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MONITORING A SUDDEN CRACK OF BEAM-LIKE BRIDGE DURING EARTHQUAKE EXCITATION

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Abstract. This paper presents a wavelet spectrum technique for monitoring a sudden crack of a beam-like bridge structure during earthquake excitation. When there is a sudden crack caused by earthquake excitation the stiffness of the structure is changed leading to a sudden change in natural frequencies during vibration. It is difficult to monitor this sudden change in the frequency using conventional approaches such as Fourier transform because in Fourier transform the time information is lost so that it is impossible to analyse short time events. To overcome this disadvantage, wavelet spectrum, a time-frequency analysis, is used for monitoring a sudden change in frequency during earthquake excitation for crack detection. In this study, a model of 3D crack is applied. The derivation of the stiffness matrix of a 3D cracked beam element with rectangular section adopted from fracture mechanics is presented. Numerical results showed that the sudden occurrence of the crack during earthquake excitation can be detected by the sudden change in frequency using wavelet power spectrum. When the crack depth increases, the instantaneous frequency (IF) of the structure is decreased.

Keywords: Earthquake, crack, crack monitoring, sudden crack, 3D crack, wavelet spectrum.

1. INTRODUCTION

In practice, cracks in a structure may gradually develop due to repeated low amplitude loads or may suddenly appear due to high amplitude loads caused by, for example, natural calamities such as storms or earthquakes. The cracks of certain structures such as building, oil-rig structure or bridge can be catastrophic, therefore, the detection of cracks in such structures is essential to ensure further use of structures. Structural Health Monitoring (SHM) methods have been developed to ensure early detection of such cracks and, hence, prevention of catastrophic failure. Structural health monitoring is a system which comprises sensors, instrumentation and methods for in situ monitoring of the integrity of structures [1]. Among methods for SHM, the vibration-based method has emerged as one possible approach to the problem of structural damage identification and localization. The structural dynamic characteristics such as frequencies, mode shapes, flexibility, etc. can be extracted from the dynamic response and analyzed to track the changes of these parameters caused by cracks [1, 2]. Other non-modal-based methods have been applied such as auto-regressive approaches [3, 4], fuzzy logic and neural networks [5, 6], time-series dimensionality [7], and genetic algorithms [8].

In practice, the structures most often damaged by natural calamities are buildings and bridges. Of the studies conducted many focus on extracting the relevant information from dynamic responses of damaged buildings and bridges under ambient excitation such as storm, earthquake etc. for damage detection. Bassam et al. [9] reported a simple quantitative approach for post earthquake damage assessment of flexure dominant reinforced concrete bridges that considers the effect of cyclical loading on the state of damage. Park et al. [10] presented a moving time window technique to monitor abrupt structural damage induced by an earthquake. Todorovska et al. [11] presented a structural health monitoring method using changes in wave travel times during earthquake excitation. In other research, Sakellariou et al. [12] proposed a stochastic output error vibration-based method for damage detection of structures under earthquake excitation. The authors reported the method based on stochastic output error model identification, statistical hypothesis testing procedures for damage detection, and a geometric method for damage assessment. Magalhaes et al. [13] presented algorithms to perform the continuous on-line identification of modal parameters based on structural responses to ambient excitation based on modal parameters tracking for structural health monitoring of bridges. Limongelli [14] proposed a method for damage detection based on the accuracy of a spline function in interpolating the operational mode shapes of frames under earthquake excitation. Yinfeng et al. [15] applied an unscented Kalman filter for time varying spectral analysis of earthquake ground motions for damage detection of a building.

In SHM methods, the frequency based methods are most interesting because the frequency is a global parameter of the structure which can easily be measured in practice. The change in natural frequency of a damaged structure can be used for damage detection. By conventional approach, the natural frequency can be extracted by Fourier transform method. In general, this method is only useful for analysing stationary responses, while the responses of structures such as bridges, especially cracked bridges subjected to ambient loads, are not stationary. Moreover, in this transform the information of the time when the frequency changes will be lost. Therefore, Fourier transform method for damage detection in these cases is not suitable. Recently, time-frequency based methods which can analyse the frequency change while the information of time is still kept, called time-frequency analysis, have been applied widely for SHM such as Short Time Fourier transform (STFT), Wigner-Ville Transform (WVT), Auto Regressive (AR), Moving Average (MA), Auto Regressive Moving Average, and Wavelet Transform (WT) wavelets [16-19]. In these methods, the wavelet transform has emerged as an efficient tool for signal processing due to its flexibility and precision in time and frequency resolution. However, most of the current research in the field of SHM focuses on existing cracks. A limited amount of research considers the sudden crack detection problem [11, 12], however, in these studies the damage models were not presented.

To this end, the present study aims to extend the SHM techniques by proposing a wavelet based technique to monitor a sudden crack of a beam-like bridge induced during earthquake excitation. The sudden crack is monitored by using the IF of the bridge. The method is simple since only one acceleration signal is needed and the change in frequency can be tracked without the base line information. The wavelet power spectrum and the derivation of 3D crack model adopted from fracture mechanics are presented. In this work, acceleration signal obtained from numerical simulation is used for wavelet analysis and the results are also provided.

2. VIBRATION OF THE BEAM-LIKE STRUCTURE SUBJECTED TO EARTHQUAKE EXCITATION

2.1. Intact beam like structure

In this study, the beam-like bridge is considered as a 3D Timoshenko beam subjected to earthquake excitation as shown in Fig. 1. The beam is modeled as R elements in finite element analysis. Under these assumptions, and applying the finite element method, the governing equation of motion of the beam can be deduced as follows [20]

$$\mathbf{M}\ddot{\mathbf{u}}_r + \mathbf{C}\dot{\mathbf{u}}_r + \mathbf{K}\mathbf{u}_r = -\mathbf{M}\mathbf{I}\ddot{d}_g \tag{1}$$

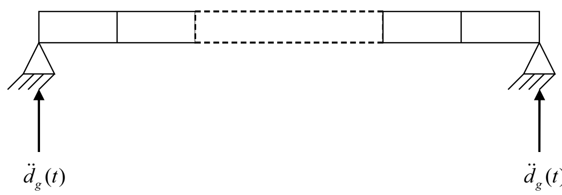

Fig. 1. A beam-like bridge under ground excitation

Here $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are structural mass, damping and stiffness matrices respectively; $\mathbf{I}$ is a vector of order N with each element equal to unity, where N is the number of DOFs; $\ddot{d}_g(t)$ is the ground acceleration time history; $\mathbf{u}_r$ is a column vector which denotes the relative nodal displacement of the beam in comparison with the ground.

Let us consider a linear elastic structure subjected to static loads. The response of such structure can be calculated by using finite element method. From the compatibility condition, the response of an element of the beam structure can be written as the following equation [21]

$$\boldsymbol{\varepsilon}_e^T = \mathbf{D}_e\mathbf{u}_e \tag{2}$$

Where $\boldsymbol{\varepsilon}_e$ is the vector of element deformation or generalized strain; $\mathbf{u}_e$ is the vector of nodal displacement; $\mathbf{D}_e$ is the compatibility matrix, and

$$\mathbf{D}_e = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1/l & 0 & 1 & 0 & 0 & 0 & 1/l & 0 & 0 & 0 \\ 0 & 0 & -1/l & 0 & 0 & 0 & 0 & 0 & 1/l & 0 & 1 & 0 \\ 0 & 1/l & 0 & 0 & 0 & 1 & 0 & -1/l & 0 & 0 & 0 & 0 \\ 0 & 1/l & 0 & 0 & 0 & 0 & 0 & -1/l & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

From the equilibrium condition, the relationship between external nodal forces $\mathbf{Q}_e$ and internal forces $\mathbf{P}_e$ can be expressed as follows

$$\mathbf{D}_e^T \mathbf{Q}_e = \mathbf{P}_e \tag{4}$$

The nodal force vector is defined as follows (see in Fig. 2)

$$\mathbf{P}_e^T = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} & P_{11} & P_{12} \end{bmatrix} \tag{5}$$

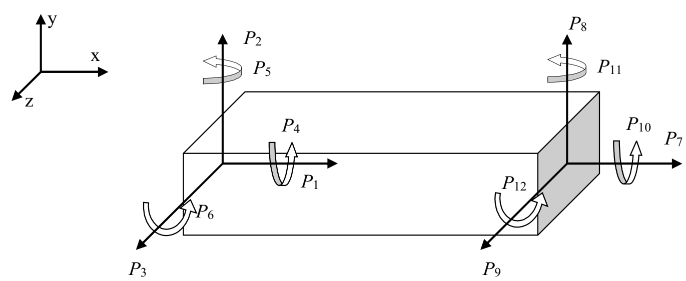

Fig. 2. Three-dimension Timoshenko beam element and its nodal forces

Where P_1 and P_7 are axial forces, P_2, P_3, P_8, P_9 are shear forces, P_5, P_6, P_{11}, P_{12} are bending moments, and P_4, P_{10} are torques acting at the crack cross section.

The constitutive condition expressing the relation of the generalized strains and internal forces or generalized stresses can be presented in the form of

$$\boldsymbol{\varepsilon}_e = \mathbf{C}_e \mathbf{Q}_e \tag{6}$$

where $\mathbf{C}_e$ is the compliance matrix.

From Eqs. (2) and (6) we have

$$\mathbf{Q}_e = \mathbf{C}_e^{-1} \mathbf{D}_e \mathbf{u}_e \tag{7}$$

Substituting Eq. (7) into Eq. (4), the relation between the external force vector and displacement vector is derived

$$\mathbf{P}_e = \mathbf{D}_e^T \mathbf{C}_e^{-1} \mathbf{D}_e \mathbf{u}_e \tag{8}$$

Therefore, from (7) the stiffness matrix $\mathbf{k}_e$ of an element can be obtained as

$$\mathbf{k}_e = \mathbf{D}_e^T \mathbf{C}_e^{-1} \mathbf{D}_e \tag{9}$$

The components of the compliance matrix $\mathbf{C}_e$ of an element can be calculated from Castingliano's theorem

$$c_{ij} = \frac{\partial^2 W^{(0)}}{\partial P_i \partial P_j}\,; \quad i, j = 1, 2, ..., 6 \tag{10}$$

Where $W^{(0)}$ is the elastic strain energy of the intact element and can be expressed as follows

$$W^{(0)} = \frac{1}{2} \int\limits_0^l \left(\frac{P_1^2}{AE} + \frac{\kappa P_2^2}{GA} + \frac{\kappa P_3^2}{GA} + \frac{(P_2 x + P_6)^2}{EI_z} + \frac{(P_3 x - P_5)^2}{EI_y} + \frac{P_4^2}{GJ} \right) dx \tag{11}$$

2.2. Cracked beam like structure

Consider the cracked element with rectangular cross section as presented in Fig. 3. It is assumed that the crack influences on the stiffness matrix only, not the mass matrix. Due to the presence of the crack, the compliance of the cracked element is expected to increase. The axial force N_c gives an additional elongation Δu_c and together with an additional rotation $\Delta\theta_{zc}$. While, bending moment M_{zc} causes an additional rotation $\Delta\theta_{zc}$ together with an additional elongation Δu_c. Besides, bending moment M_{yc} gives rise to an

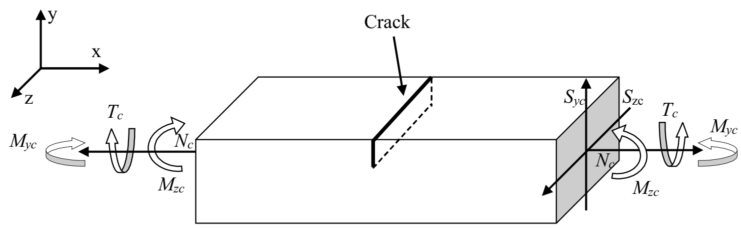

Fig. 3. Three-dimension element with edge crack

additional rotation $\Delta\theta_{yc}$. While the shear force S_{yc} causes and additional deflection Δv_c, the shear force S_{zc} causes an additional Δw_c together with an additional rotation $\Delta\theta_{xc}$. Finally, the torsion T_c will cause an additional rotation $\Delta\theta_{xc}$. This leads to the following relation [21]

$$\mathbf{\Delta u}_c = \begin{bmatrix} \Delta u_c \\ \Delta v_c \\ \Delta w_c \\ \Delta\theta_{xc} \\ \Delta\theta_{yc} \\ \Delta\theta_{zc} \end{bmatrix} = \tilde{\mathbf{C}}_e^c \mathbf{P}_c = \tilde{\mathbf{C}}_e^c \begin{bmatrix} N_c \\ S_{yc} \\ S_{zc} \\ T_c \\ M_{yc} \\ M_{zc} \end{bmatrix} \tag{12}$$

Where $\tilde{\mathbf{C}}_e^c$ is the local compliance matrix of the cracked element and

$$\tilde{\mathbf{C}}_e^c = \begin{bmatrix} c_N & 0 & 0 & 0 & 0 & c_{NM_z} \\ 0 & c_{S_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{S_z} & c_{S_zT_x} & 0 & 0 \\ 0 & 0 & c_{S_zT_x} & c_{T_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{M_y} & 0 \\ c_{NM_z} & 0 & 0 & 0 & 0 & c_{M_z} \end{bmatrix} \tag{13}$$

Where c_N is the axial compliance related to the axial force N; c_{Sy} is the shear compliance related to the shear force S_z; c_{Sz} is the shear compliance related to the shear force S_z; c_{My} is the bending compliance related to the bending moment M_z, c_T is the torsional compliance related to the torque T_x, c_{Mz} is the coupled compliance related to the shear force S_z and the torque T_x, and c_{NMz} is the coupled compliance related to the axial force S_z and bending moment M_z.

The nodal forces acting on the cracked element can be related to the internal forces as follows

$$\mathbf{P}_c = \mathbf{TQ}_e \tag{14}$$

The additional strains at the cracked section can be calculated from the additional displacements by the following equation

$$\boldsymbol{\varepsilon}_c = \mathbf{T}^T \boldsymbol{\Delta}\mathbf{u}_c \tag{15}$$

Where the transformation matrix $\mathbf{T}$ depends on the crack location $\xi = x/l$, where l is the length of the element

$$\mathbf{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/l & 1/l & 0 \\ 0 & -1/l & -1/l & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\xi+1 & -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi-1 & \xi & 0 \end{bmatrix} \tag{16}$$

Obviously, the additional strains due to the crack can be expressed in the form of Eq. (2)

$$\boldsymbol{\varepsilon}_c = \mathbf{C}_e^c \mathbf{Q}_e \tag{17}$$

From Eqs. (12)-(17) we have

$$\mathbf{C}_e^c \mathbf{Q}_e = \mathbf{T}^T \boldsymbol{\Delta}\mathbf{u}_c = \mathbf{T}^T \tilde{\mathbf{C}}_e^c \mathbf{P}_c = \mathbf{T}^T \tilde{\mathbf{C}}_e^c \mathbf{T} \mathbf{Q}_e \tag{18}$$

Therefore, the overall additional compliance due to the crack can be derived as follows

$$\mathbf{C}_e^c = \mathbf{T}^T \tilde{\mathbf{C}}_e^c \mathbf{T} \tag{19}$$

The total compliance of the cracked element is the sum of the compliance of the intact element and the overall additional compliance due to crack

$$\hat{\mathbf{C}}_e^c = \mathbf{C}_e + \mathbf{C}_e^c \tag{20}$$

Therefore, if the local additional compliance matrix $\tilde{\mathbf{C}}_e^c$ is known, the total compliance of the cracked element can be obtained by using Eqs. (19) and (20). The components of the local additional compliance matrix $\tilde{\mathbf{C}}_e^c$ can be calculated from the fracture mechanics. Using Castingliano's theorem

$$\tilde{c}_{ij}^c = \frac{\partial^2 W^{(1)}}{\partial P_i \partial P_j}\,; \quad i, j = 1, 2, ..., 6 \tag{21}$$

Where $W^{(1)}$ is the additional strain energy due to crack

$$W^{(1)} = \int\limits_A \frac{1}{E'} \left[\left(\sum_1^6 K_{Ii} \right)^2 + \left(\sum_1^6 K_{IIi} \right)^2 + \mu \left(\sum_1^6 K_{IIIi} \right)^2 \right] dA \tag{22}$$

where $E' = \dfrac{E}{1-\nu^2}$ and $\mu = 1+\nu$, and K_{II}, K_{IIi}, K_{IIIi} are stress intensity factors for opening type, sliding type and tearing type cracks respectively; $i = 1, 2, ..., 6$

Substituting Eq. (21) into Eq. (20) we have

$$\tilde{c}_{ij}^{c} = \frac{1}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\left\{\int\limits_{-b/2}^{b/2}\int\limits_{0}^{a}\left[\left(\sum_{1}^{6} K_{Ii}\right)^2 + \left(\sum_{1}^{6} K_{IIi}\right)^2 + \mu\left(\sum_{1}^{6} K_{IIIi}\right)^2\right] d\bar{a}dz\right\} \quad (23)$$

Where b is the width of the beam, a is the crack depth. The intensity factors K_I, K_{II} and K_{III} available in the references [22, 23] will be applied in this study.

Finally, applying Eq. (9), the stiffness matrix of the cracked element can be obtained as follows

$$\mathbf{k}_e^c = \mathbf{D}_e^T\left[\hat{\mathbf{C}}_e^c\right]^{-1}\mathbf{D}_e \quad (24)$$

3. WAVELET SPECTRUM

The continuous wavelet transform is defined as follows [24]

$$W(a,b) = \int\limits_{-\infty}^{+\infty} f(t)\psi_{a,b} dt \quad (25)$$

Where $\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi^*\left(\frac{t-b}{a}\right)$, a is a real number called scale or dilation, b is a real number called position, $W(a,b)$ are wavelet coefficients at scale a and position b, $f(t)$ is the input signal, $\psi\left(\frac{t-b}{a}\right)$ is the wavelet function and $\psi^*\left(\frac{t-b}{a}\right)$ is a complex conjugate of $\psi\left(\frac{t-b}{a}\right)$. In order to be classified as a wavelet a function must satisfy the following mathematical criteria

1) A wavelet must have finite energy

$$E = \int\limits_{-\infty}^{+\infty} \left|\psi\left(t\right)\right|^2 dt < \infty \quad (26)$$

2) If $\hat{\psi}\left(\omega\right)$ is Fourier transform of $\psi\left(t\right)$, i.e.

$$\hat{\psi}\left(\omega\right) = \int\limits_{-\infty}^{+\infty} \psi\left(t\right) e^{-i\omega t} dt \quad (27)$$

then the following condition must be satisfied

$$C_g = \int\limits_{0}^{\infty} \frac{\left|\hat{\psi}\left(\omega\right)\right|^2}{\omega} d\omega < \infty \quad (28)$$

This implies that the wavelet has no zero frequency component: $\hat{\psi}(0) = 0$,

$$\int\limits_{-\infty}^{+\infty} \psi(t) e^{-j\omega t} dt = 0 \;\; \text{when } \omega = 0 \quad (29)$$

or in other words, the wavelet must have a zero mean

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \tag{30}$$

3) An additional criterion is that, for complex wavelets, the Fourier transform must be both real and vanish for negative frequencies.

In order to monitor the IF of the beam bridge during vibration, the wavelet power spectrum $S(a, b)$ is used in this study. The wavelet power spectrum is defined simply as the square modulus of the wavelet transform

$$S(a, b) = |W(a, b)|^2 \tag{31}$$

The frequency of the vibration with main energy in time can be tracked using the wavelet power spectrum, the variation of the frequency during vibration or the IF can then be determined.

Instantaneous frequency

A sudden crack will lead to a sudden change in the stiffness of the beam and as a result the frequency of the beam will be changed. Therefore, monitoring the change in frequency can answer the questions whether there is a crack in the beam and when the crack occurs. However, this change in frequency and the moment when the crack is induced are generally very difficult to detect by visually inspecting the vibration signal in the time domain, even in the frequency domain. Meanwhile, the wavelet analysis transforms the signal into the frequency domain while the time information is still kept. Therefore, the wavelet transform is useful for monitoring the change in frequency of the beam during vibration. For this purpose the wavelet spectrum as defined in (31) is applied. The energy of a vibration signal is mainly concentrated on the time-scale plane around the ridges of the wavelet spectrum. Thus, the instantaneous frequency (IF) of the signal can be monitored by tracking the change in ridges of the time-scale plane. In wavelet spectrum, the scale is use instead of the frequency, so for the purpose of convenience, the time-scale is converted to the time-frequency plane by relating the scale to the pseudo-frequency as follows

$$F_a = \frac{F_c}{a\Delta} \tag{32}$$

where a is a scale, Δ is the sampling period, F_c is the center frequency of a wavelet function in Hz, F_a is the pseudo-frequency corresponding to the scale a, in Hz.

4. THE PROCEDURE FOR SUDDEN CRACK DETECTION

The procedure for monitoring a sudden crack in the beam-like bridge is performed in the following steps:

- In the first half of the excitation, bridge is modeled as an intact beam with the global matrices $\mathbf{M}$ and $\mathbf{K}$ assembled from element matrices $\mathbf{m}_e$ and $\mathbf{k}_e$ in finite element analysis. Rayleigh damping in the form of $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ is used for the beam. Where α and β are calculated from [25].

- In the second half of the excitation, the bridge is modeled as a cracked beam with the stiffness matrix of the cracked element $\mathbf{k}_e^c$ is calculated from Eq. (24). The global stiffness matrix $\mathbf{K}$ of the cracked beam is assembled from the intact element matrix $\mathbf{k}_e$ and cracked element stiffness matrix $\mathbf{k}_e^c$.

- Substituting matrices **M, K** and **C** into Eq. (1) and solving this equation by Newmark algorithms, the vertical acceleration of the beam will be obtained.

- Extracting the IF from the wavelet spectrum of the vertical acceleration. The existence of the crack is monitored by a decrease in the IF during the excitation. The moment of appearance of the crack is the time at which the IF starts to decrease.

5. SIMULATION RESULTS AND DISCUSSIONS

A numerical simulation of a beam-like bridge under earthquake excitation has been carried out. Parameters of the beam are: Mass density is 7855 kg/m^3; modulus of elasticity $E = 2.1 \times 10^{11}$ N/m^2; $L = 50$ m; $b = 1$ m; $h = 2$ m. Modal damping ratios for all modes are equal to 0.002. The crack is located at the location $L_c = L/2$.

Five levels of the crack from zero to 40% were examined. These five cases are numbered as in Tab. 1.

Table 1. Five cases with cracks of varying depths

Case	Crack depth (%)
1	0
2	10
3	20
4	30
5	40

Fig. 4 presents the 1500 sampling points for the first duration of $T = 30$ s of the records El Centro (1940) in N–S, E-W and vertical directions with $dt = 0.02$ s. It is assumed that the N-S direction coincides with the z-axis, E-W direction coincides with the x-axis, and vertical direction coincides with the y-axis of the beam bridge (see Fig. 2). In order to setup the sudden crack, the amplitude of displacement of the intact beam is first investigated. As can be seen from Fig. 5, the amplitude of displacement becomes significant in the duration from 7 s to 22 s. Therefore, the crack might be occurred in this duration. In this study, it is assumed that the crack is appeared suddenly at the moment $t = 15$ s. During the first half of the excitation, the stiffness matrix of the intact beam is used and in the second half of the excitation, the stiffness matrix of a cracked element is applied to Eq. (1). Solving this equation by Newmark algorithms, the dynamic response of the beam is obtained. In practice, the acceleration signal is easily measured, thus the vertical acceleration time history at the centre of the bridge is obtained during earthquake excitation for crack detection purposes.

Wavelet coefficients are usually used to detect small and local changes in the signal. However, the acceleration signal of the bridge depends on the earthquake excitation and

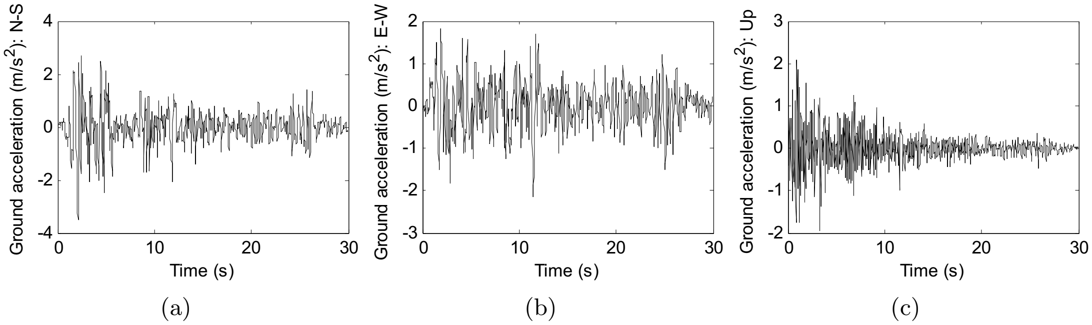

Fig. 4. Ground acceleration: a) N-S direction; b) E-W direction; c) Vertical direction

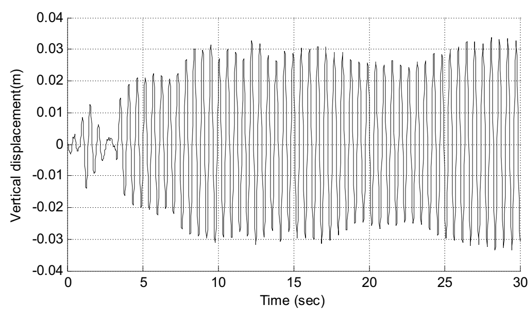

Fig. 5. Displacement of the beam under ground excitation, crack depth is 0%

may have local changes which are more significant than the local change caused by the crack. While, the change in natural frequencies of the bridge presented in this study depends only on the appearance of crack, not the excitation. Therefore, instead of using wavelet coefficients the IF extracted from the wavelet power spectrum is applied in order to monitor the sudden crack during earthquake excitation. In this study, wavelet families have been tested to select the best wavelet. Experience has shown that the Wavelet Symlet is most suitable for this work. Therefore, the wavelet functions "Symlet" is chosen as the most suitable for signal processing. The vertical acceleration of the bridge during the ground excitation is first used to calculate the wavelet power spectrum the IF is then extracted from the main ridge of the wavelet power spectrum.

Fig. 6 presents the Fourier spectrum calculated from the vertical acceleration of the beam during earthquake excitation when the crack with depth of 40% appeared at moment $t = T/2$. In this figure the frequency of 1.78 Hz corresponds to the first frequency in the first half of the excitation when there the crack depth is zero. While the frequency of 1.3 Hz corresponds to the first frequency in the second half of the excitation when there is a crack with depth of 40%.

Fig. 7 presents the IF calculated from the vertical acceleration of the beam during the ground excitation with the crack depth from zero to 40% of the beam height. In graph a) of this figure when the crack depth is zero the IF is about 1.78 Hz corresponds to the

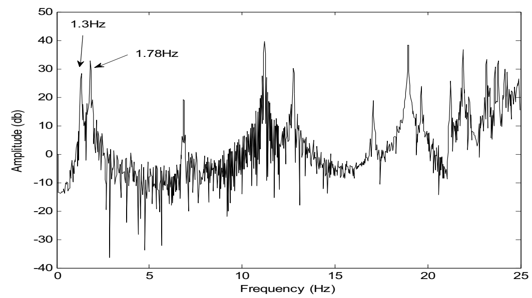

Fig. 6. Fourier spectrum of the vertical acceleration, crack depth is 40%

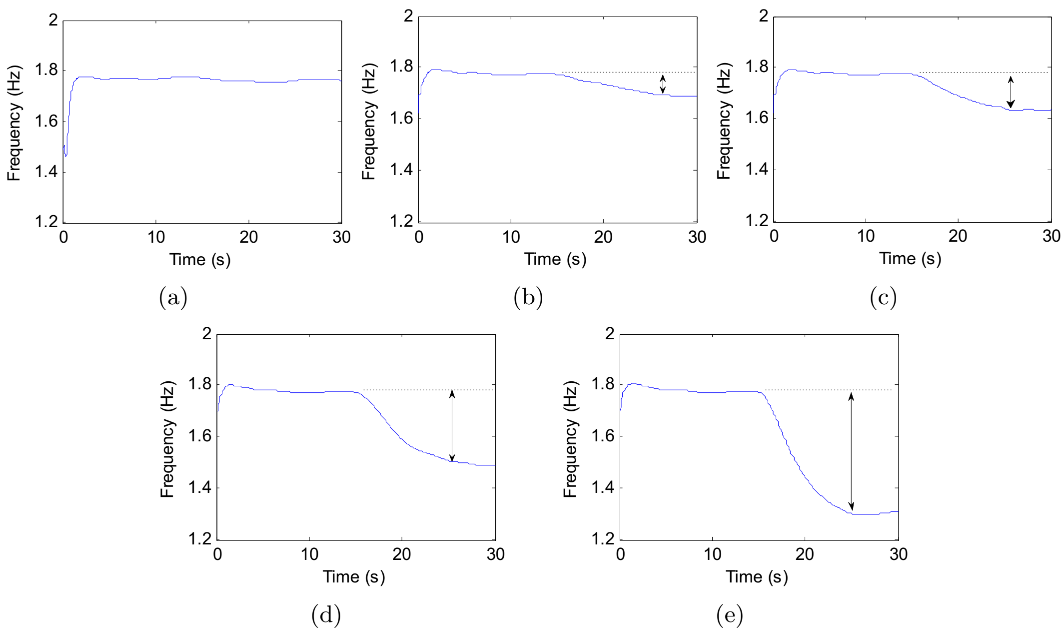

Fig. 7. The IF vs. time: a) crack depth = 0%; b) crack depth = 10%; c) crack depth = 20%; d) crack depth = 30%; e) crack depth = 40%;

first frequency in case of intact beam as can be seen in Fig. 6. This value of 1.78Hz of the IF remains the same during the entire time of excitation because there is no crack appearing during earthquake excitation.

However, when there is a small crack of 10% of the beam height appearing at $t = T/2$ as can be seen from graph b) of Fig. 7, the IF is about 1.78 Hz in the first half of the ground excitation duration, while in the second half of the ground excitation duration the IF moves decreases slightly to the smaller value of about 1.7 Hz. The reason is that, in the first half of the excitation when there is no crack present, the beam is intact beam so the IF remains at about 1.78 Hz. Meanwhile, in the second half of the excitation the beam is cracked leading to a reduction of the stiffness of the beam and as a result, the IF of the beam decreases to 1.78 Hz and remains at this value during the whole second half of the excitation.

As can be seen from graphs c) to e) of Fig. 7, when the crack depth increases to 20%, 30% and 40% respectively, in the first half of the excitation the IF remains within the frequency range centered at about 1.78 Hz which corresponds to the case of an intact beam. Meanwhile, in the second half of the wavelet spectrum, the IF decrease to about 1.3 Hz as the crack depth increases up to 40%. Therefore, the moment of appearance of a sudden crack in the bridge can be determined by the time at which the IF starts to decrease. A conclusion can be drawn from these Figs is that, when the crack depth increases, the IF in the second half of the excitation duration decreases.

Influence of noise

In order to simulate the polluted measurements, white noise is added to the calculated responses of the bridge. The noisy response is calculated as following formula [26]

$$a_{noisy} = a + E_p N \sigma(a) \tag{33}$$

where a is the vertical acceleration of the bridge obtained from the numerical simulation. E_p is the noise level and N is a standard normal distribution vector with zero mean value and unit standard deviation. a_{noisy} is the noisy acceleration, and $\sigma(a)$ is its standard deviation.

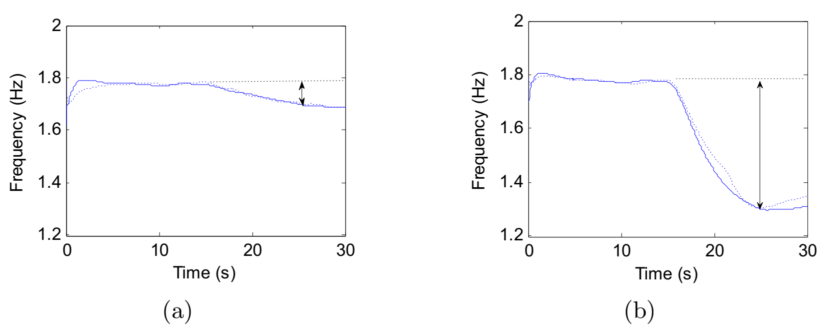

Fig. 8. The IF vs. time: a) Crack depth = 0%; solid line: 0% noise; dotted line: 10% noise; b) Crack depth = 10%; solid line: 0% noise; dotted line: 15%

Fig. 8 presents the IFs of the noisy and unnoisy responses of the bridge with the crack depth of 10% and 40%. When the crack depth is as small as 10%, the crack can be detected with the noise level of 10%. When the crack depth is 40%, the crack can be detected with the noise level up to 15%.

6. SUMMARY AND CONCLUSION REMARKS

In this study a technique for monitoring a sudden crack induced in earthquake excitation based on the wavelet power spectrum has been presented. Some remarks can be presented as follows:

- The existence of the crack is monitored by a decrease in the IF during the excitation. The moment of appearance of the crack is the time at which the IF starts to decrease.
- When the crack depth increases, the change in the IF increases. The level of change in the IF can be used to describe the crack extent.
- The advantage of the proposed method is that it does not require any information of the intact structure sine the change in IF can be detected from only one measured time history signal.
- Another advantage of the proposed method is that the crack with depth as small as 10% of the beam height can be detected.
- Since the IF, a global parameter of structures, is used for monitoring the crack, it can be extended to apply to more complicated structures.
- The proposed method can be applied with the noise level up to 15%.

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