Vietnam Journal of Mechanics, VAST, Vol. 34, No. 3 (2012), pp. 203 – 210

A NONLILEAR CONTROLLER FOR SHIP AUTOPILOTS

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Abstract. Conventional ship autopilots are designed based on a linear ship model using pole - placement technique or linear optimal theory. However, in operation, the ship kinematical parameters can go out the linear limits. In this paper, a nonlinear optimal control law based on aggregated variables is presented. The criterion is chosen so that the dynamic characteristics of object are included. The stability of the closed-loop system is global according to the Lyapunov stability theory. The control law depends explicitly on ship model parameters, so that it is can be easily to tune when the parameters change.

Key words: Ship autopilot, nonlinear control, aggregated regulator, aggregated variable.

1. INTRODUCTION

Conventional ship autopilots are designed based on a linear ship model using pole - placement technique or linear optimal theory without considering steering dynamics. However, in operation, the ship kinematical parameters can go out the linear limits; the steering mechanism has limit speed of deflection. In such a case the stability of the closed - loop system may not be guaranteed. More ever, neglecting the nonlinearity and dynamics of steering machine may effect on system performance, usually reduces it. For a nonlinear system the back stepping or state feedback linearization techniques can be used [1, 2], but no optimal criterion is considered in these approaches. In this paper, a nonlinear optimal control law based on so - called aggregated variables is presented. The criterion is chosen so that the dynamic characteristics of object are included. The stability of the closed - loop system is global according to the Lyapunov stability theory. The control law depends explicitly on ship model parameters, so that it is can be easily to tune when the parameters change. The structure of the paper is organized as follows. A brief description of controller design method and ship motion model is presented in the next section. Controller designing and simulation are presented in the Sections 3 and 4 respectively. Conclusion is presented in the last section.

2. A BRIEF DESCRIPTION ON CONTROLLER DESIGN METHOD, SHIP STEERING EQUATIONS OF MOTION AND WAVE MODEL

2.1. Controller design method based on aggregated variables

A framework of design method based on so - called aggregated variables is proposed by Kolesnikov [3]. In this section, the theoretical basics is taken from [4]. Consider a dynamic system

$$\begin{aligned} &\dot{x}_i = f_i(x), i = 1 \div n - m, \\ &\dot{x}_{n-m+j} = f_{n-m+j}(x) + b_j u_j, j = 1 \div m, \\ &x = (x_1, x_2, ..., x_n)^T, \end{aligned} \tag{1}$$

where $x = (x_1, x_2, ..., x_n)^T$ - vector of state variables, u_j - control actions.

Let $\Psi_j(x) = 0,\ j = 1 \div m$ - aggregated macro variables. The motion of synthesis system has to satisfy minimum of cost functions defined as

$$J_j = \int_{t_0}^{t} \left(\dot{\Psi}_j^2 + \phi_j^2(\Psi_j)\right) d\tau, \tag{2}$$

where $\phi_j(\Psi_j)$ - a function of $\Psi_j(x)$, which must be chosen so that, the differential equation $\dot{\Psi}_j + \phi_j(\Psi_j) = 0$ has stable solution $\Psi_j(x)$.

Stable minimum of the cost function is solution of the following differential equations

$$\dot{\Psi}_j + \phi_j(\Psi_j) = 0. \tag{3}$$

Define $\dot{\Psi}_j$ as

$$\dot{\Psi}_j = \sum_{i=1}^{n} \frac{\partial \Psi_j}{\partial x_i} f_i(x) + \sum_{i=n-m+1}^{n} \frac{\partial \Psi_j}{\partial x_i} b_{n-i} u_{n-i}. \tag{4}$$

We have the control law

$$u_j = -\left(\frac{\partial \Psi_j}{\partial x_{n-m+j}} b_j\right)^{-1} \left(\sum_{i=1}^{n} \frac{\partial \Psi_j}{\partial x_i} f_i(x) + \phi_j(\Psi_j) + \sum_{\substack{i = n-m+1 \\ i \neq n-m+j}}^{n} \frac{\partial \Psi_j}{\partial x_i} b_{n-i} u_{n-i} \right) \tag{5}$$

2.2. Ship steering equations of motion

For heading problem, the most used ship mathematical models are following:

The models of Nomoto et al [1, 5]

For small rudder angles, the transfer function between the rudder angle δ and the yawing rate ω of a surface ship can be described by the linear models of Nomoto *et al.* Nomoto's 2$^{\text{nd}}$ order model is written as

$$\frac{\omega(s)}{\delta(s)} = \frac{K(1 + T_3 s)}{(1 + T_1 s)(1 + T_2 s)}, \tag{6}$$

where s is used to denote the variable of Laplace operator, K is the gain constant and T_i $(i = 1, 2, 3)$ are three time constants. A first - order approximation is obtained by defining the effective time constant as: $T = T_1 + T_2 - T_3$. Hence,

$$\frac{\omega(s)}{\delta(s)} = \frac{K}{1 + Ts}. \tag{7}$$

The yaw angle (course) $\varphi(t)$ is related to yaw rate $\omega(t)$ as

$$\dot{\varphi}(t) = \omega(t). \tag{8}$$

The model of Bech and Wagner Smith [1, 6]

The linear ship steering equations of motion can be modified to describe large rudder angles and course - unstable ships by simply adding a nonlinear maneuvering characteristic to Nomoto's 2^{nd} order model. Bech and Wagner proposed the model

$$T_1 T_2 \varphi^{(3)} + (T_1 + T_2)\ddot{\varphi} + K H_B(\dot{\varphi}) = K(\delta + T_3 \dot{\delta}), \tag{9}$$

where the function $H_B(\dot{\varphi})$ describes the nonlinear maneuvering characteristic produced by Bech's reverse spiral maneuver, that is

$$H_B(\dot{\varphi}) = b_3 \dot{\varphi}^3 + b_2 \dot{\varphi}^2 + b_1 \dot{\varphi} + b_0. \tag{10}$$

For a course - stable ship, the $b_1 > 0$. A single screw propeller or ship with asymmetry in hull has b_2 of non - zero value. Hence, $b_2 = 0$ for a symmetrical hull.

The model of Norrbin [1, 7]

An extension of Nomoto's 1^{st} order model can be made by defining

$$T\varphi^{(2)} + H_N(\dot{\varphi}) = K\delta, \tag{11}$$

where the maneuvering characteristic $H_N(\dot{\varphi})$ is defined as

$$H_N(\dot{\varphi}) = n_3 \dot{\varphi}^3 + n_2 \dot{\varphi}^2 + n_1 \dot{\varphi} + n_0. \tag{12}$$

The Norrbin coefficients: $n_i (i = 0 \div 3)$ are related to those of Bech's model by: $n_i = b_i / |b_1|$. $n_1 = \pm 1$ for stable and unstable - course ship respectively. Since a constant rudder angle is required to compensate for constant or slowly - varying wind and current disturbances, the bias term n_0 could be treated as an additional rudder off - set. That is, a large number of ships can be described by

$$H_N(\dot{\varphi}) = n_3 \dot{\varphi}^3 + n_1 \dot{\varphi}. \tag{13}$$

2.3. The wave model

The following wave model is adopted from Paulsen et al [8]

$$\dot{x}_w = A_w x_w + b_w \eta, \quad w = C^T x_w, \tag{14}$$

where

$$A_w = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}, \quad b_w = \begin{bmatrix} 0 \\ K_w \end{bmatrix}, \quad C_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{15}$$

η is a zero mean Gaussian white noise sequence, ω_n - the dominating wave frequency, ξ - the relative damping ratio of the wave, K_w is the gain that is dependt on the wave energy. In the transfer function form the model is

$$w = C^T(sI - A_w)^{-1}B = \frac{K_w s}{s^2 + 2\xi\omega_n s + \omega_n^2}. \tag{16}$$

3. CONTROLLER DESIGN

For designing of control law, the model of Norrbin is chosen. By combining the steering mechanism dynamics, described by equation $\dot{\delta} = u$, the ship steering equation of motion, wave model and denoting $x_1 = \varphi,\ x_2 = \omega = \dot{\varphi},\ x_3 = \delta$, we have the augmented system dynamics on the wave as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{n_1}{T}x_2 - \frac{n_3}{T}x_2^3 + \frac{K}{T}(x_3 + w), \\ \dot{x}_3 &= u. \end{aligned} \tag{17}$$

To design the control law we use the system dynamics on the still water $w = 0$. Then, the autopilot performance is tested in 2 cases: still water and in wave. Define the aggregated variable as

$$\psi = c_1 x_1 + c_{21} x_2 + c_{23} x_2^3 + c_3 x_3. \tag{18}$$

Choose the cost function as

$$J = \int_{t_0}^{t} \Big(T_1^2 \dot{\Psi}^2 + \Psi^2)\Big) d\tau,\ T_1 > 0. \tag{19}$$

Then, the control signal u is defined from the equation $T_1\dot{\psi} + \psi = 0$ as

$$\begin{aligned} u = -\frac{1}{c_3}\Bigg[c_1\frac{1}{T_1}x_1 + (c_1 - c_{21}\frac{n_1}{T} + \frac{c_{21}}{T_1})x_2 + (\frac{c_{23}}{T_1} - c_{21}\frac{n_3}{T} - 3c_{23}\frac{n_1}{T})x_2^3 - \\ -3c_{23}\frac{n_3}{T}x_2^5 + (\frac{c_3}{T_1} + c_{21}\frac{K}{T})x_3 + 3c_{23}\frac{K}{T}x_2^2 x_3 \Bigg]. \end{aligned} \tag{20}$$

Under the action of this control signal, the point defining system position in state space is moved into neighborhood of the surface $\Psi = 0$. In this case, system motion along it is described by a set of differential equations as following

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{K}{T}\frac{c_1}{c_3}x_1 - (\frac{n_1}{T} + \frac{K}{T}\frac{c_{21}}{c_3})x_2 - (\frac{n_3}{T} + \frac{K}{T}\frac{c_{23}}{c_3})x_2^3. \end{aligned} \tag{21}$$

The design parameters c_1, c_{21}, c_{23}, c_3 must be chosen so that the motion is stable. Consider a Lyapunov function candidate as

$$V = \frac{1}{2}p_1 x_1^2 + \frac{1}{2}p_2 x_2^2,\ p_i > 0,\ i = 1, 2. \tag{22}$$

Time differentiation gives

$$\dot{V} = p_1\dot{x}_1x_1 + p_2\dot{x}_2x_2 = x_1x_2(p_1 - p_2\frac{K}{T}\frac{c_1}{c_3}) - x_2^2(\frac{n_1}{T} + \frac{K}{T}\frac{c_{21}}{c_3})p_2 - x_2^4(\frac{n_3}{T} + \frac{K}{T}\frac{c_{23}}{c_3})p_2. \quad (23)$$

By the choice of

$$\frac{c_1}{c_3} = \frac{p_1}{p_2}\frac{T}{K}, \ \ \frac{c_{21}}{c_3} > -\frac{n_1}{K}, \ \ \frac{c_{23}}{c_3} > -\frac{n_3}{K}. \quad (24)$$

we have

$$\frac{n_1}{T} + \frac{K}{T}\frac{c_{21}}{c_3} > 0, \ \ \frac{n_3}{T} + \frac{K}{T}\frac{c_{23}}{c_3} > 0. \quad (25)$$

Hence $V \geq 0$

$$\dot{V} = -x_2^2(\frac{n_1}{T} + \frac{K}{T}\frac{c_{21}}{c_3})p_2 - x_2^4(\frac{n_3}{T} + \frac{K}{T}\frac{c_{23}}{c_3})p_2 \leq 0. \quad (26)$$

Obvioualy, when $t \to \infty, x_2 \to 0$, from the 2^{nd} equation of (21) we get

$$\frac{K}{T}\frac{c_1}{c_3}x_1 = 0. \quad (27)$$

Since $\frac{K}{T}\frac{c_1}{c_3} \neq 0$, then $x_1 = 0$. Then the global stability is guaranteed.

4. SIMULATION STUDY

The ship model used in the simulation study is adopted from Fossen [1]. The ship parameters are following: $K = 0.5(1/\text{s}),\ T = 31(\text{s}),\ n_1 = 1,\ n_3 = 0.4(\text{s}^2)$.

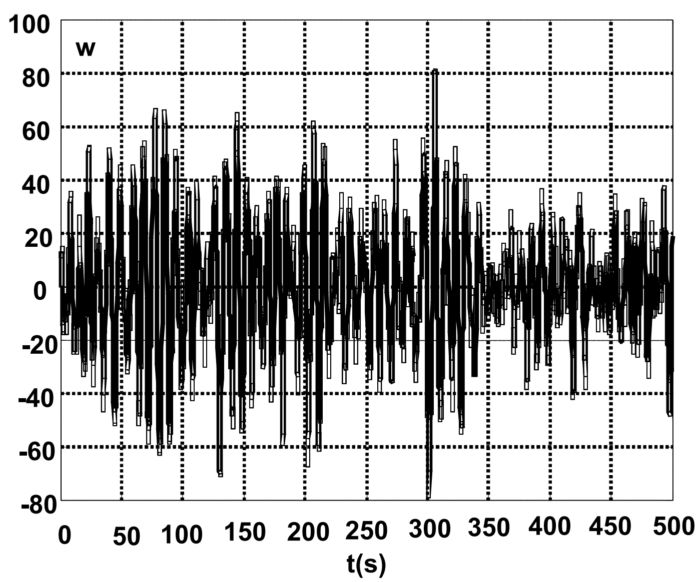


Fig. 1. Wave disturbance w versus time

Clearly it is a course - stable ship. The parameters of the wave model are chosen as: $\omega_n = 0.7,\ \xi = 1,\ K_w = 1.0$ [8], the wave characteristic in the time domain is presented in Fig. 1. The domain of design parameters variation and the phase portrait of the closed - loop system are shown in Figs. 2, 3 respectively.

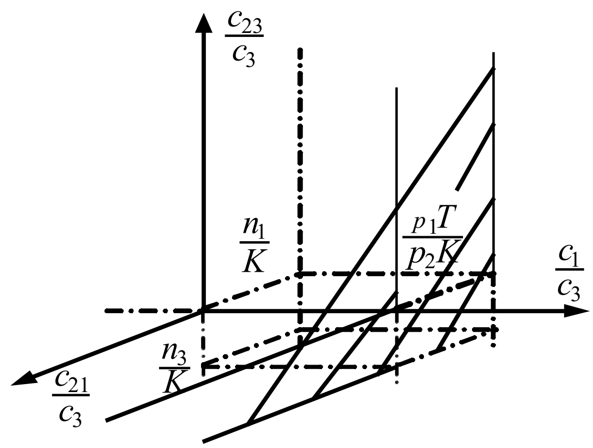


Fig. 2. Domain of design parameters

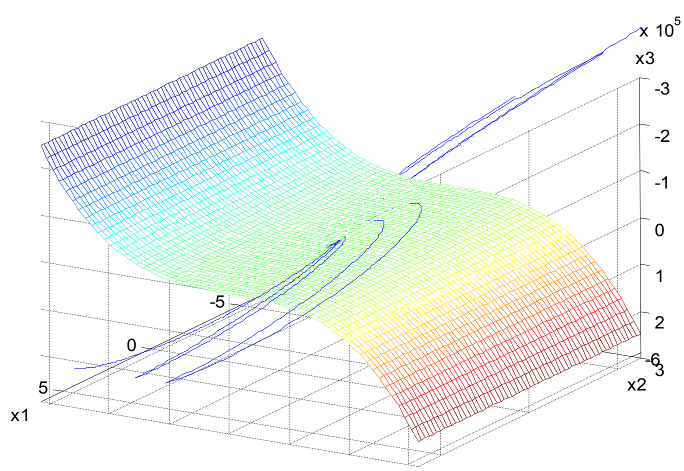
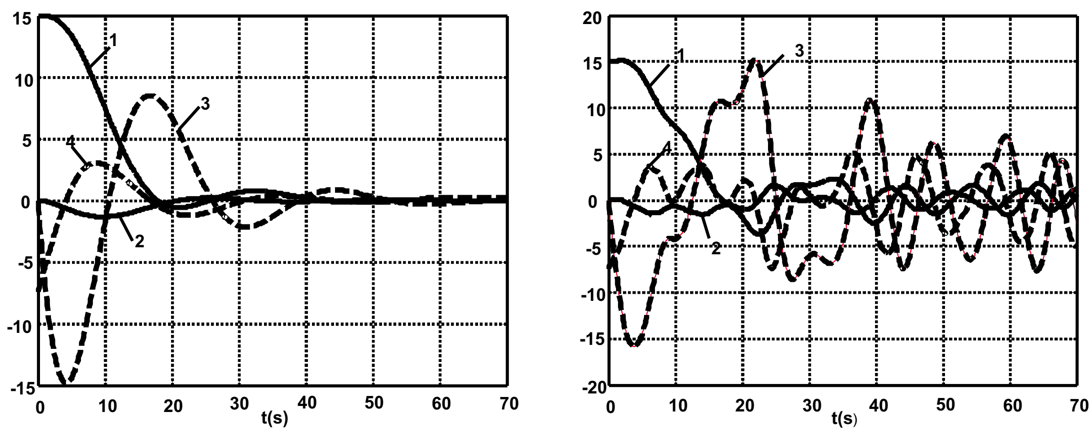


Fig. 3. Phase portrait of closed - loop system

The desired yaw angle is $\varphi_r = 0$. The design parameters are chosen to be:

$$p_1 = 0.06,\ p_2 = 1,\ c_1 = 0.0037,\ c_{21} = 0.008,\ c_{23} = 0.992,\ c_3 = 0.001,\ T_1 = 7.5.$$

At first we assume no wave disturbance, that is $w = 0$. The transition characteristics of closed - loop system are shown in Fig. 4a. We see that the yaw angle and yaw rate converge to the desired value in finite time.



(a) without wave disturbance w (b) with wave disturbance w

Fig. 4. Closed - loop system characteristics: 1 - yaw angle φ (deg), 2 - yaw rate ω (deg/s), 3 - rudder angle δ(deg), 4 - control signal u(deg/s)

In the Fig. 4b, the change of state variables and control signal in presence of the disturbance is presented. We see that, the yaw angle is bounded approximately by the value of 1 deg after about 70 s, the rudder angle - 5 deg after the same time.

5. CONCLUSIONS

A nonlinear autopilot based on aggregated macro variables has been derived. The domain of design parameters has been defined. The control law depends explicitly on the ship parameters so that it is flexible to change. The performance of the controller was illustrated through a simulation study. Global stability of closed - loop system was proven by applying Lyapunov stability theory.

ACKNOWLEDGEMENTS

This work has been partially completed by supports of the scientific research program "Research and development technology applied for mini submarine" (coded: BK-02.03) at Hanoi University of Science and Technology.

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Received August 6, 2011