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AN ANALYTICAL NONLINEAR DISPLACEMENT MODEL OF ELECTROTHERMAL V-SHAPED ACTUATOR

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Abstract. This work presents an analytical model to determine nonlinear displacements of electrothermal V-shaped actuators. The nonlinear displacement model of V-shaped beams fixed at both ends is established based on considering the axial deformation of the beam. The 3D model of the V-shaped microactuator was established to verify the theoretical nonlinear model. The evaluation shows that the displacement deviation between the analytical nonlinear model and simulation is approximately 7.7% at the driving voltage of 16 V. This confirms the advantages of the proposed model to predict more precisely the displacement of the electrothermal V-shaped actuator.

Keywords: nonlinear strain-displacement relation, nonlinear displacement, electrothermal V-shaped actuator (EVA), thermal-mechanical model.

1. INTRODUCTION

Electrothermal actuator (ETA) is a type of MEMS (Micro Electro-Mechanical Systems) device that works on the principle of converting electrical energy into heat and thermal expansion. Among them, the V-shaped beam is the most widely being used, due to the outstanding advantages such as large driving force, small driving voltage, simple structure, larger displacement-to-size ratio compared to others as mentioned in [1,2]. Currently, the electrothermal V-shaped actuators (EVA) are frequently used in driving micro-mechanical systems such as micro grippers [3], micro motors [4], nanomaterial testing devices [5] or thermally safe devices [6], etc.

Research on using mathematical methods to describe the heat transfer process of EVA has been developed recently. Direct analytical methods have been used to model steady state heat transfer [7], or transient heat transfer [8]. The finite difference heat transfer model was used to determine the change and distribution of temperature on the V-beam as shown in [9, 10]. These works have provided methods for calculating more precisely the temperature distribution on the thin beam. However, the nonlinear deformation of the V-beam has not been considered when determining the displacement of the EVA. In publications [11, 12], we have applied the finite difference model to calculate more exactly the thermal expansion force of the EVA, but the displacement is examined by the linear formula, so the tolerance is still quite large.

In practice, the displacement of EVA often needs to be large and leads to a nonlinear stress-displacement relationship of the V-beams. The development of a nonlinear displacement model of the V-beam allows to determine more accurately the displacement of the actuator. This is essential as well as valuable to predict accurately the displacement of micro devices, likes in measuring mechanical properties of micron specimens [13] or in electrical switches [14]. In [15], the authors have proposed a thermal strain model by direct analysis method, in which has considered the first-order nonlinear strain-displacement relationship. The results of this model are used to predict the maximum stress on the beam to avoid plastic deformation. However, the nonlinear model can only determine the displacement according to the average temperature on the beam and is applicable to the V-beam made of nickel material.

This work establishes a nonlinear displacement model of the EVA by analytical method. Additionally, this nonlinear model is also combined with the nonlinear heat transfer model developed by the finite difference method as mentioned in [11,12]. The integration of two models will allow determining more accurately the displacement of the EVA when taking into account the nonlinear deformation of the V-beam system fabricated from silicon by traditional SOI-MEMS technology. The result helps the designers easily select the applied voltages as well as accurate dimensions of the driving V-shaped actuator, aims to obtain an appropriate displacement for the working requirements of MEMS devices such as microgripper, micromotor or micro conveyor systems.

2. LINEAR AND NONLINEAR MODELS OF THE EVA

2.1. Configuration and linear displacement formula

The structure of a typical EVA is shown in Fig. 1. This actuator consists of the central shuttle (1) is suspended by pairs of V-shaped inclined beams (2); the other ends of these beams are attached to two fixed electrodes (3). The dimensions and geometrical parameters are denoted as follows: L, w and h are the length, width and thickness of the beam, respectively; θ is the inclined angle of the beam relative to X direction; g_a is the gap between the structural layer and the substrate (i.e. the SiO_2 buffer layer of the Silicon on Insulator-SOI wafer); n is the number of V-beam pairs.

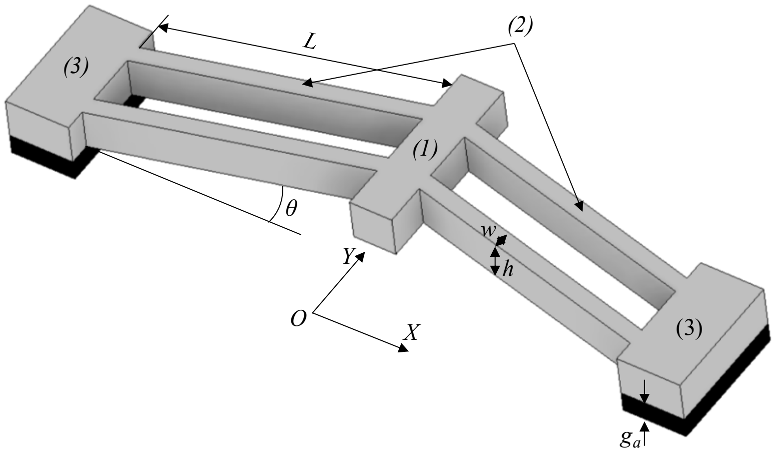

Fig. 1. Configuration of the EVA

When a voltage is applied on two fixed electrodes, the current will transmit through the beams and generate heat as well as induce the expansion of V-beam, a total expansion force pushes the central shuttle to move in Y direction. If the voltage is down to zero, the temperature on the beams gradually decreases; the beams will shrink and pull the shuttle back to its original position.

Let K be the equivalent stiffness of the V-beam system, it is determined as follows [12]

$$K = \frac{2nE(12I\cos^2\theta + AL^2\sin^2\theta)}{L^3}, \tag{1}$$

where, $I = \dfrac{hw^3}{12}$ is the inertia moment of beam cross-sectional area, $A = hw$ is the area of beam cross-section, $E = 169 \times 10^3$ MPa is a Young's modulus of silicon.

The static displacement of the EVA is determined by

$$Y_0 = \frac{F}{K}. \tag{2}$$

Here, F is the total heat expansion force acting on the shuttle in Y direction [12]

$$F = 2nAE\frac{\Delta L}{L}\sin\theta, \tag{3}$$

ΔL is the thermal expansion of a beam.

By substituting (3) and (1) into (2), we have a linear displacement as

$$Y_0 = \frac{AL^2}{12I\cos^2\theta + AL^2\sin^2\theta}\Delta L \sin\theta. \tag{4}$$

It is clear that the linear displacement Y_0 depends on the thermal expansion length ΔL as well as the geometry parameters of beam like the length L, the width w and the inclined angle θ.

2.2. Calculation of nonlinear displacement

In case the V-beam fixed at both ends, when the displacement is large enough, the length of the deformation curve is longer than the original length of the beam. Hence, there is an additional axial force against the long deformation and causes a greater stiffness of the beam. In other words, the practical displacement of the beam will be smaller than the value calculated by the linear formula (4).

Considering the fixed-end V-beam is loaded by a concentrated force with magnitude $2P$ at the center of the shuttle as shown in Fig. 2.

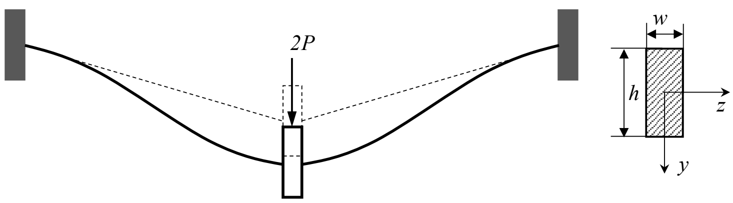

Fig. 2. Deformation of the V-beam

Due to symmetrical structure, we need only to consider the forces acting on the left half of the V-beam (Fig. 3).

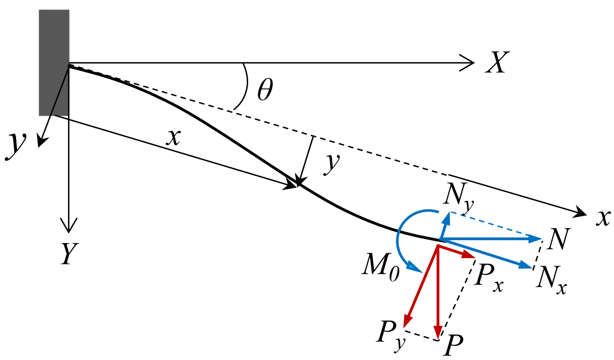

Fig. 3. Scheme of load and reaction forces acting on a half of the V-beam

The displacement differential equation of the beam in y-direction according to local coordinate Oxy (see Fig. 3) is established

$$EI\frac{d^2y}{dx^2} = (P\cos\theta - N\sin\theta)(L - x) - (N\cos\theta + P\sin\theta)\,y - M_0, \tag{5}$$

where M_0 is a reaction moment between the half-beam and the shuttle, N is an axial reaction force due to beam stretched.

The general solution of the differential equation (5) will be

$$y(x) = C_1\cosh\lambda x + C_2\sinh\lambda x + A_1 x + B_1. \tag{6}$$

From (5) and (6), we have

$$A_1 = -\frac{P\cos\theta - N\sin\theta}{N\cos\theta + P\sin\theta},\ B_1 = -\frac{M_0 - (P\cos\theta - N\sin\theta)\,L}{N\cos\theta + P\sin\theta},\ \lambda = \sqrt{\frac{N\cos\theta + P\sin\theta}{EI}}.$$

Here, C_1 and C_2 are the constants determined from boundary conditions: $\left.\frac{dy}{dx}\right|_{x=0} = 0$; $\left.\frac{dy}{dx}\right|_{x=L} = 0$; and can be expressed

$$C_1 = -\frac{P\cos\theta - N\sin\theta}{\lambda(N\cos\theta + P\sin\theta)}\tanh\left(\frac{\lambda L}{2}\right),\quad C_2 = -\frac{A_1}{\lambda} = \frac{P\cos\theta - N\sin\theta}{\lambda(N\cos\theta + P\sin\theta)}.$$

To find the value of M_0, we use the boundary condition: $\left.y\right|_{x=0} = 0$.

$$M_0 = \frac{(P\cos\theta - N\sin\theta)}{\lambda}\left(\lambda L - \tanh\left(\frac{\lambda L}{2}\right)\right).$$

The function of y-displacement can be determined by substituting A_1, B_1, C_1, C_2 and M_0 into (6)

$$y(x) = \frac{P\cos\theta - N\sin\theta}{\lambda(N\cos\theta + P\sin\theta)}\left[\sinh\lambda x - \tanh(\lambda L/2)(\cosh\lambda x - 1) - \lambda x\right]. \tag{7}$$

When the deformation of the beam element is small enough, the ratio $\frac{dy}{dx} \ll 1$, the axial expansion of the beam can be calculated as

$$\Delta L = \frac{1}{2}\int_0^L \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \mathrm{d}x. \tag{8}$$

According to scheme in Fig. 3, the axial reaction force is determined as

$$N = \frac{1}{\cos\theta}\left(AE\frac{\Delta L}{L} - P\sin\theta\right). \tag{9}$$

From (7), (8) and (9), we have

$$(N\cos\theta + P\sin\theta)^3 = \frac{AE(P\cos\theta - N\sin\theta)^2}{2}\left(\frac{3}{2} - \frac{1}{2}\tanh^2 u - \frac{3}{2}\frac{\tanh u}{u}\right), \tag{10}$$

where: $u = \dfrac{\lambda L}{2}$. From ($N\cos\theta + P\sin\theta = EI\lambda^2$) and (10), we have

$$P\cos\theta - N\sin\theta = \frac{8EI(2I/A)^{\frac{1}{2}}}{L^3}u^3\left(\frac{3}{2} - \frac{1}{2}\tanh^2 u - \frac{3}{2}\frac{\tanh u}{u}\right)^{-\frac{1}{2}}, \tag{11}$$

with $\left[N\sin\theta = \left(\dfrac{4EIu^2}{L^2} - P\sin\theta\right)\tan\theta\right]$ and Eq. (11), we infer

$$P = \frac{4EIu^2}{L^2}\sin\theta + \frac{8EI(2I/A)^{\frac{1}{2}}u^3\cos\theta}{L^3}\left(\frac{3}{2} - \frac{1}{2}\tanh^2 u - \frac{3}{2}\frac{\tanh u}{u}\right)^{-\frac{1}{2}}. \tag{12}$$

By substituting (10) and (11) into (7), the maximum y-displacement at the center of V-beam is determined as

$$y_{\max} = 2\left(\frac{2I}{A}\right)^{\frac{1}{2}}(u - \tanh u)\left(\frac{3}{2} - \frac{1}{2}\tanh^2 u - \frac{3}{2}\frac{\tanh u}{u}\right)^{-\frac{1}{2}}. \tag{13}$$

From (12) the variable u will be solved and then substituting u into (13), we will obtain the value of $y_{\max}$.

Finally, the displacement of the shuttle in Y-direction (i.e. in vertical direction) can be calculated as

$$Y_1 = \frac{y_{\max}}{\cos\theta} = \frac{2}{\cos\theta}\left(\frac{2I}{A}\right)^{\frac{1}{2}}(u - \tanh u)\left(\frac{3}{2} - \frac{1}{2}\tanh^2 u - \frac{3}{2}\frac{\tanh u}{u}\right)^{-\frac{1}{2}}. \tag{14}$$

It shows that the nonlinear displacement Y_1 also depends on the geometry dimensions of the beam as L, w, h, and θ. Besides, it is influenced by eigenvalue u or λ determined from transcendental equation (12).

3. RESULT AND DISCUSSION

To evaluate the reliability of the analytical nonlinear model, one EVA structure is considered with geometry dimensions as Table 1.

Table 1. Geometry dimensions of EVA

L (µm)	h (µm)	w (µm)	g_a (µm)	θ (0)	n
750	30	6	4	2	10

Table 2. Material parameters of silicon

d (kg/m^3)	E (MPa)	ka (W/m.K)	C_p (J/kg.K)	ρ_0 (Ω.m)	λ (1/K)
2330	1.69×10^5	0.0257	712	148×10^{-6}	1.25×10^{-3}

Table 3. Material parameters depend on temperature [16]

T (K)	300	400	500	600	700	800	900	1000	1100	1200	1300
k (W/m.K)	156	105	80	64	52	43	36	31	28	26	25
α (10^{-6}/K)	2.62	3.25	3.61	3.84	4.02	4.15	4.18	4.26	4.32	4.38	4.44

The material parameters of the silicon device layer are listed in Tables 2 and 3.

The 3D finite element model of the V-shaped beam (with the dimensions, material properties and ambient given in Tables 1, 2 and 3) was established by ANSYS WORKBENCH 15. The model is meshed with a body size of 10 micrometers in both Thermal-Electric and Static-Structural to simulate the nonlinear displacement of the V-shaped beam in the settings of the Static-Structural model and selecting the large deformation function. This is a development/difference from the linear simulation of the previous publications.

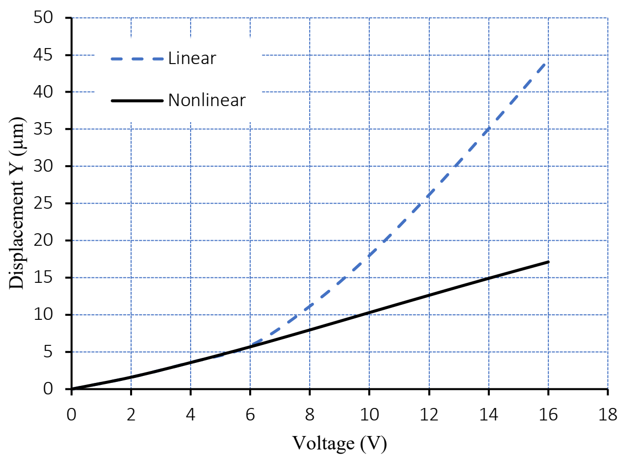

Fig. 4. Comparison between linear and nonlinear displacements

The calculation results and comparison of displacements in both of linear and nonlinear are shown in Fig. 4. The simulation results of nonlinear displacement with thermal expansion force acting on the beam at the voltage of 16 V as shown in Fig. 5. Finally, the comparison of nonlinear displacements calculated by the formula (14) and ANSYS-simulated are shown in Fig. 6.

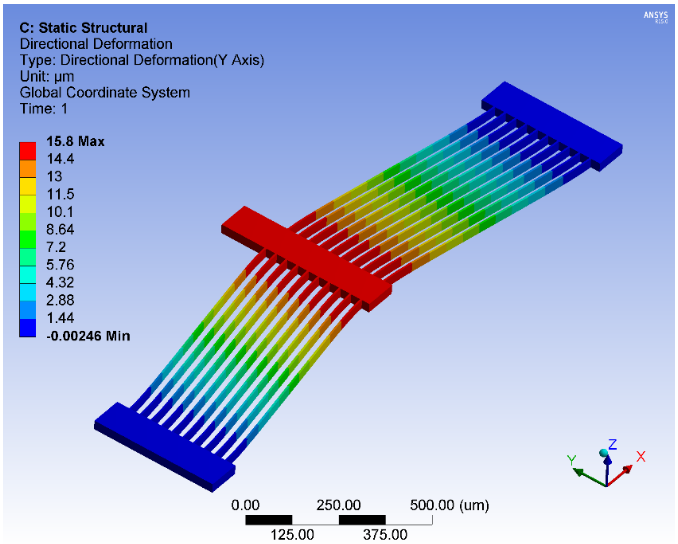

Fig. 5. Simulating displacement at the voltage of 16 V

As the displacement results are shown in Fig. 4, it is easy to see that there is a significant deviation between linear and nonlinear formulas while applied voltage is larger than 6 V. The calculated nonlinear displacement (14) increases gradually with the driving voltage and is much smaller when compared with the linear displacement (formula number 4) at the same applying voltage. As an example, at the voltage of 12 V, the linear displacement is almost twice larger than the nonlinear displacement. It can be explained by the fact that the stiffness of the V-beam system with two fixed ends increases rapidly with the displacement value Y_1. Due to the large axial internal force, the stiffness of V-beams in the moving direction of the shuttle (i.e. in Y direction) will increase.

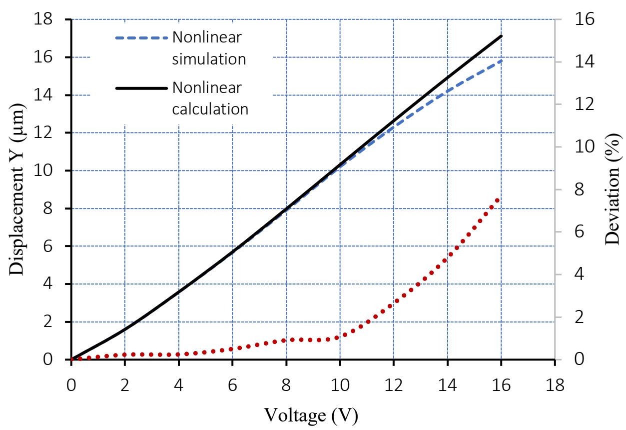

Fig. 6. Comparison between nonlinear calculation and simulation

In contrast, the results of calculation in case of nonlinear displacement and simulation have negligible deviations (approximately 7.7% at 16 V). The reason of this error is that in to simplify the analytical model some assumptions was used such as: ignoring the square term in the differential equation (5), considering only heat transfer on V-shaped beams in one direction (along the beam), ignoring the effects of convection and heat radiation. Especially at the voltages are lower than 12 V, the displacements almost match together (see Fig. 6). In other words, the proposed formula (i.e. formula number 14) has a much higher accuracy than the traditional linear formula. It enables to attain a more accurate calculation of micro devices using the V-shaped actuators.

4. CONCLUSION

The article has proposed and contributed a analytical nonlinear model for calculating more precisely the displacement of the EVA. By solving the differential equations, the formula for calculating the nonlinear displacement of the EVA has been figured out when considering the axial deformation of the V-beam. The linear, nonlinear and simulation displacements of the same EVA structure are compared to confirm the advantages of the nonlinear displacement formula. This nonlinear model can be used to design and optimize the micro systems driven by the EVA structure, aim to obtain high-precision requirements when working such as: in micro transportation system, material testing micro-size device, the positioning system during the assembly and movement of micro-specimen, etc.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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