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MOTION OF MECHANICAL SYSTEMS WITH NON-IDEAL CONSTRAINTS

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Abstract. In the paper a new method for mechanical systems with non-ideal constraints is presented. It is proved that a mechanical system subjected to physical non-ideal constraints cannot be determined purely by theoretical analysis because the reaction forces depend on the physical parameters of interactive environment, which are identified only by measurement. The principle of compatibility is shown to be an effective tool in combination with experience to investigate such a problem. For illustration the dynamics of a digging machine is investigated.

Keywords: The principle of compatibility, non-ideal constraints, physical constraints, interactive environment, method of transmission matrix.

1. INTRODUCTION

The motion of mechanical systems under both physical and program constraints is paid more and more attention in technology and in automatic control industry. In the physical constraints, both the ideal and non-ideal constraints are attracted a lot of interests but only ideal constraints are considered. The non-ideal constraints are rarely discussed due to the fact that it is impossible to define the work (of virtual displacements) of the reaction forces corresponding to the non-ideal constraints. Therefore, the corresponding generalized forces cannot be determined. Up to now only the Coulomb friction constraints are considered as a particular case of non-ideal constraints which were mentioned in papers [4, 9-13].

However, non-ideal physical constraints are often met in practical applications, for example in machine tools (milling machine, lathe, planer, honing machine, etc.) or front loaders, and scoop loaders. Working surfaces created by machine tools or tunnels and ditches dug by loaders are not often considered as ideal constraints but non-ideal constraints. The active forces (cutting or digging forces), which are not tangent to the constraint hyperplanes need to meet the problem's requirements. Thus, the constraints in those cases must be considered as non-ideal. For general non-ideal constraints such as the ones in metal fabricating machines, excavating machines, the active forces can only be measured by experiments and given in engineering notebooks [12].

This paper presents a new method to analyse mechanical systems with non-ideal physical constraints and apply to the dynamics analysis of digging machines. Furthermore, this method can be extended for the cases of friction and program constraints.

2. EQUATIONS OF MOTION OF A MECHANICAL SYSTEM WITH CONSTRAINTS

Let consider an unconstrained mechanical system of N particles M_k of mass m_k. Let denote the position vector of the particle M_k and applied force acting to the particle M_k by $\vec{r}_k(x_k, y_k, z_k)$ and $\vec{F}_k(\vec{F}_{kx}, \vec{F}_{ky}, \vec{F}_{kz}), (k = \overline{1, N})$, respectively. Suppose the constraints imposed on the considered system are of the form

$$f_\alpha(x_k, y_k, z_k) = 0; \alpha = \overline{1, r}. \tag{1}$$

Let denote the reaction forces acting to the particle M_k by $\vec{R}_k(X_k, Y_k, Z_k)$. In the case of ideal constraints, the reaction forces are directed to the normal direction of the hypersurface (1) at the contact point. In the contrary case, i.e. the case of non-ideal constraints, the reaction forces include the tangent components (the tangent and bi-tangent components), which are called the friction forces. The physical substance of these forces is very complicate, only one fact is known: they give negative works over virtual displacements. Let now consider the motion of the considered system in generalized coordinates. Suppose that the position of the system is defined by the redundant coordinates $q_j(j = \overline{1, m})$. The constraint Eqs. (1) are of the form now

$$f_\alpha(q_1, q_2, \ldots, q_m) = 0; \alpha = \overline{1, r}. \tag{2}$$

Let the kinetic energy of the system be of the form

$$T = \frac{1}{2} \sum_{i,j=1}^{m} a_{ij} \dot{q}_i \dot{q}_j.$$

In the matrix form, the kinetic energy is written as follows

$$T = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{A} \dot{\mathbf{q}}^T. \tag{3}$$

Where $\mathbf{A} = [a_{ij}]$ is an $(m \times m)$ nonsingular symmetrical matrix, but

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \ldots & \dot{q}_m \end{bmatrix}^T.$$

The symbol T denotes the transpose of matrix. Analogously, the following matrix notation is introduced

$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 & \ldots & \ddot{q}_m \end{bmatrix}^T,$$
$$\mathbf{Q} = \begin{bmatrix} Q_1 & Q_2 & \ldots & Q_m \end{bmatrix}^T.$$

The constraint Eqs. (2) in the matrix form can be written as follows

$$\mathbf{f}\ddot{\mathbf{q}} + \mathbf{f}^0 = \mathbf{0}. \tag{4}$$

Where $\mathbf{f}$ and $\mathbf{f}^0$ are the $r \times m$ and $(r \times 1)$ matrices respectively:

$$\mathbf{f} = [f_{\alpha i}]\,; f_{\alpha i} = \frac{\partial f_\alpha}{\partial q_i}; \mathbf{f}^0 = \left[\sum_{i,j=1}^{m} \frac{\partial^2 f_\alpha}{\partial q_i \partial q_j}\right]; \alpha = \overline{1, r}; i = \overline{1, m}. \tag{5}$$

From now on a vector is treated as a matrix. By the Principle of Compatibility the motion equations of the constrained system are written in the form [3-7]

$$\mathbf{A}\ddot{\mathbf{q}} = \mathbf{Q} + \mathbf{Q}^{\mathbf{0}} - \mathbf{Q}^* + \mathbf{R}, \tag{6}$$

where

$$Q_i = -\frac{\partial \pi}{\partial q_i} + \bar{Q}_i; i = \overline{1, m} \tag{7}$$

π - the potential energy of the system

$\bar{Q}_i$ - the generalized forces of non-potential forces, but

$$\mathbf{Q}^0 = \begin{bmatrix} Q_1^0 & Q_2^0 & \dots & Q_m^0 \end{bmatrix}; Q_i^0 = \frac{1}{2}\dot{\mathbf{q}}^T \partial_i \mathbf{A}\dot{\mathbf{q}}; \mathbf{Q}^* = \sum_{i=1}^{m} \partial_i \mathbf{A}\dot{\mathbf{q}}_i^*;$$

$$\partial_i \mathbf{A} = \begin{bmatrix} \frac{\partial a_{11}}{\partial q_i} & \frac{\partial a_{12}}{\partial q_i} & \dots & \frac{\partial a_{1m}}{\partial q_i} \\ \frac{\partial a_{12}}{\partial q_i} & \frac{\partial a_{22}}{\partial q_i} & \dots & \frac{\partial a_{2m}}{\partial q_i} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial a_{1m}}{\partial q_i} & \frac{\partial a_{2m}}{\partial q_i} & \dots & \frac{\partial a_{mm}}{\partial q_i} \end{bmatrix}; \dot{\mathbf{q}}_i^* = \begin{bmatrix} \dot{q}_1 \dot{q}_i \\ \dot{q}_2 \dot{q}_i \\ \vdots \\ \dot{q}_m \dot{q}_i \end{bmatrix}. \tag{8}$$

$\mathbf{R}$ - the generalized reaction forces corresponding to the i - generalized coordinate.

By the Principle of Compatibility [3-7] the reaction $\mathbf{R}$ has to satisfy the equation

$$\mathbf{F}\mathbf{R} + \mathbf{F}^0 = 0. \tag{9}$$

Where

$$\mathbf{F} = \mathbf{f}\mathbf{A}^{-1}; \mathbf{F}^0 = \mathbf{F}(\mathbf{Q} + \mathbf{Q}^0 - \mathbf{Q}^*) + \mathbf{f}^0. \tag{10}$$

By such a way we obtain r algebraic equations containing m unknowns $R_i(r < m)$. In order to determine the reactions $R_i(r < m)$ it is necessary to fill up $k = (m - r)$ equations containing only m unknowns, $R_i(r < m)$ which together with r Eqs. (9) yield a complete set of equations of unknowns $R_i(i = \overline{1, m})$, where $\mathbf{A}^{-1}$ is the inverse matrix of the inertia matrix $\mathbf{A}$.

3. THE IDEALITY AND NON-IDEALITY OF CONSTRAINTS

3.1. The ideality of constraints

As known, the motion of the system with the constraints (4) is described by the Eqs. (6), where the reactions $R_i(r < m)$ are determined by the Eqs. (9). However, the obtain system of equations is not complete yet. The problem will be solved in the case of the class of constraints, so-called ideal constraints, which can be defined by the axiom of ideality by Przeborski-Appell-Chetaev [4, 5].

By this axiom we have [3-5]

$$\mathbf{DR} = 0. \tag{11}$$

Where the $(k \times m)$ matrix $\mathbf{D}$ consists of the elements, which are the coefficients in term of the expressions of generalized accelerations represented through independent generalized accelerations. In the other words, calculating the generalized accelerations $\ddot{q}_j(j = \overline{1, m})$ from the Eqs. (2) we obtain

$$\ddot{q}_j = \sum_{\sigma=1}^{k=m-r} d_{\sigma j}\ddot{q}_\sigma + \ldots; (j = \overline{1, m}),$$

where the non-written terms are the terms which do not include the generalized accelerations. The $(k \times m)$ matrix $\mathbf{D}$

$$\mathbf{D} = [d_{\sigma i}]\,; \;\; (i = \overline{1, m}; \sigma = \overline{1, k} = m - r), \tag{12}$$

can be determined by either analytical or by numerical algorithm [2].

We obtain a closed set of m algebraic equations containing m unknowns . By solving these equations we get the reaction forces $R_i(i = \overline{1, n})$, which are the functions of the generalized coordinates and velocities as

$$\mathbf{R} = \mathbf{R}(\mathbf{q}, \dot{\mathbf{q}}). \tag{13}$$

By integrating the Eqs. (6), where the reactions $\mathbf{R}$ determined from (9) and (11), we determine the motion of the considered system. Note that in this process the reaction force $\mathbf{R} = \mathbf{R}(\mathbf{q}, \dot{\mathbf{q}})$ is calculated independently of integration of the differential Eqs. (6).

In general, the motion of the system with ideal constraints is defined by means of $2m$ Eqs. (6), (9) and (11) and by means of m algebraic equations (9) with (11) and it is possible to compute separately the reactions $R_i(q_i, \dot{q}_i)$ with respect to the equations of motion (6). However, it is possible to define the motion of the system only by r Eqs. (2) and $(m - r)$ differential equations

$$\mathbf{DA}\ddot{\mathbf{q}} = \mathbf{D}(\mathbf{Q} + \mathbf{Q}^0 - \mathbf{Q}^*). \tag{14}$$

In the other, the motion of the system with ideality constraints is determined by a set m differential algebraic Eqs. (2) and (14). By such a way the size of the problem is decreased by half.

However, in the case of non-ideal constraints, the condition (11) is not satisfied.

Thus, when the constraints are non-ideal, we have

$$\mathbf{DR} \neq 0. \tag{15}$$

3.2. The motion of the system with non-ideal constraints

Let consider the system with constraints, which are not to satisfy the condition (11). Such a constraint is called the non-ideal one and the system restricted by such constraints is called the system with non-ideal constraints.

By the principle of compatibility [3-7] the equations of such a system are written in the form (6), in which the reactions of the constraints satisfy the Eqs. (9). In this case, the condition (11) is not occurred. Instead of that we have the condition (15). In the particular case of physical constraints, the reactions of constraints spend work, we have

$$\mathbf{DR} < 0. \tag{16}$$

This character will help to the investigation of motion of the systems with the non-ideal constraints. By such a way, we obtain $(m+r)$ Eqs. (4) and (6) including $2m$ unknowns $(q_i, R_i); i = \overline{1,m}$ $(2m > m+r)$. In other words, the motion of the considered system is not defined yet, which depends on the substance of the constraints restricted to the considered system.

In the case of the physical constraints the condition (16) is arisen by acting one another between the system and constraints. It is important that these actions depend on the physic-mechanical characteristics of the environment. In other words, the reactions $R_i (i = \overline{1,n})$ are defined by experiments only. In connection with a non-ideal system, let write the Eqs. (4) and (6) in the form respectively

$$\mathbf{A\ddot{q}} = -\frac{\partial \pi}{\partial \mathbf{q}} + \mathbf{\bar{Q}} + \mathbf{Q}^{\mathbf{0}} - \mathbf{Q}^* + \mathbf{R}, \tag{17}$$

$$\mathbf{F\bar{Q}} + \mathbf{F}(-\frac{\partial \pi}{\partial \mathbf{q}} + \mathbf{Q^0} - \mathbf{Q}^* + \mathbf{R}) + \mathbf{f}_0 = 0, \tag{18}$$

where the reaction $\mathbf{R}$ can be determined by measuring machinery, based on the results of measured forces of environment acting on the system.

For this purpose let us denote the components of the force acting from the constraints to the considered system at the contact point $M_k(x_k, y_k, z_k)$ by (X_k, Y_k, Z_k). Let introduce the following notations

$$\mathbf{R}_k^{(xyz)} = \begin{bmatrix} X_k & Y_k & Z_k \end{bmatrix}^T; \quad \partial_i \mathbf{r}_k = \begin{bmatrix} \dfrac{\partial x_k}{\partial q_i} & \dfrac{\partial y_k}{\partial q_i} & \dfrac{\partial z_k}{\partial q_i} \end{bmatrix} \tag{19}$$

The reaction $\mathbf{R}$ in the Eq. (17) is written as follows

$$\mathbf{R} = \begin{bmatrix} R_1 & R_2 & \dots & R_m \end{bmatrix}^T; \quad R_i = \sum_{k=1}^{N} \partial_i \mathbf{r}_k \mathbf{R}_k^{(xyz)}; \quad i = \overline{1,m} \tag{20}$$

From the measuring result (X_k, Y_k, Z_k), the reaction $\mathbf{R}$ can be calculated by means of the formulas (20). Next, by the formula (18) it is possible to compute the drive force $\mathbf{\bar{Q}}$ for the system realizing the non-ideal constraints (2). After calculating the driven force $\mathbf{\bar{Q}}$, the motion of the system with non-ideal constraints (2) is defined by integrating the differential Eq. (17).

The algorithm for solving the system of equation of motion is following [2]:

– Determining the element X_k, Y_k, Z_k by measurement
– Calculating $R_i, i = 1, \dots, m$ by (20)
– Determining $\mathbf{\bar{Q}}$
– Determining drive forces
– Integration of equations of motion (17)
– Repeating for next time step.

In the case the constraint is the surface or the curve, the reaction forces can be expressed by components in the axes of a moving trihedral composed of a tangent, a normal, and a bi-normal to the trajectory. Its origin moves along the trajectory. The components of the reaction in the coordinate system *(Mxyz)* can be expressed in term of its component in the *(Mtnb)* coordinate axes by means of the transmission matrix $\mathbf{V}$ [8], where Mt is oriented by the tangent direction, Mn - by the normal direction, but Mb - by the bi-normal direction. By such a way, we have

$$\mathbf{R}^{(Mxyz)} = \mathbf{V}\mathbf{R}^{(Mtnb)}. \tag{21}$$

The Eq. (18) is written as follows

$$\mathbf{F}\bar{\mathbf{Q}} + \mathbf{F}(-\frac{\partial \pi}{\partial \mathbf{q}} + \mathbf{Q}^0 - \mathbf{Q}^* + \mathbf{V}\mathbf{R}^{(Mtnb)}) + \mathbf{f}_0 = 0. \tag{22}$$

By this equation we can compute the driven force $\bar{\mathbf{Q}}$ corresponding to $\mathbf{R}^{(Mtn)}$. It is noticed that from (21) we have

$$\mathbf{R}^{(Mtnb)} = \mathbf{V}^{-1}\mathbf{R}^{(Oxyz)}, \tag{23}$$

where $\mathbf{V}^{-1}$ is the inverse matrix of the transmission matrix $\mathbf{V}$.

4. EXAMPLE

Determine the drive moments M_1, M_2 of the servomotor acting on the links of the digging machine as Fig. 1. The length of the link OA and AB is equal l_1 and l_2 respectively and joined by the revolute joints. For simplicity let take $l_1 = l; l_2 = \beta l$ The mass of the links are neglected, the ditching scoop of the mass m is treated as a particle with the body coordinate $(a, 0)$ and the global coordinate *(Bxy)*. The scoop is jointed to the end point B of the link AB.

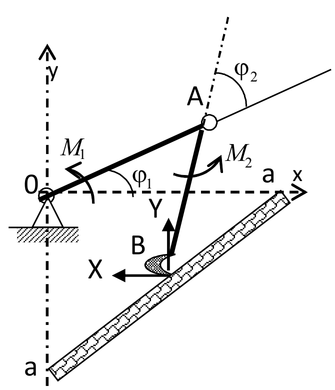

Fig. 1. Close-loop two link model of the digging machine

Suppose that the work trajectory of the scoop is of the form

$$f \equiv x - y - a = 0. \tag{24}$$

Let choose the generalized coordinates by φ_1, φ_2, where φ_1 is the angle of the link OA with respect to the fixed horizontal axis Oy, but the φ_2 - the angles between links BA and OA. The transmission matrices are of the form

$$t_1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad t_{11} = \begin{bmatrix} -s_1 & -c_1 & 0 \\ c_1 & -s_1 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$t_2 = \begin{bmatrix} c_2 & -s_2 & l \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad t_{21} = \begin{bmatrix} -s_2 & -c_2 & 0 \\ c_2 & -s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad r = \begin{bmatrix} -\beta l \\ 0 \\ 1 \end{bmatrix};$$

$$t_{12} = \begin{bmatrix} -c_1 & s_1 & 0 \\ -s_1 & -c_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad t_{22} = \begin{bmatrix} -c_2 & s_2 & 0 \\ -s_2 & -c_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Where $s_i \equiv \sin\varphi_i; c_i \equiv \cos\varphi_i; t_{i1}$ is the matrix, its elements of which are the first derivatives of the elements of the matrix t_i with respect to the variable φ_i, but t_{i2} - the second derivatives of the elements of the matrix [8]. The elements of the (2×2) matrix of inertia of the manipulator $\mathbf{A}$ are as follows [8]

$$\begin{aligned} a_{11} &= mr^T {t_2}^T {t_{11}}^T t_{11} t_2 r = ml^2(1 + 2\beta\cos\varphi_2 + \beta^2); \\ a_{22} &= mr^T t_{21}^T t_1^T t_1 t_{21} r = m\beta l^2; \\ a_{12} &= a_{21} = mr^T t_{21}^T t_1^T t_{11} t_2 r = m\beta l^2(\beta + \cos\varphi_2). \end{aligned} \tag{25}$$

The inverse matrix of the inertia matrix will be

$$\mathbf{A}^{-1} = \begin{bmatrix} \dfrac{1}{ml^2\sin^2\varphi_2} & -\dfrac{(\beta + \cos\varphi_2)}{m\beta l^2\sin^2\varphi_2} \\ -\dfrac{(\beta + \cos\varphi_2)}{m\beta l^2\sin^2\varphi_2} & \dfrac{(1 + 2\beta\cos\varphi_2 + \beta^2)}{m\beta l^2\sin^2\varphi_2} \end{bmatrix}. \tag{26}$$

The potential energy is of the form

$$\begin{aligned} \pi &= -mgl[\sin\varphi_1 - \beta\sin(\varphi_1 + \varphi_2)] \\ -\frac{\partial\pi}{\partial\varphi_1} &= mgl[\cos\varphi_1 - \beta\cos(\varphi_1 + \varphi_2)]; \\ -\frac{\partial\pi}{\partial\varphi_2} &= -mgl\beta\cos(\varphi_1 + \varphi_2). \end{aligned}$$

Therefore the potential generalized forces are calculated as follows

$$\mathbf{Q}^{(\pi)} = \begin{bmatrix} -\dfrac{\partial\pi}{\partial\varphi_1} & -\dfrac{\partial\pi}{\partial\varphi_2} \end{bmatrix}^T = \begin{bmatrix} mgl[\cos\varphi_1 - \beta\cos\varphi_1 + \varphi_2)] & -mg\beta l\cos(\varphi_1 + \varphi_2) \end{bmatrix}^T. \tag{27}$$

Suppose that the force from environment acting to the scoop of the manipulator in the axes of global coordinate is denoted by $\mathbf{R}(x, y) = [-\mathbf{X} - \mathbf{Y}]^T$, which are determined by the measure. The power of the force $\mathbf{R}$ will be

$$W = \mathbf{v}_B^T\mathbf{R} = \mathbf{r}^T\mathbf{t}_{11}^T\mathbf{t}_2^T\mathbf{R}\dot{\varphi}_1 + \mathbf{r}^T\mathbf{t}_1^T\mathbf{t}_{21}^T\mathbf{R}\dot{\varphi}_2.$$

Therefore the generalized forces R_1, R_2 corresponding to the generalized coordinates φ_1, φ_2 are of the form [8]

$$\begin{aligned} R_1 &= \mathbf{r}^T \mathbf{t}_{11}^T \mathbf{t}_2^T \mathbf{R} = l\left[\sin\varphi_1 - \beta\sin\left(\varphi_1+\varphi_2\right)\right] + X + l\left[\cos\varphi_1 - \beta\cos\left(\varphi_1+\varphi_2\right)\right] Y\,; \\ R_2 &= \mathbf{r}^T \mathbf{t}_1^T \mathbf{t}_{21}^T \mathbf{R} = l\beta\left[\sin\left(\varphi_1+\varphi_2\right) X - \cos\left(\varphi_1+\varphi_2\right) Y\right]. \end{aligned} \tag{28}$$

Where the quantities X, Y are obtained from the measure, but $\mathbf{Q}^0$ and $\mathbf{Q}^*$ will be determined as follows

By the matrix of inertia, we calculate

$$\partial_{\varphi_1}\mathbf{A} = 0; \partial_{\varphi_2}\mathbf{A} = \begin{bmatrix} -2m\beta l^2\sin\varphi_2 & -m\beta l^2\sin\varphi_2 \\ -m\beta l^2\sin\varphi_2 & 0 \end{bmatrix};$$

$$Q_1^0 = \frac{1}{2}\dot{\mathbf{q}}_1^T \partial_{\varphi_1}\mathbf{A}\dot{\mathbf{q}}_1 = 0;$$

$$Q_2^0 = \frac{1}{2}\dot{\mathbf{q}}_2^T \partial_{\varphi_2}\mathbf{A}\dot{\mathbf{q}} = -m\beta l^2\sin\varphi_2\dot{\varphi}_1^2 - m\beta l^2\sin\varphi_2\dot{\varphi}_1\dot{\varphi}_2;$$

$$\mathbf{Q}^0 = \begin{bmatrix} Q_1^0 \\ Q_2^0 \end{bmatrix} = \begin{bmatrix} 0 \\ -m\beta l^2\sin\varphi_2(\dot{\varphi}_1+\dot{\varphi}_2)\dot{\varphi}_1 \end{bmatrix}. \tag{29}$$

By means of $\partial_{\varphi_1}\mathbf{A} = 0$, in accordance to (8), we have

$$\mathbf{Q}_1^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

and

$$\begin{aligned} \mathbf{Q}_2^* &= \begin{bmatrix} -2m\beta l^2\sin\varphi_2 & -m\beta l^2\sin\varphi_2 \\ -m\beta l^2\sin\varphi_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1\dot{\varphi}_2 \\ \dot{\varphi}_2^2 \end{bmatrix} \\ &= \begin{bmatrix} -m\beta l^2\sin\varphi_2\left(2\dot{\varphi}_1+\dot{\varphi}_2\right)\dot{\varphi}_2 \\ -m\beta l^2\sin\varphi_2\dot{\varphi}_1\dot{\varphi}_2 \end{bmatrix}. \end{aligned}$$

Hence, then

$$\mathbf{Q}^* = \mathbf{Q}_1^* + \mathbf{Q}_2^* = \begin{bmatrix} -m\beta l^2\sin\varphi_2(2\dot{\varphi}_1+\dot{\varphi}_2)\dot{\varphi}_2 \\ -m\beta l^2\sin\varphi_2\dot{\varphi}_1\dot{\varphi}_2 \end{bmatrix}. \tag{30}$$

The acceleration of the scoop is calculated by the formula [8]

$${}^0\mathbf{a}_B = t_{11}t_2 r\ddot{\varphi}_1 + t_1 t_{21} r\ddot{\varphi}_2 + t_{12}t_2 r\dot{\varphi}_1^2 + t_1 t_{22} r\dot{\varphi}_2^2 + 2t_{11}t_{21} r\dot{\varphi}_1\dot{\varphi}_2$$

$$\begin{bmatrix} {}^0\ddot{x}_B \\ {}^0\ddot{y}_B \end{bmatrix} = \begin{bmatrix} -l\left[\sin\varphi_1 + \beta l\sin\left(\varphi_1+\varphi_2\right)\right]\ddot{\varphi}_1 - \beta l\sin\left(\varphi_1+\varphi_2\right)\ddot{\varphi}_2 \\ -l\left[\cos\varphi_1 + \beta\cos\left(\varphi_1+\varphi_2\right)\right]\dot{\varphi}_1^2 + \beta l\sin\left(\varphi_1+\varphi_2\right)\dot{\varphi}_2^2 - 2\beta l\cos\left(\varphi_1+\varphi_2\right)\dot{\varphi}_1\dot{\varphi}_2 \\ l\left[\cos\varphi_1 + \beta\cos\left(\varphi_1+\varphi_2\right)\right]\ddot{\varphi}_1 + \beta l\cos\left(\varphi_1+\varphi_2\right)\ddot{\varphi}_2 \\ -l\left[\sin\varphi_1 + \beta\sin\left(\varphi_1+\varphi_2\right)\right]\dot{\varphi}_1^2 - \beta l\cos\left(\varphi_1+\varphi_2\right)\dot{\varphi}_2^2 - 2\beta l\sin\left(\varphi_1+\varphi_2\right)\dot{\varphi}_1\dot{\varphi}_2 \end{bmatrix}$$

where left superscript "0" denotes the acceleration in the global frame.

Let write the constraint equation in the form

$$\ddot{x} - \ddot{y} = 0.$$

We have

$$\begin{aligned}
&\mathbf{f} = \begin{bmatrix} f_1 & f_2 \end{bmatrix};\\
&f_1 = l\,\{[\sin\varphi_1 - \cos\varphi_1] + \beta[\sin(\varphi_1+\varphi_2) - \cos(\varphi_1+\varphi_2)]\}\ ;\\
&f_2 = \beta l[\cos(\varphi_1+\varphi_2) + \sin(\varphi_1+\varphi_2)]\\
&\mathrm{f}^0 = \{l(\sin\varphi_1 - \cos\varphi_1) + \beta[\sin(\varphi_1+\varphi_2) - \cos(\varphi_1+\varphi_2)]\}\,\dot{\varphi}_1^2\\
&\qquad + \beta l[\sin(\varphi_1+\varphi_2) - \cos(\varphi_1+\varphi_2)]\dot{\varphi}_2^2\\
&\qquad + 2\beta l[\sin(\varphi_1+\varphi_2) - \cos(\varphi_1+\varphi_2)]\dot{\varphi}_1\dot{\varphi}_2.
\end{aligned} \tag{31}$$

Let denote the matrix of the drive moments M_1, M_2 acting on the links of the manipulator by $\mathbf{M}$

$$\mathbf{M} = \begin{bmatrix} M_1 & M_2 \end{bmatrix}^T. \tag{32}$$

In order to determine the driven moments M_1, M_2 we write the Eq. (18)

$$\mathbf{f}\mathbf{A}^{-1}\mathbf{M} + \mathbf{f}\mathbf{A}^{-1}(\mathbf{Q}^{(\pi)} + \mathbf{Q}^0 + \mathbf{R} - \mathbf{Q}^*) + f^0 = \mathbf{0}. \tag{33}$$

We obtain one equation of two unknownsM_1, M_2. The quantities $\mathbf{Q}^{(\pi)}, \mathbf{Q}^0, \mathbf{Q}^*$ are computed by (27), (29), (30), but $\mathbf{R}$ is provided from the measurement result by means of X, Y , which are the reaction of the constraint (the environment) to the scoop of the manipulator. In result, we have one equation included two unknowns. Therefore we can choose arbitrarily one of two quantities M_1, M_2, for example, $M_1 \equiv 0$ (corresponding to the link OA to be kept in fixed) or use the condition for optimizing some property of the scoop.

Next, the motion of the manipulator is defined by means of integrating the differential Eq. (17), where the matrix of inertia is computed by (26), the remaining quantities are in accordance with (27), (29), (30), (32).

It is necessary to notice that the problem will be solved with the help of the numerical method. In the process of numerical integral the constraints will take the part of the criterion of verification.

In some cases, it is necessary to know the normal and tangent reaction forces. For this aim, let introduce the natural moving coordinate axes Btn as shown as Fig. 2. The reaction force $\mathbf{R}$ consists of a tangent and a normal component. By such a way, the reaction force $\mathbf{R}$ can be expressed in two forms

$$\mathbf{R}^{(Oxy)} = \begin{bmatrix} X & Y \end{bmatrix}^T; \quad \mathbf{R}^{(Btn)} = \begin{bmatrix} F_{fric} & N \end{bmatrix}^T.$$

Where (X, Y) are the components of the reaction force in horizontal and vertical directions, but (N, F_{fric}) - the components in normal and tangent directions.

In the other words, it is possible to establish the relation between these quantities by means of a transmission matrix. As known [8] the transmission matrix is of the form

$$\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Where θ is the position angle between two coordinate axes

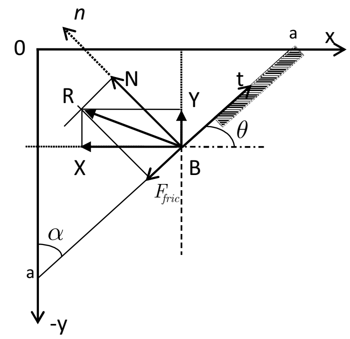

Fig. 2. Components of reaction in the natural moving frame

It is easy to obtain

$$\mathbf{R}^{(xy)} \equiv \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} -F_{fric} \\ N \end{bmatrix} \to \mathbf{R}^{(xy)} = \mathbf{V}\mathbf{R}^{(t,n)};$$
$$\mathbf{V} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}. \tag{34}$$

The Eq. (33) is written as follows

$$\mathbf{f}\mathbf{A}^{-1}\mathbf{M} + \mathbf{f}\mathbf{A}^{-1}(\mathbf{Q}^{(\pi)} + \mathbf{Q}^0 + \mathbf{V}\mathbf{R}^{(Btnb)} - \mathbf{Q}^*) + f^0 = \mathbf{0}. \tag{35}$$

This equation allows to calculate the reaction forces in normal and tangent directions. Based on these components it is possible to find out the relation between the friction force and the normal force.

<u>Note</u> :

The components of the reaction force in the (Btn) and rectangular *(Oxy)* coordinates

$$\mathbf{R}^{(Btn)} = \mathbf{V}^{-1}\mathbf{R}^{(Oxy)}; \quad \mathbf{V}^{-1} = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}. \tag{36}$$

Where $\mathbf{V}^{-1}$ denotes the inverse matrix of the matrix $\mathbf{V}$.

5. CONCLUSION

This paper presents a new general method to analyze mechanical systems with non-ideal constraints. The program constraints are applied to the mechanical system by using the Principle of Compatibility. The paper pointed out that if a mechanical system is subject to non-ideal physical constraints, its motion depends on the interaction between the constraints and the system through mechanics-physics parameters. In other words, if a mechanical system is subject to non-ideal physical constraints, its motion cannot be determined only by theoretical analysis but also experimental measurements of the interaction between the system and the environment. It is necessary to emphasize that with the help of the method of transmission matrix and measured reaction forces the generalized forces corresponding to non-ideal constraints are defined. Accordingly, the paper contributed a method to analyze the motion of mechanical systems in the interactive relation between the system and the environment. Although only the physical constraints were analyzed in the paper, this method can be extended to problems with other types of constraints such as program constraints. Of course, the presented method can be applied for the case of the non-ideal constraints of Coulomb friction type [5,9-14] and the program constraints [4-6].

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CONTENTS

Pages

1. Dang The Ba, Numerical simulation of a wave energy converter using linear generator. 103
2. Buntara S. Gan, Kien Nguyen-Dinh, Mitsuharu Kurata, Eiji Nouchi, Dynamic reduction method for frame structures. 113
3. Nguyen Viet Khoa, Monitoring breathing cracks of a beam-like bridge subjected to moving vehicle using wavelet spectrum. 131
4. Chu Anh My, Vuong Xuan Hai, Generalized pseudo inverse kinematics at singularities for developing five-axes CNC machine tool postprocessor. 147
5. Do Sanh, Dinh Van Phong, Do Dang Khoa, Motion of mechanical systems with non-ideal constraints. 157
6. N. D. Anh, Weighted Dual approach to the problem of equivalent replacement. 169