ratio R/h on non-linear dynamic responses

Fig. 7 shows the effect of dimension ratios R/h on the non-linear responses of FGM spherical cap with $k = 1$ under external load with $\Omega = 350s^{-1}$ and $Q = 1500\ N/m^2$.

Effect of dimension ratio H/r_0 on non-linear dynamic responses of FGM spherical cap under external load with $\Omega = 350s^{-1}$ and $Q = 3000N/m^2$ is presented in the Fig. 8.

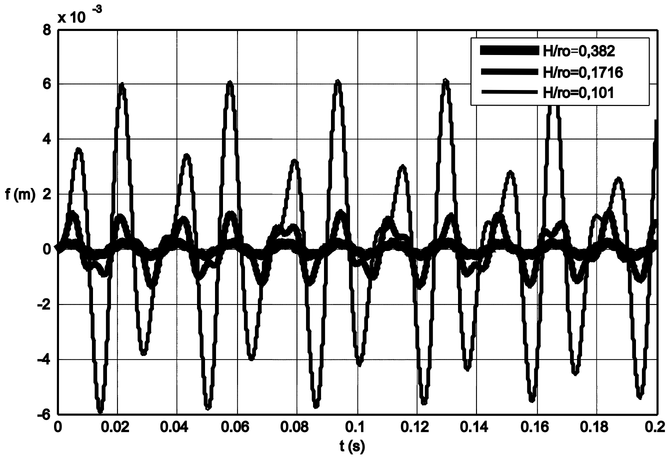

Fig. 8. Effect of dimension ratio H/r_0 on non-linear responses

It can be noted that the rate of decrease in the natural frequencies is high with the increase in dimension ration R/h; but with the decrease in dimension ratios H/r_0. What about amplitudes of non-linear dynamic responses we can see opposite phenomenon, i.e they increase when increasing ratio R/h or decreasing ratio H/r_0.

5. CONCLUSIONS

The governing equations for non-linear dynamical analysis of functionally graded shallow spherical shells, including geometric non-linearity, are derived.

Derivations are based on the classical shell theory and with the assumption of power law composition for the constituent materials.

Non-linear vibration of functionally graded shallow spherical shells is studied. Frequencies of non-linear vibration decrease, while amplitudes increase when increasing the power law index. The non-linear dynamic responses perform the phenomenon like periodic cycles when the frequency of external load is far from natural frequency of the shell and the phenomenon like harmonic beat of a linear vibration when the frequency of external load is near to natural frequency of the shell.

From the obtained results it is observed that the dynamical characteristics of FGM shallow spherical shells significantly depend on the value of material power-law index, apart from geometric shell parameters.

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REFERENCES

[1] Vel SS., Batra R.C. Three dimensional exact solution for the vibration of GFM rectangular plates. *J.Sound and vibration*, **272**, N:3 (2004) 703-730.

[2] Ferreira A..J.M., Batra R.C., Roque C.M.C Natural frequencies of FGM plates by meshless method. *Composite structures*, **75** (2006) 593-600

[3] Hiroyuki Matsunaga. Free vibration and stability of FGM plates according to a 2-D high order deformation theory, *Composite structures*, **82** (2008) 499-512.

[4] Hiroyuki Matsunaga. Free vibration and stability of functionally graded shallow shells according to a 2D higher-order deformation theory, *Composite structures,* **84** (2008), 132-146.

[5] Pradyumna S., Bandyspadhyay J.N. Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation, *Journal of sound and vibrations* **318** (2008) 176-192.

[6] Pradhan S.C., Loy C.T., Lam K.Y., Reddy J.N. Vibration characteristics of FGM cylindrical shells under various boundary conditions, *Applied Acoustics* **61** (2000) 111 -129.

[7] Loy C.T., Lam K.T., Reddy J.N. Vibration of FGM cylindrical shells, *Int. J. Mech. Sci* **41**(1999) 309-324.

[8] Yang Y., Shen H.S. Free vibration and parametric response of shear deformable FGM cylindrical panels, *J.Sound and vibrations* **261** (2003) 871-893.

[9] Reddy J.N., Cheng Z.Q. Frequency correspondence between membranes and FGM spherical shallow shell of polygonal plan form, *Int. J. Mech. Sci* **44** (2002) 967-985.

[10] Dao H.B, Vu D.L. Non-linear dynamical analysis of imperfect functionally graded material shallow shells, *Vietnam Journal of Mechanics, VAST* **32 (1)** (2010) 1-14.

[11] Sofiyev A.H., The stability of compositionally graded ceramic - metal cylindrical shells under aperiodic axial impulse loading, *Composite structures* **69** (2005) 247-257.

[12] Sofiyev A.H., Schnack E. The stability of FGM cylindrical shells under linearly increasing dynamic torsional loading, *Engineering structures* **26** (2004) 1321-1331.

[13] Ganapathi M. Dynamic stability characteristics of FGM shallow spherical shells. *Composite structures* **79** (2007) 338-343.

[14] Dao H.B., Non-linear buckling analysis of FGM shallow spherical shells. *Vietnam J. of Mechanics* **31** (2009) 17-30.

[15] Javaheri R., Eslami M.R. Buckling of FGM plates subjected to temperature rise. *Proc.of 4^{th} Int. congress on the thermal stresses, Osaka Japan*, (2001) 167-170.

[16] Volmir A.S. Non-linear dynamics of plates and shells, *Science Edition, M.* (1972).

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DAO ĐỘNG PHI TUYẾN CỦA VỎ CẦU THOẢI BẰNG VẬT LIỆU CƠ TÍNH BIẾN THIÊN

Dao động phi tuyến vỏ cầu thoải bằng vật liệu cơ tính biến thiên được đề cập nghiên cứu trong bài báo này. Cơ tính của vật liệu biến thiên theo hướng độ dày vỏ theo quy luật phân bố liên tục phụ thuộc vào tỷ phần thể tích các vật liệu cấu thành. Đã thiết lập các phương trình xác định có tính đến phi tuyến hình học trong tất cả các hệ thức biến dạng - chuyển vị. Từ các phương trình chuyển động và phương trình tương thích biến dạng nhận được hệ phương trình giải là các phương trình đạo hàm riêng phi tuyến đối với hàm ứng suất và độ võng của vỏ. Phân tích động lực của vỏ được thực hiện nhờ vào phương pháp Galerkin và phương pháp Runge - Kutta, đã nhận được biểu thức hiển của tần số riêng cơ bản và đáp ứng động lực phi tuyến của vỏ. Các kết quả số cho thấy ảnh hưởng đáng kể của các đặc trưng vật liệu và kích thước hình học lên các tính chất động lực học của vỏ.