Vietnam Journal of Mechanics, Vol. 46, No. 2 (2024), pp. 104 – 118
DOI: https://doi.org/10.15625/0866-7136/19728

WAVE PROPAGATION IN CONTEXT OF MOORE–GIBSON–THOMPSON THERMOELASTICITY WITH KLEIN–GORDON NONLOCALITY

Baljeet Singh
Department of Mathematics, Post Graduate Government College, Sector-11, Chandigarh, 160011, India
E-mail: bsinghgc11@gmail.com

Received: 21 December 2023 / Revised: 20 April 2024 / Accepted: 15 May 2024
Published online: 10 June 2024

Abstract. Understanding plane and surface waves in elastic materials is crucial in various fields, including geophysics, seismology, and materials science, as they provide valuable information about the properties of the materials they travel through and can help in earthquake detection and analysis. In the present paper, the governing equations of Moore–Gibson–Thompson (MGT) thermoelasticity are modified in context of Klein–Gordon (KG) nonlocality. For linear, homogeneous and isotropic case, the governing equations in two-dimensions are solved to obtain the dispersion relations for possible plane waves. It is found that there exists one transverse and two coupled longitudinal waves in a two-dimensional model of MGT weakly nonlocal thermoelastic medium and the speeds of these plane waves are found to be dependent on KG nonlocal parameters. The coupled longitudinal waves are also found to be dependent on conductivity rate parameter. For linear, homogeneous and isotropic case, the governing equations in two-dimensions are also solved to obtain a Rayleigh wave secular equation at thermally insulated surface. For a numerical example of aluminium material, the speeds of transverse wave, coupled longitudinal waves and the Rayleigh wave are computed and graphically illustrated to visualize the effects of KG nonlocality parameters, conductivity rate parameter and the angular frequency on the wave speeds.

Keywords: plane waves, Rayleigh wave, Moore–Gibson–Thompson thermoelasticity, secular equation, wave speed, nonlocality parameters, conductivity rate parameter.

1. INTRODUCTION

The hyperbolic-parabolic field equations of coupled thermoelasticity are developed by Biot [1]. Lord and Shulman [2] and Green and Lindsay [3] generalized this theory with the use of hyperbolic field equations. Ignaczak and Ostoja-Starzewski [4] and Hetnarski and Ignaczak [5] presented detailed review of these generalized theories with applications. Recently, Quintanilla [6] developed a theory of generalized thermoelasticity, where the heat conduction is described by the Moore–Gibson–Thompson equation.

Lord Rayleigh [7] investigated the existence of surface waves known as Rayleigh waves propagating on the free surface of an isotropic solid half-space. These waves are mainly applied for characterization of material and to investigate the structural and mechanical properties of the objects. The propagation of Rayleigh waves in thermoelastic media finds numerous uses in different engineering fields and future technologies and was explored by many investigators including Flavin [8], Chadwick and Windle [9], Ivanov [10], Abouelregal [11], Chirita [12], Bucur et al. [13], Singh [14], Singh and Verma [15] and Passarella et al. [16].

The nonlocal theory of continuum mechanics considers long-range interactions within the material which create discrepancies between the results of the classical continuum limit and the atomic theory of lattices. According to Eringen [17], the classical field theories are unable to describe various problems including stress fields at the dislocation core and at the tips of cracks, fracture of solids, sharp corners and discontinuities in bodies, singularities present at the point of application of concentrated loads and short wavelength nature of elastic waves. The theories of nonlocal elasticity were developed by Edelen and Laws [18], Edelen et al. [19] and Eringen and Edelen [20]. Some prominent researchers including Chirita [21], Iesan [22], Eringen [23], Altan [24], Nowinski [25] and Cracium [26] applied nonlocal elasticity to solve various dynamical problems. Eringen [27] also applied the linear theory of nonlocal elasticity to show the dispersive nature of elastic waves due to nonlocal parameters. Recently, Singh et al. [28] discussed the effects of nonlocal parameter on the propagation of harmonic waves in elastic solid with voids.

Eringen [29] and Balta and Suhubi [30] extended the nonlocal elasticity for thermoelastic materials. These days, the wave propagation in nonlocal thermoelasticity is a hot topic amongst various researchers. For instance, Singh [31], Pramanik and Biswas [32], Biswas [33], Lata and Singh [34], Abd-Alla et al. [35], Kumar et al. [36] and Biswas [37] studied Rayleigh surface waves in varous thermoelastic models with nonlocality in space. Jangid et al. [38] have used Moore–Gibson–Thompson thermoelasticity to study the plane harmonic waves. However, in the present paper, the Moore–Gibson–Thompson thermoelasticity with Klein–Gordon nonlocality in space and time is used to study the characteristics of Rayleigh surface waves.

Recently, the solution to the differential model in context to Eringen's theory of nonlocal elasticity has been challenged by some researchers like Romano [39] and Kaplunov et al. [40, 41] by giving some counter examples. Anh and Vinh [42] introduced a novel model of weakly nonlocal elasticity in space and studied harmonic plane waves and Stoneley waves at an interface between two weakly nonlocal half-spaces. Anh et al. [43] also studied the Rayleigh wave with impedance boundary conditions in context of weakly nonlocal elasticity.

Lazar and Agiasofitou [44] developed a weakly nonlocal elasticity theory of Klein–Gordon type for isotropic case by extending the nonlocal elasticity of Helmholtz type by including a characteristic time scale parameter in addition to a characteristic length scale parameter. In development of this theory, they used weak nonlocal elasticity instead of strong nonlocal elasticity. The nonlocality in time is of great importance due to the optic modes and frequency bad-gaps in the dispersion relations in addition to acoustic modes. Motivated by the Moore–Gibson–Thompson thermoelastic model developed by Quintanilla [6] and nonlocal elasticity of Klein–Gordon type given by Lazar and Agiasofitou [44], the governing equations of Moore–Gibson–Thompson thermoelasticity with nonlocality in space and time are developed in the present paper to explore both the plane and surface waves. The present paper is aimed to deliver some new theoretical and numerical information about the propagation of plane and surface waves in a MGT thermoelastic medium in context of KG nonlocality. The present paper is structured as follows: Section 2 deals with the two-dimensional formulation of the governing equations for an isotropic, homogeneous and linear MGT thermoelastic medium with KG nonlocality. In Section 3, the propagation of plane waves is considered and velocity equations of plane waves are obtained. In Section 4, the propagation of possible Rayleigh waves is examined and a secular equation of the Rayleigh wave is obtained. A numerical example of Aluminium material is taken in Section 5 to illustrate graphically the dependence of speeds of plane and surface waves on the KG nonlocal parameters, conductivity rate parameter and angular frequency. The concluding remarks based on theoretical derivations and numerical results are listed in Section 6.

2. GOVERNING EQUATIONS

Following Quintanilla [6] and Lazar and Agiasofitou [44], the governing equations of linear, homogeneous and isotropic Moore–Gibson–Thompson thermoelasticity with Klein–Gordon nonlocality and without body forces and heat sources, are

(a) Constitutive equations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{1}$$

$$(1 - e_0\nabla^2 + \tau^2\partial_t^2)t_{ij} = \sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \beta T)\delta_{ij}. \tag{2}$$

(b) Equations of motion

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta T_{,i} = \rho(1 - e_0\nabla^2 + \tau^2\partial_t^2)\partial_t^2 u_i. \tag{3}$$

(c) Heat Equations

$$(K^* + K\partial_t)\nabla^2 T = (\partial_t^2 + \tau_q\partial_t^3)(\rho c_v T + \beta T_0 u_{i,i}), \tag{4}$$

where σ_{ij} are the Cauchy stress tensor components for classical elasticity, t_{ij} are stress tensor components for nonlocal elasticity of Klein–Gordon type, e_{ij} are the strain tensor components, u_i are the displacement components, δ_{ij} is the Kronecker delta, λ, μ are Lame′s constants, ρ is the density of the medium, T is change in temperature with reference temperature T_0, e_0 is the characteristic internal length scale parameter due to nonlocality in space, τ is the characteristic time scale parameter due to nonlocality in time. K is the thermal conductivity, K^* is conductivity rate parameter. c_v is the specific heat at constant strain, τ_q is a non-negative parameter called as the relaxation time, $\beta = (3\lambda + 2\mu)\alpha_0$ and α_0 is the thermal expansion coefficient. $\partial_t = \partial/\partial t$ denotes the partial derivative with respect to time t. ∇^2 denotes the Laplace operator. The subscripts given after a comma symbolizes space partial differentiation. The heat equation reduces for Biot model when $K^* = 0, \tau_q = 0$, for Lord and Shulman model when $K^* = 0$, for Green–Nagdhi model of type III when $\tau_q = 0$ and for Moore–Gibson–Thompson model when $K^* \neq 0, \tau_q \neq 0$.

A thermally conducting linear, isotropic and homogeneous elastic material is considered at reference temperature T_0 in the unstrained state. A Cartesian system of axes is considered with the origin at plane surface $z = 0$ of the half-space $z \geq 0$. The positive z-axis is taken normal into the half-space. The surface $z = 0$ is assumed as stress free and without any heat transfer across the surface. The propagation direction of elastic waves is selected along the x-axis with equal displacement of particles on a line parallel to y-axis. Then, all the field quantities will not dependent on y-coordinates. Using the Helmholtz's decomposition given below

$$u_1 = \phi_{,1} - \psi_{,3}, \quad u_3 = \phi_{,3} + \psi_{,1}, \tag{5}$$

the governing equations (3) and (4) are specialized in x-z plane as under

$$(\lambda + 2\mu)\nabla_1^2\phi - \beta T = \rho(1 - e_0\nabla_1^2 + \tau^2\partial_t^2)\partial_t^2\phi, \tag{6}$$

$$(K^* + K\partial_t)\nabla_1^2 T = (\partial_t^2 + \tau_q\partial_t^3)[\rho c_v T + \beta T_0\nabla_1^2\phi], \tag{7}$$

$$\mu\nabla_1^2\psi = \rho(1 - e_0\nabla_1^2 + \tau^2\partial_t^2)\partial_t^2\psi, \tag{8}$$

where $\nabla_1^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$.

3. PLANE WAVES

The following type of plane wave solutions of Eqs. (6) to (8) are sought

$$\{\psi,\ \phi,\ T\} = \{A, B, C\} e^{\iota k(x \sin\theta + z \cos\theta) - \iota\omega t}, \tag{9}$$

where θ is the propagation angle, k is the complex wave-number and ω is the circular frequency.

Using (9) into Eqs. (6) to (8), the following velocity equations are obtained

$$V^4 + GV^2 + H = 0, \tag{10}$$

$$V^2 - \frac{e_2}{\tau_1} = 0, \tag{11}$$

where

$$\begin{gathered} V^2 = \frac{\omega^2}{k^2},\ G = \frac{\bar{K}}{i\omega} - \frac{e_1}{\tau_1} - i\omega\bar{\beta}\epsilon,\ H = -\frac{\bar{K}}{i\omega}\frac{e_1}{\tau_1},\ \epsilon = \frac{\beta T_0}{\rho c_v}, \\ c_1^2 = \frac{\lambda + 2\mu}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ e_1 = c_1^2 - e_0^2\omega^2,\ e_2 = c_2^2 - e_0^2\omega^2, \\ \tau_1 = 1 - \tau^2\omega^2,\ \bar{K} = \frac{K^* - i\omega K}{\rho c_v \tau^*},\ \tau^* = \tau_q + \frac{i}{\omega}. \end{gathered} \tag{12}$$

The two roots of velocity equation (10) show the existence of two coupled longitudinal waves with distinct speeds $v_1 = Re(V_1)$ and $v_2 = Re(V_2)$ in a linear, isotropic and homogeneous MGT thermoelastic medium. In absence of thermal parameters, the coupled longitudinal wave with speed v_2 will not exist. The root of Eq. (11) correspond to one transverse wave with speed $v_3 = \frac{e_2}{\tau_1}$.

4. RAYLEIGH SURFACE WAVE

The Rayleigh wave propagation is considered in the x-direction and decaying in the z-direction with complex wave number k and angular frequency ω. The appropriate displacement and temperature potential functions for propagation of Rayleigh waves along the surface $z = 0$, are selected as

$$\begin{aligned} \phi(x,z,t) &= f(z)e^{i(kx-\omega t)}, \\ T(x,z,t) &= g(z)e^{i(kx-\omega t)}, \\ \psi(x,z,t) &= h(z)e^{i(kx-\omega t)}, \end{aligned} \tag{13}$$

Making use of (13) in Eqs. (6) to (8), the appropriate solutions in half-space $z > 0$ for surface wave are obtained as

$$\begin{aligned}\phi(x,z,t) &= \sum_{n=1}^{2} A_n \exp[-\lambda_n + i(kx - \omega t)],\\ T(x,z,t) &= \sum_{n=1}^{2} \zeta_n A_n \exp[-\lambda_n + i(kx - \omega t)],\\ \psi(x,z,t) &= B_0 \exp[-\lambda_3 + i(kx - \omega t)],\end{aligned} \tag{14}$$

where the unknowns A_n and B_0 are the amplitudes of displacement potentials and temperature and the coupling coefficients ζ_n are derived after substituting ϕ and T in Eqs. (6) and (7) as

$$\zeta_n/k^2 = e_1 - \tau_1\frac{\omega^2}{k^2} - e_1\frac{\lambda_n^2}{k^2},\ (n = 1, 2), \tag{15}$$

or

$$\zeta_n/k^2 = \frac{i\omega^3\epsilon({\lambda_n}^2 - k^2)}{\bar{K}{\lambda_n}^2 - k^2(\bar{K} + i\omega\frac{\omega^2}{k^2})},\ (n = 1, 2), \tag{16}$$

where λ_1 and λ_2 are the solutions of the characteristic equation

$$\lambda^4 - R\lambda^2 + S = 0, \tag{17}$$

where

$$\begin{aligned}R/k^2 &= \left(1 - \frac{\tau_1}{e_1}\frac{\omega^2}{k^2}\right) + \left(1 + \frac{i\omega}{\bar{K}}\frac{\omega^2}{k^2}\right) + i\omega\frac{\epsilon\bar{\beta}}{e_1\bar{K}}\frac{\omega^2}{k^2},\\ S/k^4 &= \left(1 - \frac{\tau_1}{e_1}\frac{\omega^2}{k^2}\right)\left(1 + \frac{i\omega}{\bar{K}}\frac{\omega^2}{k^2}\right) + i\omega\frac{\epsilon\bar{\beta}}{e_1\bar{K}}\frac{\omega^2}{k^2},\end{aligned} \tag{18}$$

and λ_3 is the solution of the characteristic equation

$$\lambda^2 - k^2\left(1 - \frac{\tau_1}{e_2}\frac{\omega^2}{k^2}\right) = 0. \tag{19}$$

From Eqs. (17) and (19), it follows that

$$\lambda_1^2 + \lambda_2^2 = R,\ \lambda_1^2\lambda_2^2 = S,\ \lambda_3^2 = k^2\left(1 - \frac{\tau_1}{e_2}\frac{\omega^2}{k^2}\right). \tag{20}$$

The required boundary conditions at thermally insulated stress-free surface $z = 0$ are vanishing of the tangential nonlocal stress component, normal nonlocal stress component and the normal heat flux component at $z = 0$, i.e.,

$$t_{13} = 0,\ t_{33} = 0,\ \left(K^* + K\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = 0, \tag{21}$$

where

$$(1 - e_0\nabla_1^2 + \tau^2\partial_t^2)t_{13} = \sigma_{13} = \mu(\psi_{,11} - \psi_{,33} + 2\phi_{,13}),$$
$$(1 - e_0\nabla_1^2 + \tau^2\partial_t^2)t_{33} = \sigma_{33} = \lambda(\phi_{,11} + \phi_{,33}) + 2\mu(\phi_{,33} + \psi_{,13}) - \gamma T. \tag{22}$$

The appropriate potential functions given by (14) satisfy the conditions (21) and a homogeneous system of three equations in A_1, A_2 and B_0 are obtained after using relations given by (22). The non-trivial solution of the homogeneous system require the vanishing of the determinant of the coefficients matrix, i.e.,

$$a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}) = 0, \tag{23}$$

where

$$a_{1j} = -\lambda + (\lambda + 2\mu)\frac{\lambda_j^2}{k^2} - \beta\frac{\zeta_j}{k^2},\ j = 1,2,\ a_{13} = -2i\mu\frac{\lambda_3}{k},$$
$$a_{2j} = 2i\frac{\lambda_j}{k},\ j = 1,2,\ a_{23} = 1 + \frac{\lambda_3^2}{k^2},\ a_{3j} = (K^* - i\omega K)\frac{\lambda_j}{k}\frac{\zeta_j}{k^2},\ a_{33} = 0. \tag{24}$$

Eq. (23) is dispersion equation of Rayleigh waves along the stress-free thermally insulated surface of an isotropic and homogeneous MGT thermoelastic solid half-space with KG nonlocality. In absence of thermal effects, Eq. (23) reduces to

$$\left[2 - \frac{\tau_1}{e_1}\frac{\omega^2}{k^2}(\frac{\lambda}{\mu} + 2)\right]\left[2 - \frac{\tau_1}{e_2}\frac{\omega^2}{k^2}\right] - 4\sqrt{1 - \frac{\tau_1}{e_1}\frac{\omega^2}{k^2}}\sqrt{1 - \frac{\tau_1}{e_2}\frac{\omega^2}{k^2}} = 0. \tag{25}$$

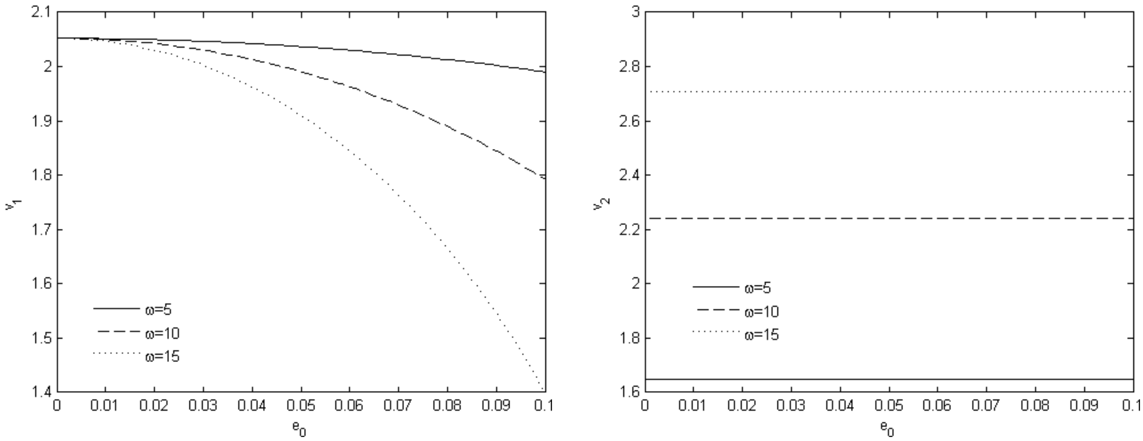
In absence of thermal and nonlocal effects, Eq. (23) reduces to

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 - 4\sqrt{1 - \frac{c^2}{c_1^2}}\sqrt{1 - \frac{c^2}{c_2^2}} = 0, \tag{26}$$

where $c^2 = \omega^2/k^2$. Eq. (26) is the secular equation of the Rayleigh wave along the free surface of an isotropic elastic solid half-space as obtained by Lord Rayleigh [7].

5. NUMERICAL ILLUSTRATIONS

For numerical illustrations of the wave speeds of homogeneous plane waves and the Rayleigh surface wave, the following relevant parameters of aluminium material at $T_0 = 300K$ are taken
$\rho = 2.7 \times 10^3$ Kg.m^{-3}, $\lambda = 7.59 \times 10^{10}$ N.m^{-2}, $\mu = 1.89 \times 10^{10}$ N.m^{-2},
$K = 0.5 \times 10^2$ W.m^{-1}.deg^{-1}, $K^* = 0.4 \times 10^2$ W.m^{-1}.deg^{-1}, $c_E = 0.2 \times 10^2$ J.Kg^{-1}.deg^{-1},
$\tau_q = 0.000005$ s., $\beta = 0.002$.
For above numerical values of material parameters, the velocity equations (10) an (11) for plane waves and the secular equation (23) for Rayleigh surface wave is solved numerically to illustrate the effects of KG nonlocal parameters, conductivity rate and angular frequency on the wave speeds of plane waves and Rayleigh wave.



(a) (b)

Fig. 1. The effect of nonlocal space parameter e_0 on the wave speed v_1 and v_2 of longitudinal wave and thermal wave, respectively when $\omega = 5, 10$ and 15

Figs. 1 and 2 illustrate the effect of nonlocal space parameter e_0 on the wave speeds v_1, v_2 and v_3 of longitudinal, thermal and transverse wave, respectively when angular frequency $\omega = 5, 10$ and 15. The speeds of longitudinal and transverse wave decreases with increasing value of e_0. The rate of decrease in value of speeds becomes faster as the value of ω increases. The speed of thermal wave is not affected significantly due to the presence of nonlocality in space.

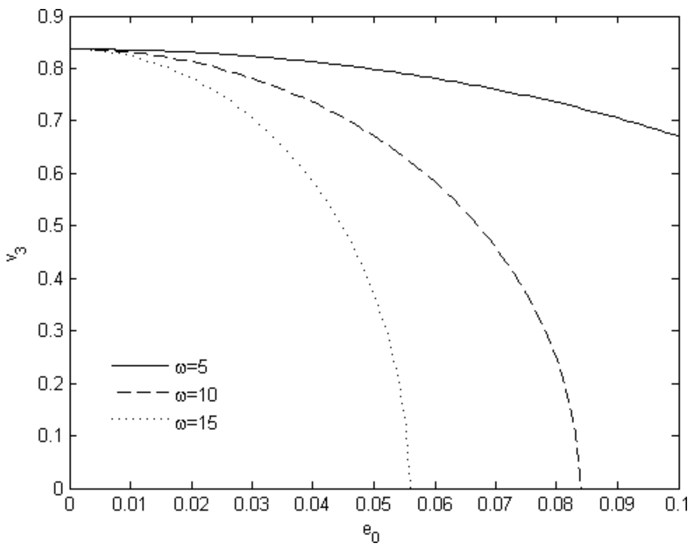
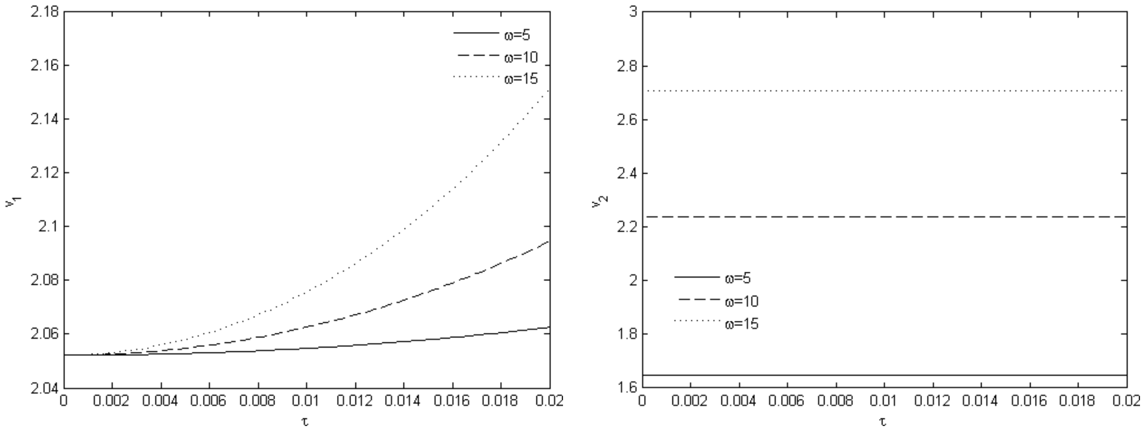


Fig. 2. The effect of nonlocal space parameter e_0 on the wave speed v_3 of transverse wave, when $\omega = 5, 10$ and 15



(a) (b)

Fig. 3. The effect of nonlocal time parameter τ on the wave speed v_1 and v_2 of longitudinal wave and thermal wave, respectively when $\omega = 5, 10$ and 15

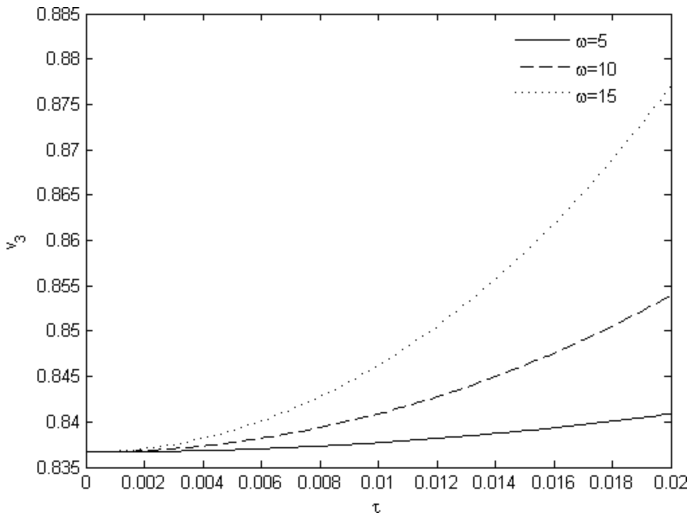
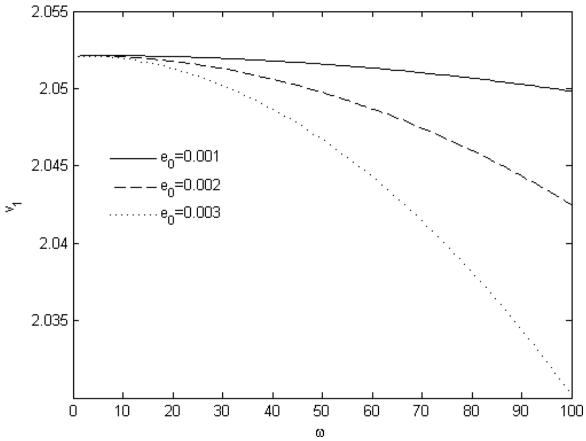
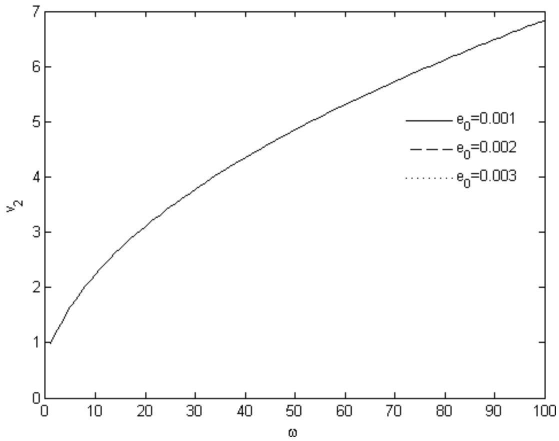


Fig. 4. The effect of nonlocal time parameter τ on the wave speed v_3 of transverse wave, when $\omega = 5, 10$ and 15

Figs. 3 and 4 illustrate the effect of nonlocal time parameter τ on the wave speeds v_1, v_2 and v_3 of longitudinal, thermal and transverse wave, respectively when $\omega = 5, 10$ and 15. The speeds of longitudinal and transverse wave increases with increasing value of τ. The rate of increase in value of speeds becomes faster as the value of ω increases. The speed of thermal wave is not affected significantly due to the presence of nonlocality in time.



(a)



(b)

Fig. 5. The effect of angular frequency ω on the wave speed v_1 and v_2 of longitudinal wave and thermal wave, respectively when $e_0 = 0.001, 0.002$ and 0.003

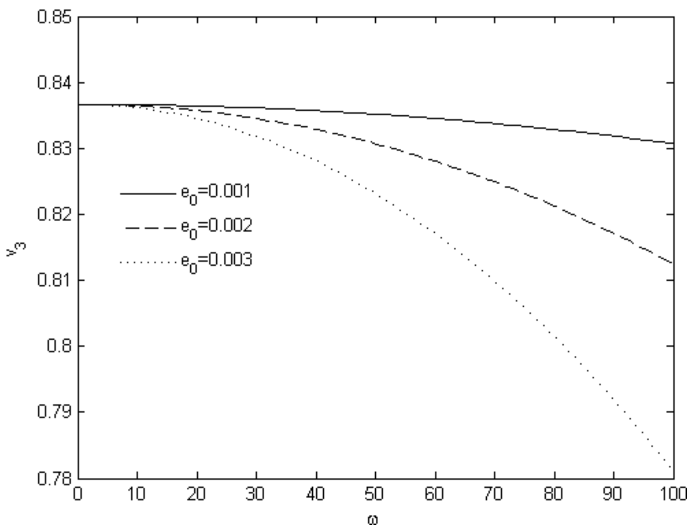
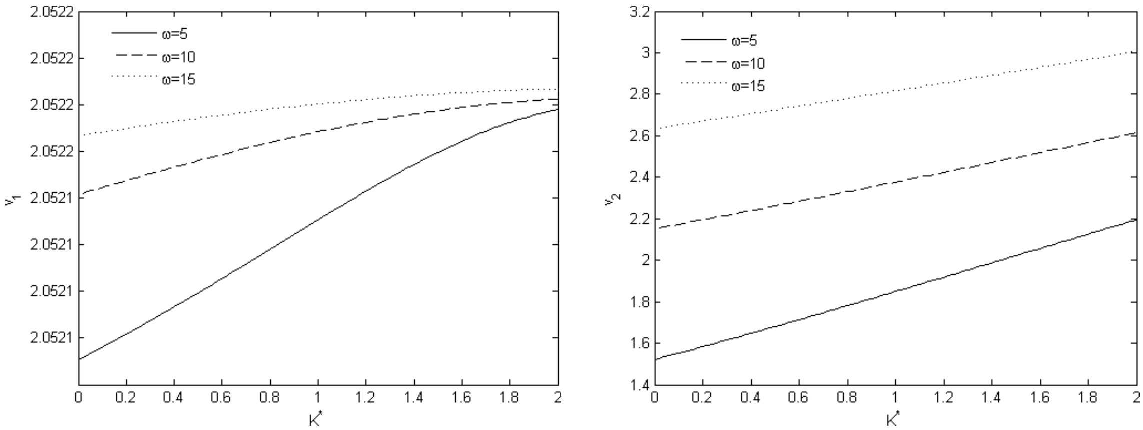


Fig. 6. The effect of angular frequency ω on the wave speed v_3 of transverse wave, when $\omega = 5, 10$ and 15

Figs. 5 and 6 illustrate the effect of angular frequency ω on the wave speeds v_1, v_2 and v_3 of longitudinal, thermal and transverse wave, respectively when $e_0 = 0.001, 0.002$ and 0.003. The speeds of longitudinal and transverse wave decreases with increasing value of ω. The rate of decrease in value of speeds becomes faster as the value of e_0 increases. The speed of thermal wave increases as the value of angular frequency increases. This rate of increase remains same for different values of e_0.



(a) (b)

Fig. 7. The effect of conductivity rate parameter K^* on the wave speed v_1 and v_2 of longitudinal wave and thermal wave, respectively when $\omega = 5, 10$ and 15

Fig. 7 illustrates the effect of conductivity rate parameter K^* on the wave speeds v_1 and v_2 of longitudinal and thermal wave, respectively when $\omega = 5, 10$ and 15. The speeds of these waves increase as the value of K^* increase. The rate of increase in speeds of these wave is slower for high frequency value.

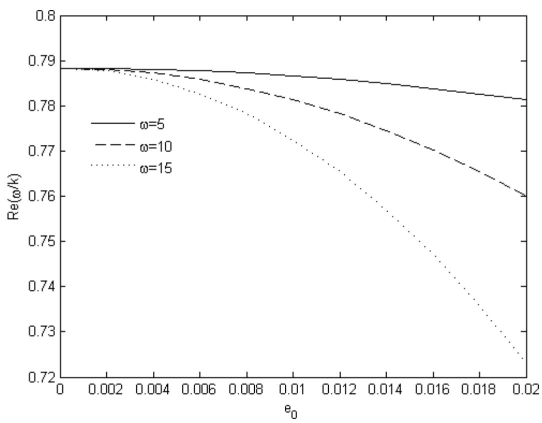


Fig. 8. The effect of nonlocal space parameter e_0 on the wave speed of Rayleigh wave, when $\omega = 5, 10$ and 15

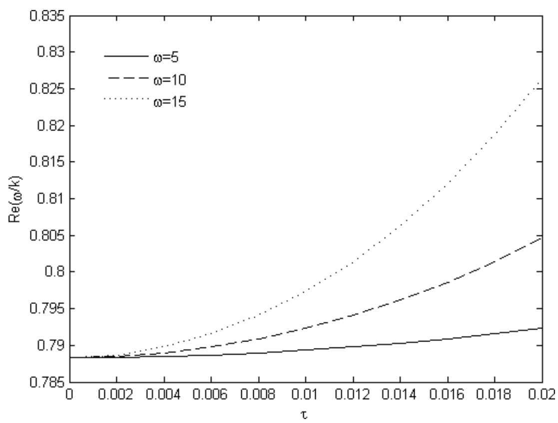


Fig. 9. The effect of nonlocal time parameter τ on the wave speed of Rayleigh wave, when $\omega = 5, 10$ and 15

Figs. 8 and 9 illustrate the effect of nonlocal parameters e_0 and τ on the wave speed of Rayleigh wave when $\omega = 5, 10$ and 15. The speeds of Rayleigh wave decreases with increasing value of e_0, This rate of decrease in value of speeds becomes faster as the value of ω increases. The speeds of Rayleigh wave increases with increasing value of τ. The rate of increase in value of speed becomes faster as the value of ω increases.

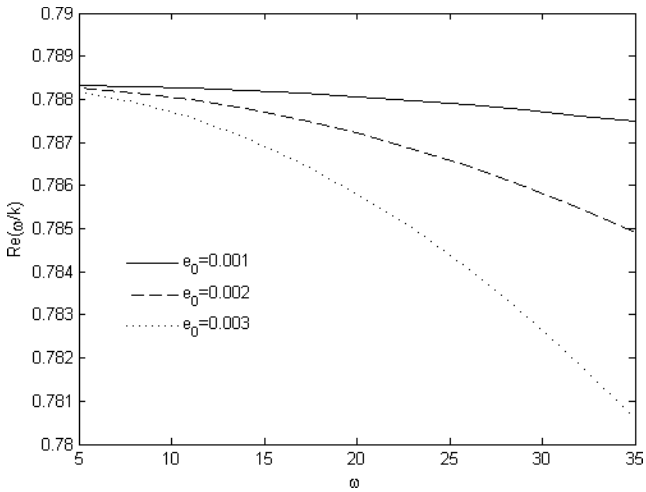


Fig. 10. The effect of angular frequency ω on the wave speed of Rayleigh wave, when $e_0 = 0.001, 0.002$ and 0.003

Fig. 10 illustrates the effect of angular frequency ω on the wave speed of Rayleigh wave when $e_0 = 0.001, 0.002$ and 0.003. The speed of Rayleigh wave decreases with increasing value of ω, The rate of decrease in value of speeds becomes faster as the value of e_0 increases.

The speed of Rayleigh wave is also computed against K^* and it decrease very slowly (constant up to 4 decimals) as the value of K^* increases.

6. CONCLUSIONS

Moore–Gibson–Thompson thermoelasticity with Klein–Gordon nonlocality is applied to study the homogeneous plane waves and the Rayleigh surface wave in a generalized thermoelastic medium. It is found that there exists two coupled longitudinal waves and one transverse wave in a plane of MGT thermoelastic medium. A dispersion equation of Rayleigh wave is derived along a stress-free thermally insulated surface of a half-space of a MGT thermoelastic material. The numerical computations and illustrations of wave speeds of plane and Rayleigh wave based on a particular material show the significant effects of KG nonlocality parameters, conductivity rate parameter and the angular frequency on the wave speeds. The present theoretical predictions on plane and surface waves in context of Moore–Gibson–Thompson thermoelasticity with Klein–Gordon nonlocality may be applied in possible experimental studies on the heat conduction in nano-scale materials.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

FUNDING

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

REFERENCES

[1] M. A. Biot. Thermoelasticity and irreversible thermodynamics. *Journal of Applied Physics*, **27**, (1956), pp. 240–253. https://doi.org/10.1063/1.1722351.

[2] A. E. Green and K. A. Lindsay. Thermoelasticity. *Journal of Elasticity*, **2**, (1972), pp. 1–7. https://doi.org/10.1007/bf00045689.

[3] H. W. Lord and Y. Shulman. A generalized dynamical theory of thermoelasticity. *Journal of the Mechanics and Physics of Solids*, **15**, (1967), pp. 299–309. https://doi.org/10.1016/0022-5096(67)90024-5.

[4] J. Ignaczak and M. Ostoja-Starzewski. *Thermoelasticity with finite wave speeds*. Oxford University Press, (2009). https://doi.org/10.1093/acprof:oso/9780199541645.001.0001.

[5] R. B. Hetnarski and J. Ignaczak. Generalized thermoelasticity. *Journal of Thermal Stresses*, **22**, (4-5), (1999), pp. 451–476. https://doi.org/10.1080/014957399280832.

[6] R. Quintanilla. Moore–Gibson–Thompson thermoelasticity. *Mathematics and Mechanics of Solids*, **24**, (2019), pp. 4020–4031. https://doi.org/10.1177/1081286519862007.

[7] L. Rayleigh. On waves propagated along the plane surface of an elastic solid. *Proceedings of the London Mathematical Society*, **s1-17**, (1885), pp. 4–11. https://doi.org/10.1112/plms/s1-17.1.4.

[8] J. N. Flavin. Thermo-elastic Rayleigh waves in a prestressed medium. *Mathematical Proceedings of the Cambridge Philosophical Society*, **58**, (1962), pp. 532–538. https://doi.org/10.1017/s0305004100036811.

[9] P. Chadwick and D. W. Windle. Propagation of Rayleigh waves along isothermal and insulated boundaries. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, **280**, (1380), (1964), pp. 47–71. https://doi.org/https://doi.org/10.1098/rspa.1964.0130.

[10] T. P. Ivanov. On the propagation of thermoelastic Rayleigh waves. *Wave Motion*, **10**, (1988), pp. 73–82. https://doi.org/10.1016/0165-2125(88)90007-8.

[11] A. E. Abouelregal. Rayleigh waves in a thermoelastic solid half space using dual-phase-lag model. *International Journal of Engineering Science*, **49**, (2011), pp. 781–791. https://doi.org/10.1016/j.ijengsci.2011.03.007.

[12] S. Chiriţă. On the Rayleigh surface waves on an anisotropic homogeneous thermoelastic half space. *Acta Mechanica*, **224**, (2012), pp. 657–674. https://doi.org/10.1007/s00707-012-0776-z.

[13] A. V. Bucur, F. Passarella, and V. Tibullo. Rayleigh surface waves in the theory of thermoelastic materials with voids. *Meccanica*, **49**, (2013), pp. 2069–2078. https://doi.org/10.1007/s11012-013-9850-4.

[14] B. Singh. Rayleigh wave in a thermoelastic solid half-space with impedance boundary conditions. *Meccanica*, **51**, (2015), pp. 1135–1139. https://doi.org/10.1007/s11012-015-0269-y.
[15] B. Singh and S. Verma. On propagation of Rayleigh type surface wave in five different theories of thermoelasticity. *International Journal of Applied Mechanics and Engineering*, **24**, (2019), pp. 661–673. https://doi.org/10.2478/ijame-2019-0041.
[16] F. Passarella, V. Tibullo, and G. Viccione. Rayleigh waves in isotropic strongly elliptic thermoelastic materials with microtemperatures. *Meccanica*, **52**, (2016), pp. 3033–3041. https://doi.org/10.1007/s11012-016-0591-z.
[17] A. C. Eringen. *Nonlocal continuum field theories*, Vol. 56. Springer Verlag, New York, (2001).
[18] D. G. B. Edelen and N. Laws. On the thermodynamics of systems with nonlocality. *Archive for Rational Mechanics and Analysis*, **43**, (1971), pp. 24–35. https://doi.org/10.1007/bf00251543.
[19] D. G. B. Edelen, A. E. Green, and N. Laws. Nonlocal continuum mechanics. *Archive for Rational Mechanics and Analysis*, **43**, (1971), pp. 36–44. https://doi.org/10.1007/bf00251544.
[20] A. C. Eringen and D. G. B. Edelen. On nonlocal elasticity. *International Journal of Engineering Science*, **10**, (1972), pp. 233–248. https://doi.org/10.1016/0020-7225(72)90039-0.
[21] S. Chirita. Nonlocal effects on the stress distribution in an elastic half space. *Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche,Matematiche e Naturali. Rendiconti, Serie 8*, **60**, (6), (1976), pp. 802–807.
[22] D. Ieşan. Reciprocal theorems and variational theorems in nonlocal elastodynamics. *International Journal of Engineering Science*, **15**, (1977), pp. 693–699. https://doi.org/10.1016/0020-7225(77)90019-2.
[23] A. C. Eringen. On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves. *Journal of Applied Physics*, **54**, (1983), pp. 4703–4710. https://doi.org/10.1063/1.332803.
[24] B. S. Altan. Uniqueness in linear theory of nonlocal elasticity. *Bulletin of the Technical University of Istanbul*, **37**, (1984), pp. 373–385.
[25] J. L. Nowinski. On a three-dimensional Kelvin problem for an elastic nonlocal medium. *Acta Mechanica*, **84**, (1990), pp. 77–87. https://doi.org/10.1007/bf01176089.
[26] B. Cracium. On nonlocal thermoelasticity. *Analele Stiintifice ale Universitatii Ovidius Constanta*, **5**, (1996), pp. 29–36.
[27] A. C. Eringen. Linear theory of nonlocal elasticity and dispersion of plane waves. *International Journal of Engineering Science*, **10**, (1972), pp. 425–435. https://doi.org/10.1016/0020-7225(72)90050-x.
[28] D. Singh, G. Kaur, and S. K. Tomar. Waves in nonlocal elastic solid with voids. *Journal of Elasticity*, **128**, (2017), pp. 85–114. https://doi.org/10.1007/s10659-016-9618-x.
[29] A. C. Eringen. Theory of nonlocal thermoelasticity. *International Journal of Engineering Science*, **12**, (1974), pp. 1063–1077. https://doi.org/10.1016/0020-7225(74)90033-0.
[30] F. Balta and E. S. Şuhubi. Theory of nonlocal generalised thermoelasticity. *International Journal of Engineering Science*, **15**, (1977), pp. 579–588. https://doi.org/10.1016/0020-7225(77)90054-4.
[31] B. Singh. Rayleigh-type surface waves in a nonlocal thermoelastic solid half space with voids. *Waves in Random and Complex Media*, **31**, (2020), pp. 2103–2114. https://doi.org/10.1080/17455030.2020.1721612.
[32] A. S. Pramanik and S. Biswas. Surface waves in nonlocal thermoelastic medium with state space approach. *Journal of Thermal Stresses*, **43**, (2020), pp. 667–686. https://doi.org/10.1080/01495739.2020.1734129.

[33] S. Biswas. Rayleigh waves in a nonlocal thermoelastic layer lying over a nonlocal thermoelastic half-space. *Acta Mechanica*, **231**, (2020), pp. 4129–4144. https://doi.org/10.1007/s00707-020-02751-2.

[34] P. Lata and S. Singh. Rayleigh wave propagation in a nonlocal isotropic magneto-thermoelastic solid with multi-dual-phase lag heat transfer. *GEM - International Journal on Geomathematics*, **13**, (2022). https://doi.org/10.1007/s13137-022-00195-5.

[35] A. M. Abd-Alla, S. M. Abo-Dahab, S. M. Ahmed, and M. M. Rashid. Effect of magnetic field and voids on Rayleigh waves in a nonlocal thermoelastic half-space. *The Journal of Strain Analysis for Engineering Design*, **57**, (2021), pp. 61–72. https://doi.org/10.1177/03093247211001243.

[36] M. Kumar, P. Kaswan, N. Sarkar, X. Liu, and M. Kumari. Rayleigh waves in nonlocal generalized thermoelastic media. *International Journal of Numerical Methods for Heat & Fluid Flow*, **33**, (2023), pp. 2049–2072. https://doi.org/10.1108/hff-08-2022-0468.

[37] S. Biswas. Rayleigh waves in porous nonlocal orthotropic thermoelastic layer lying over porous nonlocal orthotropic thermoelastic half space. *Waves in Random and Complex Media*, **33**, (2021), pp. 136–162. https://doi.org/10.1080/17455030.2021.1876279.

[38] K. Jangid, M. Gupta, and S. Mukhopadhyay. On propagation of harmonic plane waves under the Moore–Gibson–Thompson thermoelasticity theory. *Waves in Random and Complex Media*, **34**, (2021), pp. 1976–1999. https://doi.org/10.1080/17455030.2021.1949071.

[39] G. Romano, R. Barretta, M. Diaco, and F. Marotti de Sciarra. Constitutive boundary conditions and paradoxes in nonlocal elastic nanobeams. *International Journal of Mechanical Sciences*, **121**, (2017), pp. 151–156. https://doi.org/10.1016/j.ijmecsci.2016.10.036.

[40] J. Kaplunov, D. A. Prikazchikov, and L. Prikazchikova. On non-locally elastic Rayleigh wave. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **380**, (2022). https://doi.org/10.1098/rsta.2021.0387.

[41] J. Kaplunov, D. A. Prikazchikov, and L. Prikazchikova. On integral and differential formulations in nonlocal elasticity. *European Journal of Mechanics - A/Solids*, **100**, (2023). https://doi.org/10.1016/j.euromechsol.2021.104497.

[42] V. T. N. Anh and P. C. Vinh. Expressions of nonlocal quantities and application to Stoneley waves in weakly nonlocal orthotropic elastic half-spaces. *Mathematics and Mechanics of Solids*, **28**, (2023), pp. 2420–2435. https://doi.org/10.1177/10812865231164332.

[43] V. T. N. Anh, P. C. Vinh, T. T. Tuan, and L. T. Hue. Weakly nonlocal Rayleigh waves with impedance boundary conditions. *Continuum Mechanics and Thermodynamics*, **35**, (5), (2023), pp. 2081–2094. https://doi.org/10.1007/s00161-023-01235-7.

[44] M. Lazar and E. Agiasofitou. Nonlocal elasticity of Klein–Gordon type: Fundamentals and wave propagation. *Wave Motion*, **114**, (2022). https://doi.org/10.1016/j.wavemoti.2022.103038.