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WAVE PROPAGATION ANALYSIS IN TRANSVERSELY ISOTROPIC PIEZOELASTIC MEDIUM BASED ON NONLOCAL STRAIN GRADIENT THEORY

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Abstract. The purpose of this research is to study the propagation of surface waves in transversely isotropic piezoelastic medium based on nonlocal strain gradient theory. A characteristics equation for the existence of surface waves is discussed. This equation could be easily reduced to the ones of the gradient strain theory, nonlocal theory, and classical theory. It has also been concluded that there exist cut-off frequency for the wave propagating in size-dependent materials based on the nonlocal strain gradient theory. The dispersion equation which surface wave speed satisfies is derived from the free traction condition on the surface of half-space with consideration of electrically open circuit conditions. The effect of the nonlocal parameter, the strain gradient parameter on the existence of surface waves as well as the Rayleigh wave propagation is illustrated through some numerical examples.

Keywords: dispersion equation, nonlocal, gradient, transversely isotropic, piezoelectric.

1. INTRODUCTION

The piezoelectric medium has found many applications in the area of signal processing, transduction, and frequency control. Both theoretical and experimental studies on wave propagation in piezoelectric materials have attracted the attention of scientists and engineers during last two decades. The survey of literature can be found in many related texts and books. One of the most critical problems in designing Seismic Acoustic Wave

(SAW) devices is the observation and investigation of the properties of surface waves such as Rayleigh waves, leaky waves, etc.

Nanoscale structures are of significance in the field of nano-mechanics, so it is crucial to account for small size influences in their mechanical analysis. The lack of a scale parameter in the classical continuum theory makes it impossible to describe the size effects. In addition, there exist certain phenomena (e.g., dispersion of elastic waves, crack propagation, dislocations, and so on) that cannot be explained using local theory of continuum mechanics. Therefore, some continuum mechanics theories have been developed to capture such effects, such as the nonlocal Eringen theory [1,2], the modified couple stress theory [3], the micropolar theory [4], the strain gradient theory [5] and others. Tung [6,7] used the nonlocal Eringen theory to investigate wave propagation in nonlocal orthotropic micropolar elastic solids, in nonlocal transversely isotropic liquid-saturated porous solid.

Researchers find out that the nonlocal Eringen theory is not powerful enough to estimate the behavior of small structures completely. In other words, the stiffness-hardening behavior of nanostructures is neglected in this theory and only the stiffness softening effect is included. It is reported that the nonlocal elasticity theory predicts a decrease of the structural stiffness when the scale parameter increases while the strain gradient theory prompts the stiffening of the nanostructures with the non-classical parameter. Recently, Lim et al. [8] have proposed a nonlocal elasticity and strain gradient theory (NSGT) for the wave propagation analysis of size-dependent structures with the objective of eliminating the disadvantages of the last aforementioned theories, where both stiffening and softening effects of the material could be well investigated. In NSGT the stress field accounts for not only the nonlocal stress field but also the strain gradients stress field. This theory contains two non-classical material parameters (the nonlocal parameter and the strain gradient parameter) and is able to reproduce both the increase and decrease of structural stiffness [9]. Based on NSGT, Arefi [10] considered the propagation wave in a functionally graded magneto-electroelastic nano-rod using nonlocal elasticity model subjected to electric and magnetic potentials. Ma et.al [11] investigated the wave propagation characteristics in magneto-electro-elastic nanoshells using nonlocal strain gradient theory.

In this paper, we considered the propagation wave in transversely isotropic piezoelastic medium based on the nonlocal strain gradient theory. Two kinds of scale parameters, namely, the nonlocal parameter and the strain gradient parameter are introduced to account for the size effect of mechanical properties of nanostructures. The constitutive equations and the equations of motion are then established and used to investigate the plane waves propagating in transversely isotropic piezoelectric media. The new characteristics equations of plane waves are then obtained and the cut-off frequency of each wave are derived. The effect of the nonlocal parameter, the strain gradient parameter on the Rayleigh wave propagation as well as the existence of surface waves is considered.

2. BASIC EQUATIONS

We consider homogeneous transversely isotropic piezoelastic solid. It is assumed that the medium is transversely isotropic in such a way that planes of isotropy are perpendicular to x_3 axis. We take the origin of the coordinate system (x_1, x_2, x_3) at any point on the plane surface and x_3-axis pointing vertically downward into the half-space. For two-dimensional problem in which the plane wave is in the plane x_1x_3, the displacement field u_1, u_3, the electric potential ϕ have form

$$u_1 = u_1(x_1, x_3), \quad u_3 = u_3(x_1, x_3), \quad \phi = \phi(x_1, x_3). \tag{1}$$

The constitutive equations for the homogeneous transversely isotropic piezoelastic solid are given as [12, 13]

$$\begin{aligned}
\sigma_{11} &= c_{11}u_{1,1} + c_{13}u_{3,3} + e_{31}\phi_{,3},\\
\sigma_{33} &= c_{13}u_{1,1} + c_{33}u_{3,3} + e_{33}\phi_{,3},\\
\sigma_{13} &= c_{44}(u_{1,3} + u_{3,1}) + e_{15}\phi_{,1},\\
D_1 &= e_{15}(u_{1,3} + u_{3,1}) - \epsilon_{11}\phi_{,1},\\
D_3 &= e_{13}u_{1,1} + e_{33}u_{3,3} - \epsilon_{33}\phi_{,3},
\end{aligned} \tag{2}$$

where σ_{ij}, D_i are stress and electrical displacement components, respectively.

In the absence of body forces, the equations of motion and Gaussian equations for the electric are [12, 13]

$$\begin{aligned}
\sigma_{11,1} + \sigma_{13,3} &= \rho\ddot{u}_1,\\
\sigma_{13,1} + \sigma_{33,3} &= \rho\ddot{u}_3,\\
D_{1,1} + D_{3,3} &= 0,
\end{aligned} \tag{3}$$

where ρ is density of mass and a dot over a quantity represents differentiation with respect to time t.

According to [14, 15], the differential form of constitutive equation with the nonlocal strain gradient can be obtained as follows

$$\begin{aligned}
\tau_{ij} &= (1 - l_1^2\nabla^2)\sigma_{ij} = (1 - l_2^2\nabla^2)\big(c_{ijkl}\varepsilon_{kl} - e_{mij}E_m\big),\\
d_i &= (1 - l_1^2\nabla^2)D_i = (1 - l_2^2\nabla^2)\big(e_{ikl}\varepsilon_{kl} + \epsilon_{im}E_m\big),
\end{aligned} \tag{4}$$

where e_{ijk} the piezoelectric moduli, $E_j = -\phi_{,j}$ and the higher-order nonlocal parameters l_1 and the nonlocal gradient length coefficients l_2 are introduced to account for the size-dependent characteristics of nonlocal gradient materials at nanoscale.

Substituting (4) into (3) and taking in account (2), we have

$$\begin{aligned}
&(1-l_2^2\nabla^2)\Big(c_{11}u_{1,11}+c_{13}u_{3,13}+c_{44}(u_{1,33}+u_{3,13})+(e_{13}+e_{15})\phi_{,13}\Big)=(1-l_1^2\nabla^2)\rho\ddot{u}_1,\\
&(1-l_2^2\nabla^2)\Big(c_{44}(u_{1,13}+u_{3,11})+c_{13}u_{1,13}+c_{33}u_{3,33}+e_{15}\phi_{,11}+e_{33}\phi_{,33}\Big)=(1-l_1^2\nabla^2)\rho\ddot{u}_3,\\
&(1-l_2^2\nabla^2)\Big((e_{13}+e_{15})u_{1,13}+e_{15}u_{3,11}+e_{33}u_{3,33}-\epsilon_{11}\phi_{,11}-\epsilon_{33}\phi_{,33}\Big)=0.
\end{aligned} \tag{5}$$

3. CHARACTERISTIC EQUATION OF PLANE WAVES

For the waves propagating in the plane $x_3 = 0$, we take the form of relevant components of displacement and the electric potential ϕ as [6,7,16]

$$\begin{cases} u_1 = a_1 e^{ik(x_1+\xi x_3-ct)}, \\ u_3 = a_3 e^{ik(x_1+\xi x_3-ct)}, \\ \phi = A e^{ik(x_1+\xi x_3-ct)}, \end{cases} \tag{6}$$

where a_1, a_3, A are unknown amplitudes of the displacement, k is x_1-component of wavenumber, ξ is unknown ratio of wave vector components along x_3 and x_1 direction, c is phase velocity along x_1.

Substituting the expressions for displacements from (6) into (5), we obtain the three homogeneous equations in three unknowns a_1, a_3, A, namely

$$\begin{aligned}
&\Big((c_{11}+c_{44}\xi^2)(1+k^2l_2^2+k^2l_2^2\xi^2)-\rho c^2(1+k^2l_1^2+k^2l_1^2\xi^2)\Big)a_1\\
&\quad+\Big((c_{13}+c_{44})\xi(1+k^2l_2^2+k^2l_2^2\xi^2)\Big)a_3+(e_{13}+e_{15})\xi(1+k^2l_2^2+k^2l_2^2\xi^2)A=0,\\
&\Big((c_{13}+c_{44})\xi(1+k^2l_2^2+k^2l_2^2\xi^2)\Big)a_1+\Big((c_{44}+c_{33}\xi^2)(1+k^2l_2^2+k^2l_2^2\xi^2)\\
&\quad-\rho c^2(1+k^2l_1^2+k^2l_1^2\xi^2)\Big)a_3+(e_{15}+e_{33}\xi^2)(1+k^2l_2^2+k^2l_2^2\xi^2)A=0,\\
&\Big((e_{13}+e_{15})\xi(1+k^2l_2^2+k^2l_2^2\xi^2)\Big)a_1+(e_{15}+e_{33}\xi^2)(1+k^2l_2^2+k^2l_2^2\xi^2)a_3\\
&\quad-\Big((\epsilon_{11}+\epsilon_{33}\xi^2)(1+k^2l_2^2+k^2l_2^2\xi^2)\Big)A=0.
\end{aligned} \tag{7}$$

The necessary condition for the existence of a non-trivial solution a_1, a_3, A for above system equations is vanishing of the determinant of the corresponding coefficients matrix, which yields a twelfth equation for ξ

$$t_{12}\xi^{12} + t_{10}\xi^{10} + t_8\xi^8 + t_6\xi^6 + t_4\xi^4 + t_2\xi^2 + t_0 = 0, \tag{8}$$

where $t_{12}, t_{10}, t_8, t_6, t_4, t_2, t_0$ are given by Appendix. According to the form of the above characteristics equation, we can predict that six different modes can be generated in nonlocal strain gradient piezoelastic solid. A known consequence is the existence of new wave modes which could not be observed in the classical piezoelastic solids.

It is well known that in an anisotropic medium there are generally three body-waves propagating with velocities which vary with the direction of phase propagation. Their polarizations are orthogonal and fixed for the particular direction of phase propagation. The waves are called quasi-waves (qP, qSV, qSH waves) because polarizations may not be along the dynamic axes. In this paper, for two-dimensional problem in which the plane wave is in the plane , these waves are qP, qSV [17]. It has also been concluded that there exist cut-off frequency and escape frequency for wave propagating in size-dependent materials based on the higher-order nonlocal strain gradient model. In some waveguides, the wavenumber will be purely imaginary to start with and becomes real or complex after certain frequency. In such cases, the wave will be evanescent to start with and will start propagating only after certain frequency. This frequency at which the change from evanescent mode to propagating mode happens is called the cut-off frequency, ω^c [18, 19]. The values of these frequencies can easily be obtained by substituting $\xi = 0$ in (8). Therefore, the cut-off frequency of qP wave and qSV wave are

$$\omega_1^c = k\sqrt{\frac{c_{11}(1+k^2l_2^2)}{\rho(1+k^2l_1^2)}}, \quad \omega_2^c = k\sqrt{\frac{(e_{15}^2+\epsilon_{11}c_{44})(1+k^2l_2^2)}{\rho\epsilon_{11}(1+k^2l_1^2)}}. \tag{9}$$

When the wavenumbers become infinite at a particular frequency, which is referred here as the escape frequency, ω^e. The expressions for the escape frequencies can be obtained by forcing the coefficient of ξ^{12} equal to zero in (8) [18, 19]. From the expression of ξ^{12}, it is clear that the escape frequency does not exist in this case.

4. DISPERSION EQUATION OF SURFACE WAVE IN NONLOCAL TRANSVERSELY ISOTROPIC PIEZOELASTIC MEDIUM

As the first step in our research, in this paper, we are only interested in three solutions with positive imaginary parts out of six possible solutions of the characteristics equation (8) by the choice of material constants and the parameters l_1, l_2. The existence of exactly three wave solutions out of six ones in the general case requires fulfil boundary conditions which will be studied in subsequent works. Let ξ_1, ξ_2, ξ_3 be the three roots of Eq. (8) with positive imaginary part, the filed displacment has form

$$\begin{cases} u_1 = \sum_{j=1}^{3} a_{1j} e^{ik(x_1+\xi_j x_3 - ct)}, \\ u_3 = \sum_{j=1}^{3} a_{3j} e^{ik(x_1+\xi_j x_3 - ct)}, \\ \phi = \sum_{j=1}^{3} A_j e^{ik(x_1+\xi_j x_3 - ct)}, \end{cases} \tag{10}$$

and $a_{1j} = \alpha_j A_j, a_{3j} = \beta_j A_j, (j = 1, 2, 3)$ where

$$\begin{aligned}
\alpha_j &= \frac{\delta_{1j}}{\delta_j}, \quad \beta_j = \frac{\delta_{3j}}{\delta_j}, \\
\delta_j &= \Big((c_{11} + c_{44}\xi_j^2)(1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) - \rho c^2 (1 + k^2 l_1^2 + k^2 l_1^2 \xi_j^2) \Big) \Big((c_{44} \\
&\quad + c_{33}\xi_j^2)(1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) - \rho c^2 (1 + k^2 l_1^2 + k^2 l_1^2 \xi_j^2) \Big) \\
&\quad - \Big((c_{13} + c_{44}) \xi_j (1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) \Big)^2, \\
\delta_{1j} &= -\Big((c_{44} + c_{33}\xi_j^2)(1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) - \rho c^2 (1 + k^2 l_1^2 + k^2 l_1^2 \xi_j^2) \Big) (e_{13} + e_{15}) \xi_j (1 \\
&\quad + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) + (e_{15} + e_{33}\xi_j^2)(1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) \Big((c_{13} + c_{44}) \xi_j (1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) \Big), \\
\delta_{3j} &= -\Big((c_{11} + c_{44}\xi_j^2)(1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) - \rho c^2 (1 + k^2 l_1^2 + k^2 l_1^2 \xi_j^2) \Big) (e_{15} + e_{33}\xi_j^2)(1 \\
&\quad + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) + \Big((c_{13} + c_{44}) \xi_j (1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2) \Big) (e_{13} + e_{15}) \xi_j (1 + k^2 l_2^2 + k^2 l_2^2 \xi_j^2).
\end{aligned} \tag{11}$$

In the present problem, boundary conditions appropriate for particle motion in the $x_1 x_3$ plane are considered at the plane surface $x_3 = 0$. Since the boundary surface of the half-space is mechanically stress free, therefore all the components of stresses must vanish.

$$\tau_{13} = 0, \quad \tau_{33} = 0. \tag{12}$$

Another condition is required to represent that the surface of half-space is maintained at charge free condition (open circuit-surface), namely

$$d_3 = 0. \tag{13}$$

By making use of (11) in the boundary condition (12) and (13), we have three equations of A_1, A_2, A_3, namely

$$\begin{cases} \big(c_{44}(\xi_1\alpha_1+\beta_1)+e_{15}\big)(1+k^2l_1^2+k^2l_1^2\xi_1^2)A_1+\big(c_{44}(\xi_2\alpha_2+\beta_2)+e_{15}\big)(1+k^2l_1^2 \\ +k^2l_1^2\xi_2^2)A_2+\big(c_{44}(\xi_3\alpha_3+\beta_3)+e_{15}\big)(1+k^2l_1^2+k^2l_1^2\xi_3^2)A_3=0, \\ (c_{13}\alpha_1+c_{33}\beta_1\xi_1+e_{33}\xi_1)(1+k^2l_1^2+k^2l_1^2\xi_1^2)A_1+(c_{13}\alpha_2+c_{33}\beta_2\xi_2 \\ +e_{33}\xi_2)(1+k^2l_1^2+k^2l_1^2\xi_2^2)A_2+(c_{13}\alpha_3+c_{33}\beta_3\xi_3+e_{33}\xi_3)(1+k^2l_1^2+k^2l_1^2\xi_3^2)A_3=0, \\ (e_{15}\alpha_1+e_{33}\xi_1\beta_1-\epsilon_{33}\xi_1)(1+k^2l_1^2+k^2l_1^2\xi_1^2)A_1+(e_{15}\alpha_2+e_{33}\xi_2\beta_2 \\ -\epsilon_{33}\xi_2)(1+k^2l_1^2+k^2l_1^2\xi_2^2)A_2+(e_{15}\alpha_3+e_{33}\xi_3\beta_3-\epsilon_{33}\xi_3)(1+k^2l_1^2+k^2l_1^2\xi_3^2)A_3=0. \end{cases} \tag{14}$$

Determinant of coefficients leads to the dispersion equation, namely

$$\Delta.\det\begin{bmatrix} \xi_1^* & \xi_2^* & \xi_3^* \\ \xi_1^{**} & \xi_2^{**} & \xi_3^{**} \\ \xi_1^{***} & \xi_2^{***} & \xi_3^{***} \end{bmatrix}=0, \tag{15}$$

where

$$\begin{aligned} &\Delta=(1+k^2l_1^2+k^2l_1^2\xi_1^2)(1+k^2l_1^2+k^2l_1^2\xi_2^2)(1+k^2l_1^2+k^2l_1^2\xi_3^2), \\ &\xi_j^*=c_{44}(\xi_j\alpha_j+\beta_j)+e_{15}, \quad \xi_j^{**}=c_{13}\alpha_j+c_{33}\xi_j\beta_j+e_{33}\xi_j, \\ &\xi_j^{***}=e_{15}\alpha_j+e_{33}\xi_j\beta_j-\epsilon_{33}\xi_j, \quad (j=1,2,3). \end{aligned} \tag{16}$$

This is the dispersion equations for the propagation of Rayleigh-type waves in the transversely isotropic piezoelastic medium based on NSGT.

To facilitate proceed numerical calculations, dimensionless material parameters defined by

$$\begin{aligned} &f_{01}=k^2l_1^2, \quad f_{02}=k^2l_2^2, \quad f_1=\frac{c_{11}}{c_{44}}, \quad f_2=\frac{c_{13}}{c_{44}}, \quad f_3=\frac{c_{33}}{c_{44}}, \\ &e_1=\frac{e_{13}}{e_{33}}, \quad e_2=\frac{e_{15}}{e_{33}}, \quad e=\frac{e_{33}}{\sqrt{c_{44}\epsilon_{33}}}, \quad f=\frac{\epsilon_{11}}{\epsilon_{33}}, \quad X=\frac{\rho c^2}{c_{44}}. \end{aligned} \tag{17}$$

In addition for the propagation of waves with phase velocity v in the direction making an angle θ_0 with the vertical axis, the plane harmonic wave of the form is rewritten by

$$\begin{cases} u_1=a_1e^{ik_0(p_1x_1+p_3x_3-vt)}, \\ u_3=a_3e^{ik_0(p_1x_1+p_3x_3-vt)}, \\ \phi=Ae^{ik_0(p_1x_1+p_3x_3-vt)}, \end{cases} \tag{18}$$

where k_0 is wavenumber, $p_1 = \sin\theta_0$, $p_3 = \cos\theta_0$ are components of slowness. It is note that $k_0 = \dfrac{k}{p_1}$ and $v = p_1 c$. Substituting (18) into (5) leads to

$$\begin{aligned}
&\Big((c_{11}p_1^2 + c_{44}p_3^2)(1 + k_0^2 l_2^2) - \rho v^2(1 + k_0^2 l_1^2)\Big)a_1 + (c_{13} + c_{44})p_1 p_3(1 \\
&\qquad + k_0^2 l_2^2)a_3 + (e_{13} + e_{15})p_1 p_3(1 + k_0^2 l_2^2)A = 0, \\
&(c_{13} + c_{44})p_1 p_3(1 + k_0^2 l_2^2)a_1 + \big((c_{44}p_1^2 + c_{33}p_3^2)(1 + k_0^2 l_2^2) - \rho v^2(1 + k_0^2 l_1^2)\big)a_3 \\
&\qquad + (e_{15} + e_{33}\xi^2)(1 + k_0^2 l_2^2)A = 0, \\
&(e_{13} + e_{15})p_1 p_3(1 + k_0^2 l_2^2)a_1 + (e_{15}p_1^2 + e_{33}p_3^2)(1 + k_0^2 l_2^2)a_3 \\
&\qquad - (\epsilon_{11}p_1^2 + \epsilon_{33}p_3^2)(1 + k_0^2 l_2^2)A = 0.
\end{aligned} \tag{19}$$

The determinant of their coefficients vanishes leads to a quadratic equation in v^2. The roots of this equation give two values of v. Each value of v corresponds to a wave if v^2 is real and positive. The waves with velocities v_1, v_2 correspond to longitudinal qP and transverse qSV waves.

5. NUMERICAL SIMULATION AND DISCUSSION

In order to illustrate theoretical results obtained in the preceding sections, the material chosen for the numerical calculations is CdSe (6 mm class) of hexagonal symmetry, which is transversely isotropic material. The physical data for a single crystal of CdSe material is given below [20,21]

$$\begin{aligned}
&c_{11} = 7.41 \times 10^{10}\ \text{Nm}^{-2}, c_{13} = 3.93 \times 10^{10}\ \text{Nm}^{-2}, c_{33} = 8.36 \times 10^{10}\ \text{Nm}^{-2}, \\
&c_{44} = 1.32 \times 10^{10}\ \text{Nm}^{-2}, \rho = 5504\ \text{kgm}^{-3}, e_{15} = -0.138\ \text{Cm}^{-2}, e_{31} = -0.16\ \text{Cm}^{-2}, \\
&e_{33} = 0.347\ \text{Cm}^{-2}, \epsilon_{11} = 8.26 \times 10^{-11}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}, \epsilon_{33} = 9.03 \times 10^{-11}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}.
\end{aligned}$$

Fig. 1 depicts the variation of dimensionless velocities of qP, qSV wave with incident angle for which two length scale parameters l_1 and l_2. It is observed that the phase velocities of waves for $(l_1 > l_2)$ case (solid lines) are smaller than the ones for $(l_1 < l_2)$ case (dash lines) respectively.

Fig. 2 illustrates the dependence of dimensionless velocities waves on dimensionless parameters f_{01} and f_{02}. From these figures we can see that these velocities decrease gradually as the dimensionless parameter f_{01} increases (see Fig. 2(a)) and meanwhile the ones increase as the dimensionless parameter f_{02} increases (see Fig. 2(b)).

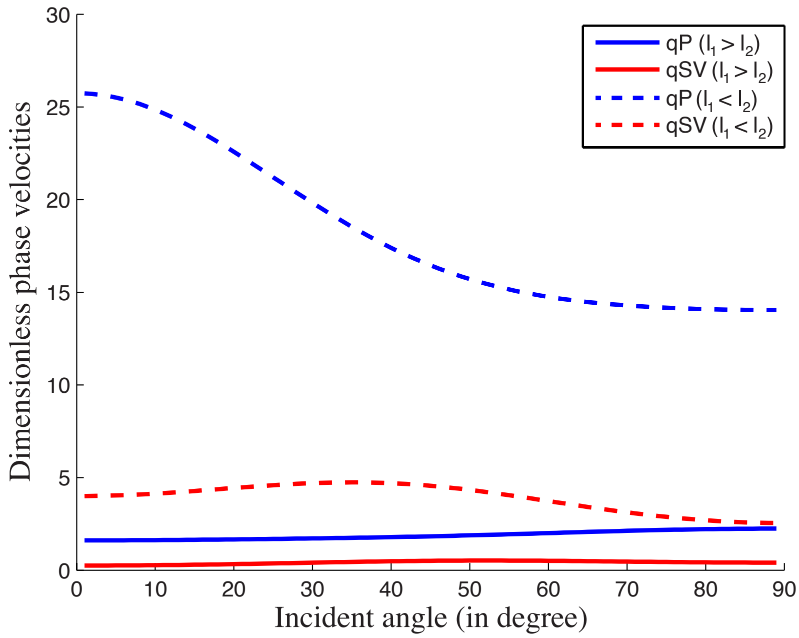
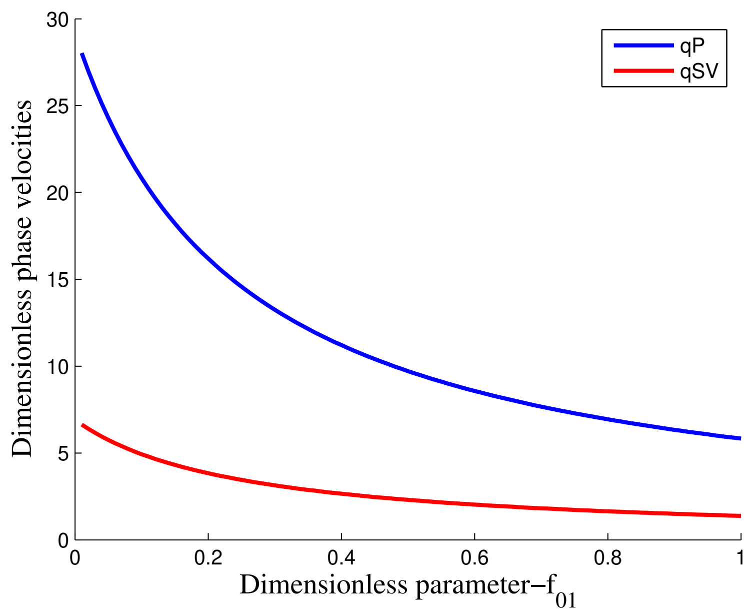
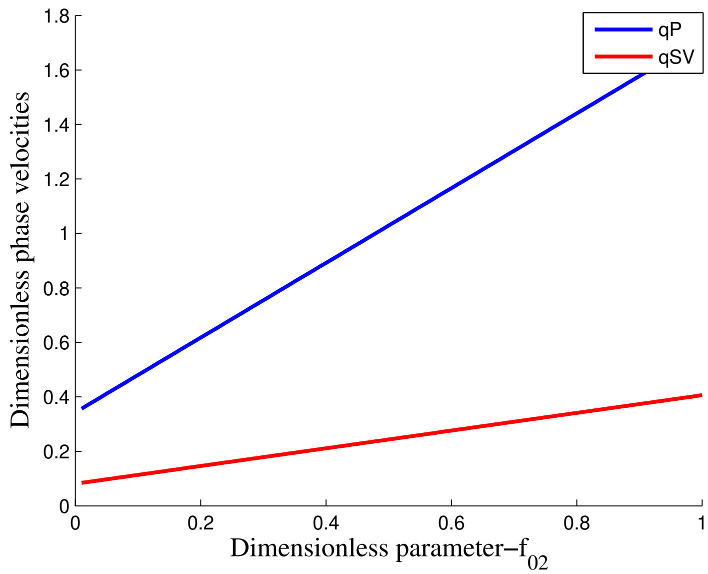


Fig. 1. Dimensionless phase velocities variation with angle directions θ

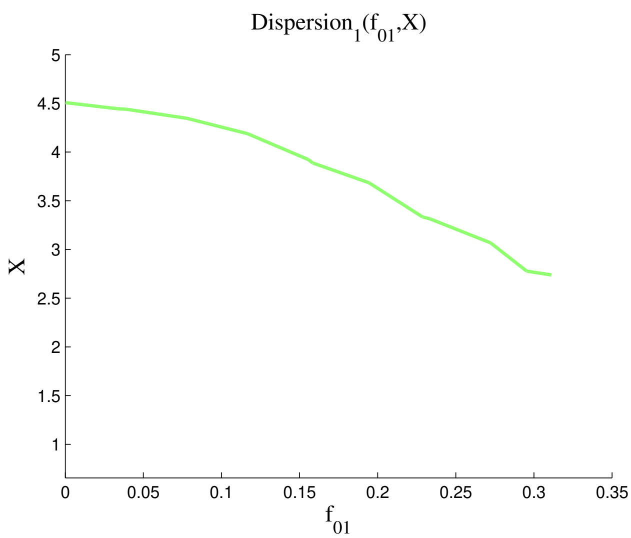


(a)



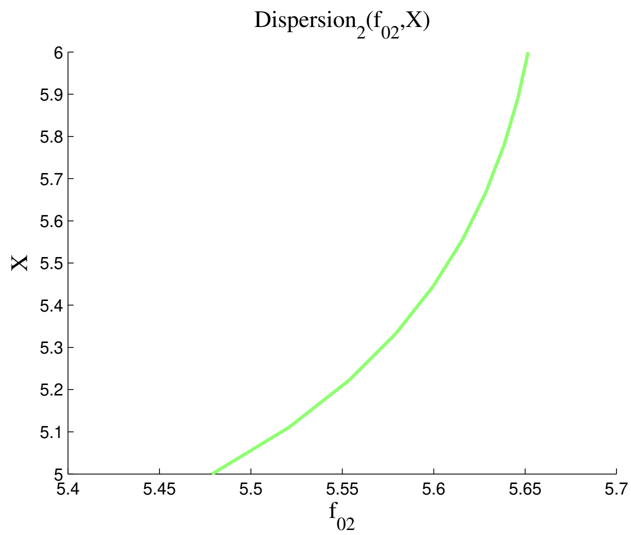
(b)

Fig. 2. Influences of dimensionless parameters f_{01} and f_{02} on dimensionless phase velocities waves



(a) $f_{02} < 1$

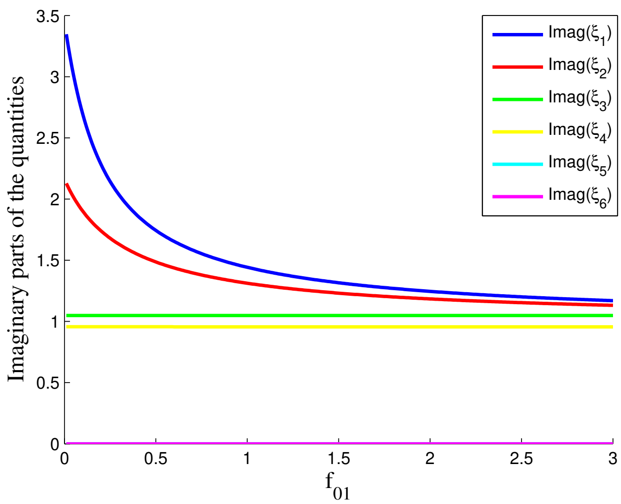
The effect of f_{01} and f_{02} parameters on the non-dimensional speed of the Rayleigh wave X are shown graphically in Fig. 3(a) and Fig. 3(b), respectively. In Fig. 3(a), the



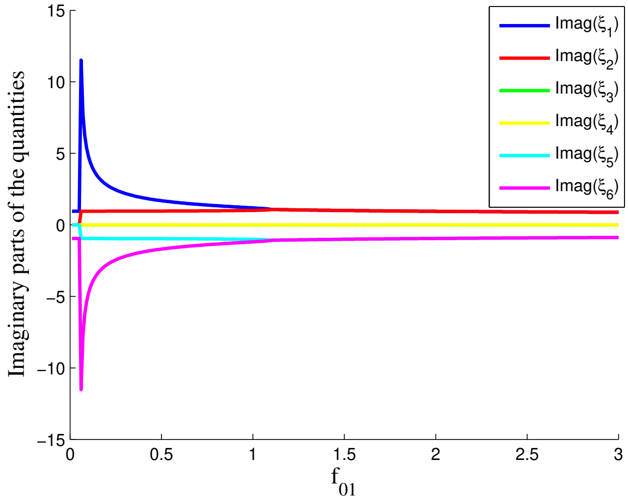
(b) $f_{01} > 1$

Fig. 3. Variation of non-dimensional speed X of Rayleigh wave against dimensionless parameters f_{01} and f_{02}

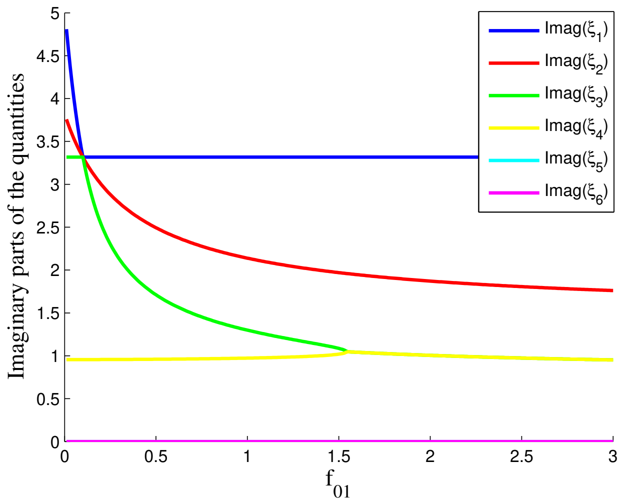
speed of Rayleigh wave is decreasing when the parameter f_{01} is increasing. The Rayleigh wave exists only in the domain $0 \leq f_{01} \leq 0.34$ with $f_{02} < 1$. On the contrary, the Rayleigh wave exists in the domain $5.0 \leq f_{02} \leq 5.67$ with $f_{01} > 1$ (see Fig. 3(b)).



(a) $f_{02} > 1$



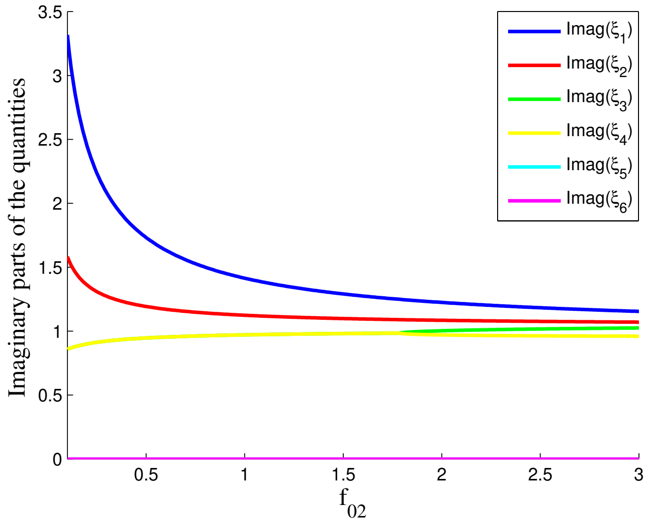
(b) $f_{02} < 1$



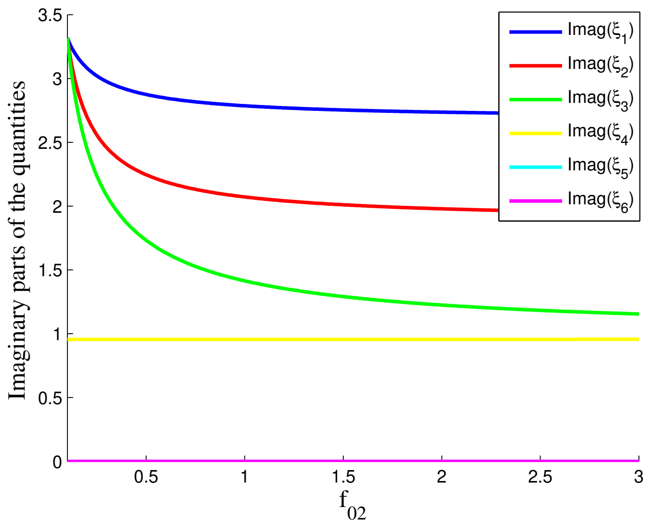
(c) $f_{02} = 0$

Fig. 4. The existence of surface waves depend on f_{01} for three cases

The existence of solutions of the characteristic equation (8) in domain $0 \leq f_{01} \leq 3$, under restrictions generated by $\mathrm{Im}(\xi) > 0$ is illustrated by Fig. 4 for three cases $f_{02} > 1, f_{02} < 1, f_{02} = 0$, respectively. Fig. 4(a) shows all imaginary parts ξ_i, $(i = 1,2,3,4)$ are positive. Hence, there are no surface waves in this case. In contrast to Fig. 4(a), in Fig. 4(b) we can always choose two surface waves satisfying the problem with $0.1 \leq f_{01} \leq 3$. For $f_{02} = 0$ (Nonlocal theory) there are three surface waves in domain $1.51 \leq f_{01} \leq 3$.



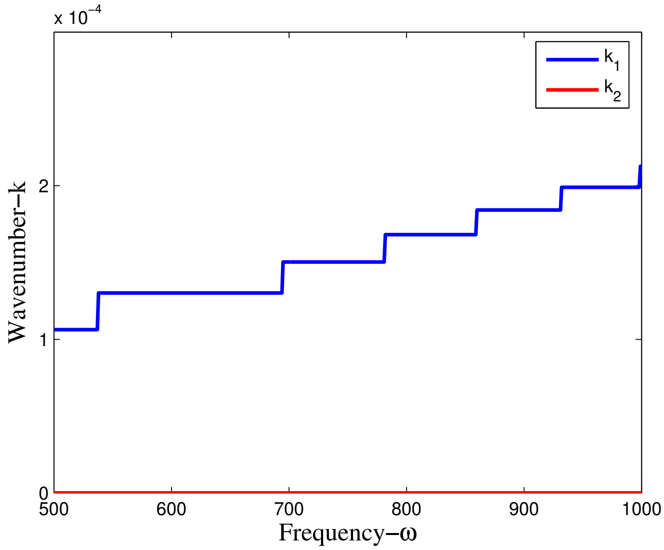
(a) $f_{01} > 1$



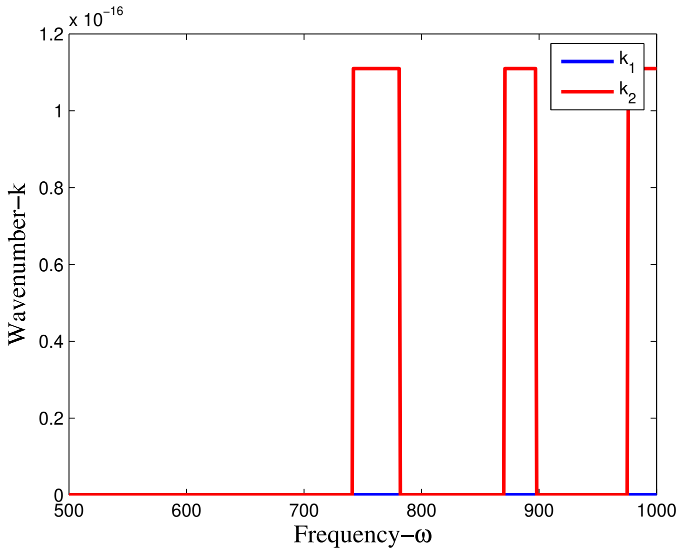
(b) $0 \leq f_{01} < 1$

Fig. 5. The existence of surface waves depend on f_{02} for two cases

The existence of surface waves depends on $0 \leq f_{02} \leq 3$ for two cases are depicted in Fig. 5. There are always three solutions with the positive imaginary in domain $0 \leq f_{02} \leq 1.75$ with $f_0 > 1$ (see Fig. 5(a)). There are no surface waves in domain $1.75 < f_{02} \leq 3$ with $f_0 > 1$ (see Fig. 5(a)) or $0 \leq f_{02} \leq 3$ with $0 \leq f_0 < 1$ (see Fig. 5(b)).



(a) $l_2 = 10l_1$



(b) $l_1 = 10l_2$

Fig. 6. Wavenumber k variation with frequency ω for two cases

The variation of the cut-off frequencies with wavenumber k variation with frequency ω are shown in Fig. 6. In Fig. 6(a), the wavenumber k_1 is dominant over k_2 for $l_2 = 10l_1$. The opposite is shown in Fig. 6(b) for $l_1 = 10l_2$.

6. CONCLUSIONS

In the present work, we have studied the propagation of surface waves in transversely isotropic piezoelastic medium based on the nonlocal strain gradient theory. The existence of the number of surface waves depends on the dimensionless nonlocal parameter f_{01}, dimensionless gradient length parameter f_{02} of the medium through the number of solutions satisfying the damping condition of the characteristic equation. It is clearly dispersive due to the appearance of the parameters f_{01} and f_{02}. Moreover, the expression of the cut-off is derived. Phase speeds of waves are computed numerically and their variation against the incident angle θ, two dimensionless length scale parameters are presented graphically. These parameters have significant effect on the velocities of propagation of Rayleigh-type waves.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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APPENDIX

The coefficients of characteristic equation for NSGT

$$
\begin{aligned}
t_{12} =\ & -f_{02}^3(e^2+f_3),\\
t_{10} =\ & f_{02}^2(-3f_3+f_{02}(f_2(2+f_2)-(3+f+f_1)f_3)+f_{01}(1+f_3)X\\
& -e^2(3+f_{02}(3+f_1-2(e_1+(e_1+e_2)f_2)+(e_1+e_2)^2f_3)-f_{01}X)),\\
t_8 =\ & f_{02}(-3f_3-f_{02}^2(f_1-(3+f)f_2(2+f_2)+3(1+f)f_3+(3+f)f_1f_3)\\
& +3f_{02}(f_2(2+f_2)-(2+f+f_1)f_3)+2f_{01}(1+f_3)X+f_{02}(1+f_3+f_{01}(4+f+f_1\\
& +(3+f)f_3))X-f_{01}^2X^2+e^2(-3+2f_{01}X+f_{02}(6e_1+6e_1f_{02}-e_1^2f_{02}-3f_{02}f_1\\
& -2e_2f_{02}f_1-3(2+f_{02}+f_1)+6e_1f_2+6e_2f_2+6e_1f_{02}f_2+6e_2f_{02}f_2+2e_1e_2f_{02}f_2\\
& +2e_2^2f_{02}f_2-3(e_1+e_2)^2(1+f_{02})f_3+(1+(3+2e_2+(e_1+e_2)^2)f_{01})X))),\\
t_6 =\ & -f_3+f_{02}(3f_2(2+f_2)-3(1+f+f_1)f_3-3f_{02}(f_1-(2+f)f_2(2+f_2)+f_3+2ff_3\\
& +(2+f)f_1f_3)-f_{02}^2(-3(1+f)f_2(2+f_2)+f_3+3ff_3+f_1(3+f+3(1+f)f_3)))\\
& +f_{01}X+(f_{01}f_3+2f_{02}(1+f_3+f_{01}(3+f+f_1+(2+f)f_3))+f_{02}^2(3+f_1+2f_3\\
& +3f_{01}(2+f_1+f_3)+f(1+f_3+f_{01}(4+f_1+3f_3))))X-f_{01}(f_{01}+2f_{02}\\
& +(3+f)f_{01}f_{02})X^2+e^2(-1+f_{01}X+f_{02}(-3(1+f_{02}+f1)+6e_1(1+f_{02})(1+f_{02}\\
& +f_2+f_{02}f_2+e_2f_{02}f_2-e_2(1+f_{02})f_3)-f_{02}^2(1+(3+e_2(6+e_2))f_1+3e_2(-2(1\\
& +e_2)f_2+e_2f_3))-6f_{02}((1+e_2)f_1+e_2(-(2+e_2)f_2+e_2f_3))+2(1+2f_{01})X\\
& +(2+3f_{01}+e_2(2+e_2+6f_{01}+4e_2f_{01}))f_{02}X+2e_1e_2(f_{02}+f_{01}(2+3f_{02}))X\\
& +e_2(6f_2-3e_2f_3+2(2+e_2)f_{01}X)+e_1^2(-3(1+f_{02})(f_{02}+f_3+f_{02}f_3)\\
& +(f_{02}+f_{01}(2+3f_{02}))X))),\\
t_4 =\ & f_2(2+f_2)-(f+f_1)f_3+f_{02}^3((1+3f)f_2(2+f_2)-ff_3-f_1(3+f_3+3f(1+f_3)))\\
& +(1+f_3+f_{01}(2+f+f_1+f_3+ff_3))X-f_{01}(2+(2+f)f_{01})X^2+f_{02}^2(-3(-(1\\
& +2f)f_2(2+f_2)+ff_3+f_1(2+f+f_3+2ff_3))+(3+2f_1+f_3+f_{01}(4+3f_1+f_3)\\
& +f(3+f_1+2f_3+3f_{01}(2+f_1+f_3)))X)+f_{02}(-3(-(1+f)f_2(2+f_2)+ff_3\\
& +f_1(1+f_3+ff_3))+2(2+f_1+f_3+f_{01}(3+2f_1+f_3)+f(1+f_3+f_{01}(3+f_1\\
& +2f_3)))X-(1+2(2+f)f_{01}+3(1+f)f_{01}^2)X^2)+e^2((1+f_{02})(-(1+f_{02}(2+f_{02}\\
& +3e_2(2+(2+e_2)f_{02})))f_1+e_2(1+f_{02})(2(1+f_{02}+3e_2f_{02})f_2-e_2(1+f_{02})f_3))\\
& +(1+f_{01}+2e_2f_{01}+e_2^2f_{01}+2(1+e_2)(1+e_2+f_{01}+3e_2f_{01})f_{02}+(1+f_{01}\\
& +e_2(4+3e_2+6(1+e_2)f_{01}))f_{02}^2)X+e_1^2(1+f_{02})(-(1+f_{02})(f_3+f_{02}(3+f_3))\\
& +(f_{01}+2f_{02}+3f_{01}f_{02})X)+2e_1(1+f_{02})((1+f_{02})(1+f_{02}+f_2+f_{02}f_2\\
& +3e_2f_{02}f_2-e_2(1+f_{02})f_3)+e_2(f_{01}+2f_{02}+3f_{01}f_{02})X)),
\end{aligned}
$$

$$
\begin{aligned}
t_2 = & -(1+f_{02})^2(-f(1+f_{02})f_2(2+f_2)+f_1(1+f_{02}+3ff_{02}+f(1+f_{02})f_3)) \\
& +(1+f_{02})((1+f_{01})(1+f_{02})(1+f_1)+f(1+f_3+f_{02}(3+2f_1+f_3) \\
& +f_{01}(2+f_1+f_3+f_{02}(4+3f_1+f_3))))X-(1+f_{01})(1+f_{01}+2ff_{01} \\
& +(1+f+f_{01}+3ff_{01})f_{02})X^2-e^2(1+f_{02})(e_2(1+f_{02})((2+2f_{02}+3e_2f_{02})f_1 \\
& -2e_2(1+f_{02})f_2)-e_2(2(1+f_{01})(1+f_{02})+e_2(1+2f_{01}+3f_{02}+4f_{01}f_{02}))X \\
& -2e_1e_2(1+f_{02})(f_2+f_{02}f_2+X+f_{01}X)+e_1^2(1+f_{02})(1+f_{02}-(1+f_{01})X)), \\
t_0 = & -(1+f_{02})((1+f_{02})f_1-(1+f_{01})X)((e^2e_2^2+f)(1+f_{02})-f(1+f_{01})X).
\end{aligned}
$$