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ADHESIVE CONTACT BETWEEN TWO-DIMENSIONAL ANISOTROPIC ELASTIC BODIES

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Abstract. Adhesion plays a vital role in the design of smart and intelligent high-tech devices such as modern optical, microelectromechanical, and biomedical systems. However, in the literature, adhesive contact is mostly considered for contact of rigid substrates and transversely isotropic and isotropic elastic materials. The composite materials are increasingly used in the mart and intelligent high-tech devices. Since the composite materials are generally anisotropic and contact bodies are all deformable, it is more practical to consider the adhesive contact of two anisotropic elastic materials. In this paper, an adhesive contact model of anisotropic elastic bodies is established, and the closed-form solutions for two-dimensional adhesive contact of two anisotropic elastic bodies are derived. The full-field solutions and the relation for the contact region and applied force are developed using the Stroh complex variable formalism, the analytical continuation method, and concepts of the JKR adhesive model. We will show that the frictionless contact of two anisotropic elastic materials is just a special case of the present contact problem, and its solutions can be obtained by setting the work of adhesion equal to zero. In addition, we also show that our present solutions are valid for the problems of indentation by a rigid punch on an elastic half-space through a proper replacement of the contact radius and the corresponding material constant. Numerical results are provided to demonstrate the accuracy, applicability, and versatility of the developed solutions.

Keywords: adhesion, anisotropic elasticity, Stroh formalism, JKR adhesive, closed-form solutions.

1. INTRODUCTION

Due to the high strength/stiffness-to-weight ratio, composite materials have been introduced in almost every industry in various forms and fashions, including aerospace, automotive, marine, civil infrastructure, biomechanical and renewable energy applications [1–5]. With the continuing demand for good quality and performance of composite devices and technical systems, contact problems with composite materials are one of the most popular and active topics to be studied in mechanical, civil and aeronautical engineering [5–12]. Many cases modeled composite materials as anisotropic elastic materials rather than transversely isotropic or orthotropic elastic materials since the composite materials are generally anisotropic [5–11]. In the studies, however, the adhesion phenomenon is usually neglected. In recent years, smart devices, such as modern optical, microelectromechanical, and biomedical systems, have become smaller and smaller in dimension [12–14]. One key concern associated with the contact problems of smart materials is the role of adhesion [12–16]. Many studies have proved that adhesion has significant influences on contact behaviors such as contact size and distribution of contact pressure [12–24]. It is, therefore, necessary to develop a solution for solving the adhesion contact with anisotropic elastic materials.

Among the different methods for solving the adhesion contact with elastic materials, the analytical approach is one of the most important methods. The analytical approach can provide exact solutions that can explicitly present the response of the materials for which the interpretation of the experiment data and nanoindentation technique is relied on [12–14]. However, due to the complexity of anisotropic materials, none of the exact solutions can be found for adhesive contact of anisotropic elastic materials. Most of the previous works on adhesive contact was limited to isotropic, transversely isotropic and orthotropic materials and focused on the adhesion of a rigid punch/indenter/sphere on an anisotropic half-plane [15–24]. In actual technical systems, all the contact bodies are deformable. It is more practical to consider the contact of two anisotropic elastic bodies rather than contact with rigid bodies.

Motivated by these considerations, we develop an exact solution for adhesive contact of two dissimilar elastic solids. Here the contact bodies are generally anisotropic elastic materials, and the two-dimensional (2D) generalized plane strain is considered. Here, the 2D generalized plane strain condition means that all displacement components are coupled and depend on the coordinate variables x_1 and x_2 only, that can reflect the situations in which two components of a smart device contact each other as shown in Fig. 1. The effect of adhesion is described by using the Johnson–Kendall–Roberts (JKR) adhesive contact model—a well-established classic theory of adhesive contact [15,17,20,23,24]. The solutions to the contact problems are obtained using the Stroh formalism [8–11, 25–27] and the analytical continuation method [8–11, 25, 28]. Using these methods, the full-field and general solutions for contact tractions and displacement are expressed in an elegant and compact mathematical form. Based on the general solutions, the solutions for some special cases will be derived and discussed. Numerical results are finally presented to demonstrate the correctness and versatility of our derived solutions. Through the numerical results, we further discussed the influence of the anisotropy (represented by the fiber orientation) of materials on the contact solutions.

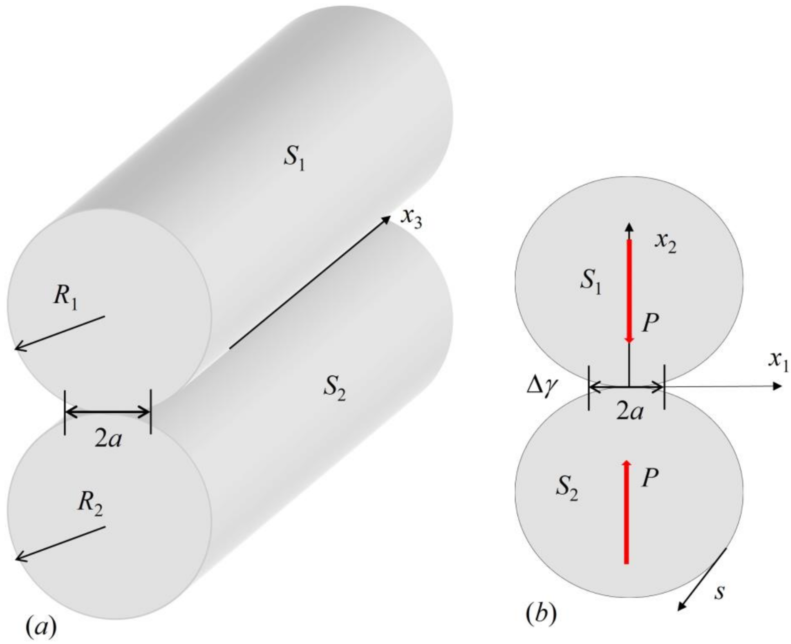

Fig. 1. (a) Adhesive contact between two elastic cylinders (b) Two-dimensional generalized plane strain conditions. Here S_1 and S_2 denote, respectively, the upper and lower contact bodies; R_1 and R_2 are the radii of the contact bodies; s denotes the tangential direction; P is the external pair force applied on the contact bodies; (x_1, x_2, x_3) is the global coordinate; a is the half-width of the contact region; $\Delta\gamma$ is the work of adhesion

2. NOMENCLATURE FOR AN ANISOTROPIC ELASTIC MATERIALS

In a fixed Cartesian coordinate system $x_i, i = 1, 2, 3$, the basic equation, including constitutive laws, the strain-displacement and the equilibrium relations for a generally anisotropic elasticity, can be written as [25, 26]

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \sigma_{ij,j} = 0, \quad i, j, k, l = 1, 2, 3, \tag{1a}$$

where u_i, σ_{ij} and ε_{ij} denote, respectively, the displacement, stress and strain; the repeated indices imply summation; a subscript comma stands for differentiation; C_{ijkl} is the elastic stiffness tensor which has the following symmetry properties

$$C_{ijkl} = C_{jikl} = C_{klij} = C_{lkij} \tag{1b}$$

It should be mentioned that with the tensor expression C_{ijkl} and the symmetry properties of the elastic tensor (1b), the anisotropic elastic materials considered in this paper have at most 21 independent elastic coefficients, which can specified to other kinds of materials, such as monoclinic, orthotropic, transversely isotropic and isotropic materials by the additional material symmetric plane [25].

Using the well-known Stroh complex variable formalism for two-dimensional anisotropic elasticity [25–27], the general solutions satisfying the basic equations (1) can be obtained as

$$\mathbf{u} = 2\mathrm{Re}\left\{\mathbf{A}\mathbf{f}(z)\right\}, \quad \boldsymbol{\phi} = 2\mathrm{Re}\left\{\mathbf{B}\mathbf{f}(z)\right\}, \tag{2a}$$

where Re denotes the real part of a complex number and

$$\mathbf{u} = \left\{\begin{array}{c} u_1 \\ u_2 \\ u_3 \end{array}\right\}, \ \boldsymbol{\phi} = \left\{\begin{array}{c} \phi_1 \\ \phi_2 \\ \phi_3 \end{array}\right\}, \quad \mathbf{f}(z) = \left\{\begin{array}{c} f_1(z_1) \\ f_2(z_2) \\ f_3(z_3) \end{array}\right\}, \ z_\alpha = x_1 + \mu_\alpha x_2, \ \ \alpha = 1,2,3,$$
$$\mathbf{A} = \left[\begin{array}{ccc} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{array}\right], \quad \mathbf{B} = \left[\begin{array}{ccc} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{array}\right]. \tag{2b}$$

In (2b), ϕ_i, $i = 1,2,3$ are the stress functions, which are related to the stresses and traction by

$$\sigma_{i1} = -\phi_{i,2}, \quad \sigma_{i2} = \phi_{i,1}, \quad t_j = \sigma_{ij} n_j = \phi_{i,s}, \quad i = 1,2,3. \tag{2c}$$

where $n_j, j = 1,2,3$ are the components of the unit outer normal $\mathbf{n}$ and s denotes the tangential direction of the surface as shown in Fig. 1. $(\mathbf{a}_i, \mathbf{b}_i)$, and $\mu_i, i = 1,2,3$, are the material eigenvectors and eigenvalues to be determined from the elastic stiffness tensor C_{ijkl}; $f_\alpha(z_\alpha), \alpha = 1,2,3$, are holomorphic complex functions of the variable z_α, which will be determined by taking the boundary conditions of the problem into consideration.

3. ADHESION BETWEEN TWO-DIMENSIONAL ANISOTROPIC ELASTIC BODIES

Consider an adhesive contact of two dissimilar anisotropic elastic bodies subjected to an external pair force P. Here P has a positive value when acting on the contact bodies, as shown in Fig. 1. When the external load acts on the two bodies, they are brought into contact on the contact region $(-a, a)$ and then produce the normal contact traction along the contact region. If we assume the Cartesian coordinate (x_1, x_2, x_3) is attached to the contact region as shown in Fig. 1, in which its origin is placed at the initial contact point of two contact bodies, the boundary conditions for surface displacement and tractions along the surfaces of contact bodies can be expressed as

$$\left.\begin{array}{l} t_1^{(1)}(x_1) = t_1^{(2)}(x_1) = 0, u_2^{(2)}(x_1) - u_2^{(1)}(x_1) = g(x_1) \\ t_3^{(1)}(x_1) = t_3^{(2)}(x_1) = 0 \end{array}\right\} \text{when } x_1 \in (-a, a) \\ t_i(x_1) = 0, i = 1, 2, 3, \text{ when } x_1 \notin (-a, a). \tag{3a}$$

where the superscripts (1) and (2) denote the value of the contact bodies S_1 and S_2, respectively (see Fig. 1); $g(x_1)$ is the gap function which is related to the profiles of the contact bodies by the following relation,

$$g(x_1) = \frac{x_1^2}{2R}, \text{ where } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{3b}$$

in which R_1 and R_2 are, respectively, the radii of the upper and lower elastic bodies.

3.1. A full-field solution

To obtain the solution that satisfies the boundary conditions (3) for the present contact problems, we may start with the general solutions of displacements and tractions in (2a). Using (2a), the solutions of displacements and tractions of each contact body can be written as follows,

$$\begin{array}{lll} \mathbf{u}_1 = 2\text{Re}\left\{\mathbf{A}_1\mathbf{f}_1(z)\right\}, & \phi_1 = 2\text{Re}\left\{\mathbf{B}_1\mathbf{f}_1(z)\right\}, & \text{when } z \in S_1, \\ \mathbf{u}_2 = 2\text{Re}\left\{\mathbf{A}_2\mathbf{f}_2(z)\right\}, & \phi_2 = 2\text{Re}\left\{\mathbf{B}_2\mathbf{f}_2(z)\right\}, & \text{when } z \in S_2, \end{array} \tag{4}$$

where the subscripts 1 and 2 denote, respectively, the values related to the contact bodies S_1 and S_2.

Knowing that the material eigen matrices $(\mathbf{A}_1, \mathbf{B}_1)$ for the upper contact body S_1 and $(\mathbf{A}_2, \mathbf{B}_2)$ for the lower contact body S_2 can be determined from their associated material properties, the only unknown functions to be determined are the holomophic complex function vectors $\mathbf{f}_1(z)$ and $\mathbf{f}_2(z)$. To find these complex functions satisfying the boundary conditions (3), we can employ the analytical continuation method due to its obvious advantage in solving contact problems [25,28–30]. Following the concept of this method, we may introduce a sectional holomorphic function $\mathbf{\Theta}'(z)$ related to the derivatives of the complex function vector $\mathbf{f}_1(z)$ and $\mathbf{f}_2(z)$ by

$$\mathbf{\Theta}'(z) = \begin{cases} -\mathbf{B}_1\mathbf{f}_1'(z), & z \in S_1 \\ \mathbf{B}_2\mathbf{f}_2'(z), & z \in S_2 \end{cases} \tag{5}$$

$\mathbf{\Theta}(z)$ defined in (5) can be proved to satisfy all the continuity of traction along the contact region and free traction conditions along the non-contact region [25,29]. By substituting

(5) into (3) and then (2c), the surface tractions and displacements along the contact region can be rewritten as follows,

$$\begin{aligned} &\mathbf{t}_1(x_1^+) = -\mathbf{t}_2(x_1^-) = \mathbf{\Theta}'(x_1^+) - \mathbf{\Theta}'(x_1^-), \\ &\mathbf{u}_1'(x_1^+) = i\mathbf{M}_1^{-1}\mathbf{\Theta}'(x_1^+) + i\bar{\mathbf{M}}_1^{-1}\mathbf{\Theta}'(x_1^-), \\ &\mathbf{u}_2'(x_1^+) = -i\mathbf{M}_2^{-1}\mathbf{\Theta}'(x_1^-) - i\bar{\mathbf{M}}_2^{-1}\mathbf{\Theta}'(x_1^+), \end{aligned} \tag{6a}$$

where $\bullet'$ denotes the derivative w.r.t to the complex variable z and

$$\begin{aligned} &\mathbf{M}_1 = -i\mathbf{B}_1\mathbf{A}_1^{-1}, \quad \mathbf{M}_2 = -i\mathbf{B}_2\mathbf{A}_2^{-1}, \\ &\mathbf{\Theta}'(x_1^+) = \lim_{x\to 0^+} \mathbf{\Theta}'(z), \quad \mathbf{\Theta}'(x_1^-) = \lim_{x\to 0^-} \mathbf{\Theta}'(z). \end{aligned} \tag{6b}$$

Subtracting $(6a)_3$ by $(6a)_2$, we obtain

$$\mathbf{u}_2'(x_1^+) - \mathbf{u}_1'(x_1^+) = -i\mathbf{M}\mathbf{\Theta}'(x_1^+) - i\bar{\mathbf{M}}\mathbf{\Theta}'(x_1^-), \tag{7a}$$

where $\mathbf{M}$ is a bimaterial matrix which is defined as [25,26]

$$\mathbf{M} = \mathbf{M}_1^{-1} + \bar{\mathbf{M}}_2^{-1}. \tag{7b}$$

With the relations $(6a)_1$ and (7a), the boundary conditions (3) can be expressed through $\mathbf{\Theta}'(z)$ as

$$\left.\begin{aligned} &\theta_1'(x_1^+) - \theta_1'(x_1^-) = 0 \\ &\left\{\begin{aligned} &m_{21}\theta_1'(x_1^+) + m_{22}\theta_2'(x_1^+) + m_{23}\theta_3'(x_1^+) + \\ &\bar{m}_{21}\theta_1'(x_1^-) + m_{22}\theta_2'(x_1^-) + \bar{m}_{23}\theta_3'(x_1^-) = ig'(x_1) \end{aligned}\right\} \\ &\theta_3'(x_1^+) - \theta_3'(x_1^-) = 0 \end{aligned}\right\}, x_1 \in (-a,a) \tag{8a}$$
$$\mathbf{\Theta}'(x_1^+) - \mathbf{\Theta}'(x_1^-) = \mathbf{0}, \text{ when } x_1 \notin (-a,a)$$

where $m_{2j}, j = 1,2,3$ is the second-row components of the bi-material matrix $\mathbf{M}$ defined in Eq. (7b); θ_1', θ_2' and θ_3' are three components of the sectional holomorphic complex function $\mathbf{\Theta}'(z)$, i.e.,

$$\mathbf{\Theta}'(z) = [\theta_1'(z), \theta_2'(z), \theta_3'(z)]^T, \tag{8b}$$

where the superscript T denotes the transpose of the vector. Note that m_{22} is real number due to $\mathbf{M}$ is a Hermitian matrix [25,31].

It can be seen from Eq. (8) that the functions $\theta_1(z)$ and $\theta_3(z)$ are holomorphic everywhere in the entire complex plane including the point at infinity and they approach zero when $z \to \infty$ because the stresses vanish here. By Liouville's theorem, we conclude that

$$\theta_1'(z) = \theta_3'(z) = 0. \tag{9}$$

With this result, Eq. (8a) can now be reduced to

$$\begin{aligned} &\theta_2'(x_1^+) + \theta_2'(x_1^-) = \frac{i}{m_{22}} g'(x_1), \quad &&\text{when } x_1 \in (-a, a) \\ &\theta_2'(x_1^+) - \theta_2'(x_1^-) = 0, &&\text{when } x_1 \notin (-a, a) \end{aligned} \tag{10}$$

(10) is a standard Hilbert problem, and its solution has proven to be [25,28,29]

$$\theta_2'(z) = \frac{\chi_0(z)}{2\pi m_{22}} \int_{-a}^{a} \frac{g'(t)}{\chi_0^+(t)(t-z)} \mathrm{d}t + \chi_0(z) d_0, \tag{11a}$$

where $\chi_0(z)$ is the basic Plemelj function and d_0 is the value derived from the overall equilibrium condition of the contact bodies and they are expressed as [25]

$$\chi_0(z) = \frac{1}{\sqrt{z^2 - a^2}}, d_0 = \frac{iP}{2\pi}. \tag{11b}$$

Substituting $g(x_1)$ given in (3b) into (11), the solution for $\theta_2'(z)$ can be evaluated by using the line integral with the aid of residue theory presented in Appendix B.3 of [25]. The solution is

$$\theta_2'(z) = \frac{i}{2m_{22}R} \left\{ z - \frac{\{z^2 - (a^2/2)\}}{\sqrt{z^2 - a^2}} \right\} + \frac{iP}{2\pi\sqrt{z^2 - a^2}}. \tag{12}$$

By substituting (9) and (12) into (6), the contact traction $t_2(x_1)$ can now be expressed as

$$t_2(x_1) = -\frac{1}{m_{22}R\sqrt{a^2 - x_1^2}} \left\{ x_1^2 - \frac{a^2}{2} - \frac{Rm_{22}P}{\pi} \right\}. \tag{13}$$

3.2. JKR-model of adhesive contact

In deriving the above solutions, we have used the assumed contact region a. To complete the above solutions, the actual contact region needs to be determined. Here the concepts of the JKR adhesive contact model [23] is adopted in which the contact region can be found via Griffith energy balance. Using the Griffith energy balance, we have

$$G = \Delta\gamma, \tag{14}$$

in which G is the energy release rate and $\Delta\gamma$ is the work of adhesion of the surface. The energy release rate for this problem can be obtained from [13,25,31]

$$G = \frac{1}{4} E_{22} K^2. \tag{15}$$

In (15), K is the stress intensity factor at the ends of the contact regions which can evaluated by using the following relation,

$$K = \lim_{x \to a} \sqrt{2\pi(a - x_1)} t_2(x_1). \tag{16a}$$

With the result of $t_2(x_1)$ in (13), we have

$$K = -\frac{\sqrt{\pi}}{m_{22}R}\left\{\frac{a^{3/2}}{2} - \frac{Rm_{22}P}{\pi\sqrt{a}}\right\}. \tag{16b}$$

E_{22} is the value related to the material properties of anisotropic elastic materials and is defined as

$$E_{22} = \mathbf{i}_2^T\mathbf{E}\mathbf{i}_2, \tag{17a}$$

and $\mathbf{E}$ is the matrix defined as [25]

$$\begin{aligned}&\mathbf{E} = \mathbf{D} + \mathbf{W}\mathbf{D}^{-1}\mathbf{W}, \quad \mathbf{D} = \mathbf{L}_1^{-1} + \mathbf{L}_2^{-1}, \quad \mathbf{W} = \mathbf{S}_1\mathbf{L}_1^{-1} - \mathbf{S}_2\mathbf{L}_2^{-1},\\ &\mathbf{L}_1 = -2i\mathbf{B}_1\mathbf{B}_1^T, \ \mathbf{L}_2 = -2i\mathbf{B}_2\mathbf{B}_2^T, \ \mathbf{S}_1 = i(2\mathbf{A}_1\mathbf{B}_1^T - \mathbf{I}), \ \mathbf{S}_2 = i(2\mathbf{A}_2\mathbf{B}_2^T - \mathbf{I}).\end{aligned} \tag{17b}$$

Substituting (16b) into (15), the contact area denoted by a can be obtained as

$$a^2 - 4Rm_{22}\sqrt{\frac{\Delta\gamma a}{\pi E_{22}}} = \frac{2Rm_{22}}{\pi}P. \tag{18}$$

In the case of no loading, i.e., $P = 0$, two parabolic cylinders are in self-equilibrium status, and the self-equilibrium contact half-width a_s can be obtained from (18) as

$$a_s = \left(\frac{16R^2m_{22}^2\Delta\gamma}{\pi E_{22}}\right)^{1/3}. \tag{19}$$

In adhesive contact, the pull-off force and pull-off contact size, defined as the maximum load required to pull two elastic bodies away from each other, are important factors and need to be studied. To derive the pull-off contact half-width for this problem, we consider [32]

$$\frac{\partial P}{\partial a} = 0 \tag{20}$$

Substituting P obtained from (18) into (20), the pull-off contact half-width can be obtained as

$$a_p = \left(\frac{R^2m_{22}^2\Delta\gamma}{\pi E_{22}}\right)^{1/3} \tag{21}$$

Comparing (21) and (19), we have

$$\frac{a_p}{a_s} = \frac{1}{2\sqrt[3]{2}}, \tag{22}$$

which is independent of the material properties of two anisotropic elastic materials. The relation (22) provides us with a simple and alternative way to calculate the pull-off contact size. The pull-off force can then be obtained by substituting back (21) to (18). The resulting expression is

$$P_p = -\frac{3}{2}\left(\frac{\pi R m_{22}(\Delta\gamma)^2}{E_{22}^2}\right)^{1/3}. \tag{23}$$

3.3. Dimensionless relations for adhesive contact

Using the value of the pull-off contact region and pull-off force given in (21) and (23), the dimensionless relations for contact region and applied load can be obtained from (18) as

$$\tilde{a}^2 - 4\sqrt{\tilde{a}} = -3\tilde{q}, \tag{24a}$$

where

$$\tilde{a} = a/a_p, \quad \tilde{q} = P/P_p. \tag{24b}$$

The formula (24) does not contain any elastic constant of the materials; therefore, it is the same for isotropic elastic materials.

4. SPECIAL CASES

The general solutions given in Section 3 are valid for general situations where the contact bodies are both deformable anisotropic elastic materials and surface energy exists on the contact surface. In the following, the solutions of some special cases such as the case of no work of adhesion, i.e., $\Delta\gamma = 0$ and adhesion contact with a rigid contact body are derived and discussed.

4.1. No work of adhesion

If there is no work of adhesion (or free surface energy) on the contact surface, we have $\Delta\gamma = 0$. Substituting this value into (18) and (13), the contact region and contact traction to this problem can be obtained as

$$a = \sqrt{\frac{2Rm_{22}}{\pi}P}, \quad t_2(x_1) = -\frac{\sqrt{a^2 - x_1^2}}{m_{22}R}. \tag{25}$$

The solutions for this case are the same as those of the frictionless contact of two anisotropic elastic bodies [8, 25]. Several similar conclusions have been reported in the literature for isotropic elastic bodies [23, 24] or transversely isotropic elastic materials [17, 20].

4.2. Indentation of a parabolic rigid punch on an anisotropic elastic half-plane

In this section, we discuss the ways in which we can use the above solutions derived for adhesive contact of two deformable contact bodies to problems of a rigid punch indented into an anisotropic elastic half-plane. Without losing the generality, we may assume the upper contact body is rigid, and the lower contact body is an anisotropic elastic half-plane.

It is known that the half-plane can be considered by letting the radius of the lower contact body be infinity, i.e., $R_2 \to \infty$. With this assumption, from (3b) we have $R = R_1$ where R_1 is the radius of the parabolic punch in this case. Since the upper body is rigid, we have $\mathbf{u}_1'(x_1^+) = 0$, which leads to $\mathbf{M}_1^{-1} = 0$ according to the equation (6a). From (7b), we have $\mathbf{M} = \bar{\mathbf{M}}_2^{-1}$ and m_{22} is now related to the material properties of $\bar{\mathbf{M}}_2^{-1}$ only.

Now, by properly replacing R by R_1 and m_{22} by $\left[\bar{\mathbf{M}}_2^{-1}\right]_{22}$, which is the component located at the second row and second component of $\bar{\mathbf{M}}_2^{-1}$, the solutions from (12)–(24) for the case of two anisotropic contact bodies can be directly applied to the problem of indentation by a rigid punch on an anisotropic elastic body. For the case of free surface energy, the solutions can be obtained from (25).

5. NUMERICAL RESULTS AND DISCUSSIONS

Consider the contact of two anisotropic elastic materials, as shown in Fig. 1. To get the numerical results, the radius of the upper contact body is selected to be $R_1 = 2$ m, whereas for the lower contact body, the radius is $R_2 = 3$ m. In case the half-plane is considered, $R_2 \to \infty$ as discussed in Section 4. The lower contact body is selected to be a unidirectional fiber-reinforced composite whose elastic properties are given as [7]

$$\begin{aligned} &E_{11} = 134\ \text{GPa},\ E_{22} = E_{33} = 11\ \text{GPa},\ G_{12} = G_{13} = 5.84\ \text{GPa},\\ &G_{23} = 2.98\ \text{MPa},\ v_{12} = v_{13} = 0.3, v_{23} = 0.49 \end{aligned} \tag{26}$$

In (26), E and G are, respectively, Young and shear moduli; v is Poisson's ratio. The subscripts 1, 2, and 3 denote the fiber orientation, transverse direction, and the x_3 direction. The angle between the fiber direction and the x_1 in the cylinder plane is denoted by the angle β, which varies as $0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° in this Section. Note that the material will appear to be anisotropic elastic material with the variation of the angle β.

The upper contact body can be a rigid or orthotropic elastic material. In case the upper contact body is made of orthotropic elastic material, the material properties are selected to be [7]

$$\begin{aligned} &C_{11} = 147.34\ \text{GPa},\ C_{22} = C_{33} = 10.78\ \text{GPa},\ C_{12} = C_{13} = 4.23\ \text{GPa},\\ &C_{23} = 3.32\ \text{GPa},\ C_{44} = C_{55} = C_{66} = 4.1\ \text{GPa},\ \text{other}\ C_{ij} = 0, i, j = 1,2,3,4,5,6 \end{aligned} \tag{27}$$

Note that in (27), the Voigt contract notation for the elastic tensor presented in Section 2 is used [25].

Figs. 2 and 3 present the contact area-applied load relation using Eq. (16) with different values of adhesion work, i.e., $\Delta\gamma = 0, 20, 50$, and 100 nJ/m. From these figures, we see that for the case of no adhesion, the present results are well matched with those obtained for frictionless contact for the case of indentation by rigid punch and the case of two elastic cylinders [8, 10], confirming our discussion for the special case discussed in Section 4. In addition, when the surface energy increases, the pull-off force denoted by the lowest point of the curves also increases. When the surface energy increases, the self-equivalent contact region a_s also increases, which is reasonable and consistent with the relation shown in (19).

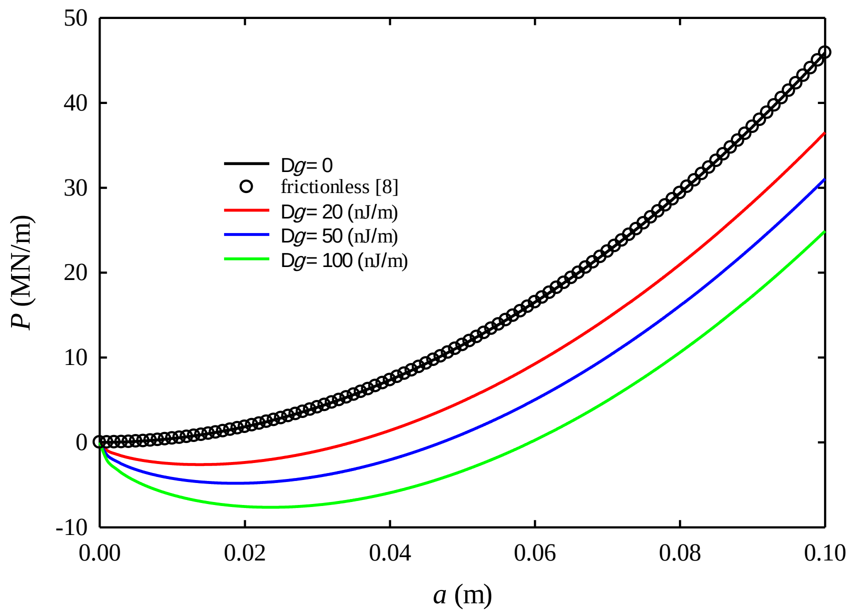

Fig. 2. The contact area-force relation when $\beta = 0$ for the adhesive contact of two elastic cylinders

Figs. 4 and 5 show the influence of the fiber orientation on the contact area-force relation. From these figures, we can see that no matter the upper contact body (rigid or elastic), the pull-off force apparently depends on the fiber orientation. It increases when the angle of fiber orientation β increases. When the force P is positive (acting in the direction shown in Fig. 1), the force required to obtain the same contact area increases when the fiber orientation angle β increases. This phenomenon is reasonable since the material properties in the x_2 direction are softer, and the rotation of fiber orientation makes the material stiffer.

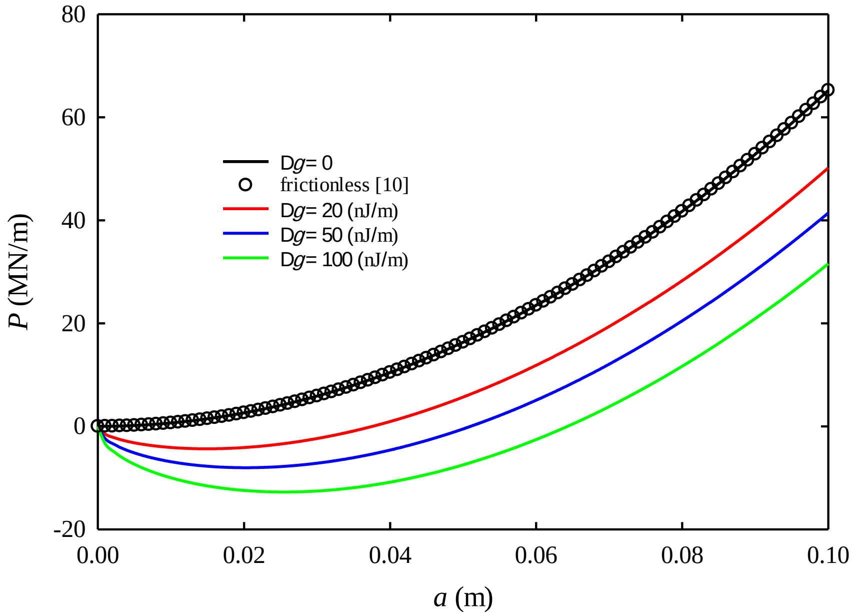

Fig. 3. The contact area-force relation when $\beta = 0$ for the adhesive indentation of a rigid punch into an anisotropic elastic half-plane

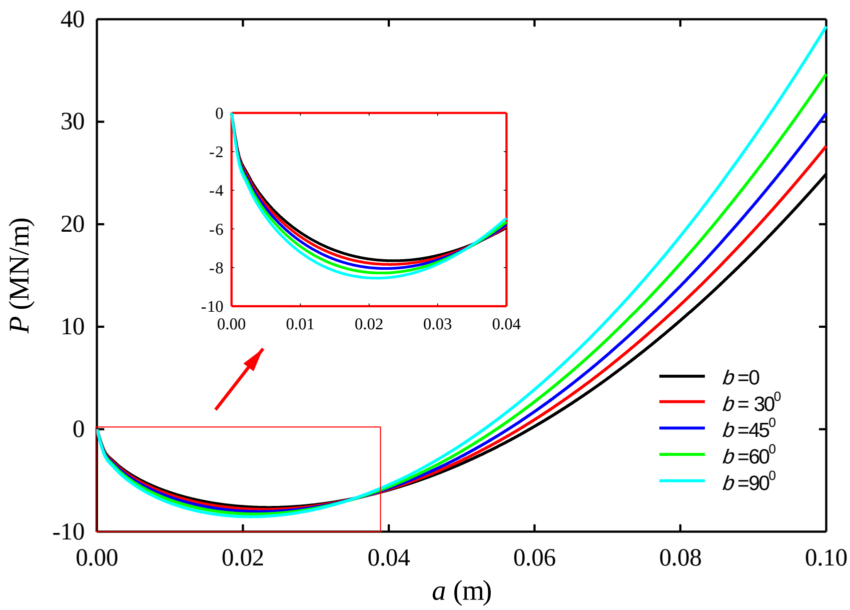

Fig. 4. Influence of the fiber orientation on the contact area force relation for adhesive contact of two anisotropic elastic bodies ($\Delta\gamma$ = 100 nJ/m)

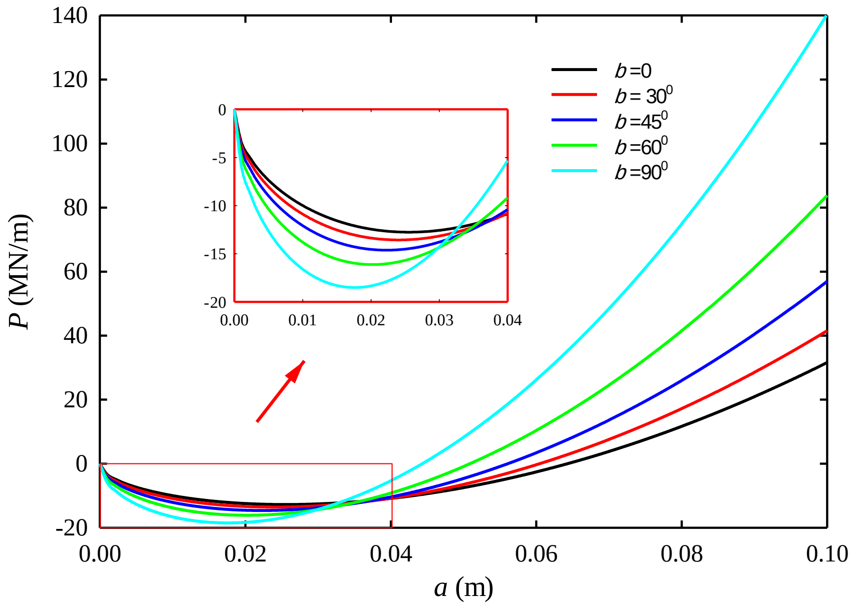

Fig. 5. Influence of the fiber orientation on the contact area force relation for adhesive indentation of a rigid punch into an anisotropic elastic half-plane ($\Delta\gamma$ = 100 nJ/m)

6. CONCLUSIONS

Motivated by the need to better understand the adhesive contact of composite materials, we derived the closed-form solutions for adhesive contact of two-dimensional anisotropic elastic bodies. The solutions are given in the elegant form by using the Stroh formalism presented in Section 2 and the analytical continuation method discussed in Section 3. The general solution given in Section 3 is then specialized for two special cases, including no work of adhesion and the adhesive indentation by a rigid punch, as shown in Section 4. It showed that the solutions for the case with no work of adhesion are the same as that of frictionless contact, and the general solutions can be easily applied to the problems of adhesive indentation by a proper replacement of contact region and the biomaterial constant m_{22}. Section 5 provides the numerical results, demonstrating the correctness and applicability of our derived solutions. The main advantages of the derived solutions include:

Unlike the studies in the literature mostly provide the solutions for transversely isotropic and isotropic elastic materials and are limited to the problems of contact with a rigid body (rigid substrate or indenter). Our developed solution can be used for more general situations in which both contact bodies are deformable, dissimilar, and generally anisotropic elastic materials. Our solutions allow us to have a comprehensive study of the influence of anisotropy denoted by the fiber orientation on the contact solutions, as illustrated in Section 5.

By proper replacement of the surface energy and biomaterial constant as well as the geometry of the contact bodies, our solutions can be reduced to frictionless contact or indentation of rigid punch on an elastic half-plane – these problems have been extensively studied in the literature.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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