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A NOVEL CRITERION FOR CRACK DETECTION IN BEAM STRUCTURES BY FREQUENCY RESPONSE FUNCTIONS

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Abstract. The frequency response function (FRF) is a fruitful attribute that includes almost all dynamical characteristics of a structure such as natural frequencies and modes, damping coefficients, or resonance and antiresonance conceptions. However, the complex feature of FRF has not been thoroughly employed for structural damage detection. In the present study, a novel indicator extracted from FRFs of beam structures is developed for crack identification. The damage indicator originated from the well-known mode assurance criterion (MAC) and therefore it is termed spectral assurance criterion (SAC). First, a coherence coefficient calculated from FRFs of intact and damaged beams and called herein spectral damage index (SDI) is analyzed for examining sensitivity of FRFs to crack. Then, SAC calculated for different FRFs of the same damaged structure is employed for crack detection by the so-called contour method. Results obtained in numerical examples of the crack detection problem show that SAC is really a novel and efficient criterion for crack identification in beams from measured FRFs.

Keywords: crack identification, frequency response function frequency domain assurance criterion, contour method.

1. INTRODUCTION

The frequency response functions have been early employed for structural damage detection problems [1–9], however, most of the studies were based on the damage-induced changes in the FRF's shape or its constituents from points to points of a discretized structure. The FRFs as functions in the frequency domain were utilized to solve the damage detection problem by the authors of Ref [10]. Indeed, FRFs in the frequency domain have been used earlier for model correlation and updating [11] through the well-known assurance criterion concept [12–14] that was developed for structural damage detection in [15–18].

In the present study, a novel indicator extracted from frequency domain FRFs of beam structures is developed for crack identification. The damage indicator originated from the well-known mode assurance criterion (MAC) and therefore it is called spectral assurance criterion (SAC). First, a coherence coefficient calculated from FRFs of intact and damaged beams and called herein spectral damage index (SDI) is analyzed for examining sensitivity of FRFs to crack. Then, SAC calculated for different FRFs of the same damaged structure is employed for crack detection by the so-called contour method. Results obtained in numerical examples of the crack detection problem show that SAC is really a novel and efficient criterion for crack identification in beams from measured FRFs.

2. FREQUENCY RESPONSE FUNCTION MULTIPLE CRACKED BEAMS

Now, let's consider a Euler-Bernoulli (EB) beam of material and geometry constants: E, G, ν, ρ are the elastic and shear modulus, Poisson coefficient and mass density; $\ell, A = b \times h, I = bh^3/12$ - the length, cross-section area and moment of inertia. Moreover, it is assumed that the beam is cracked at positions $0 \leq e_1 < e_2 < \ldots < e_{n-1} < e_n \leq \ell$ and all the cracks are transverse and open with depths respectively $(a_1, \ldots, a_n)$ as shown in Fig. 1 [19].

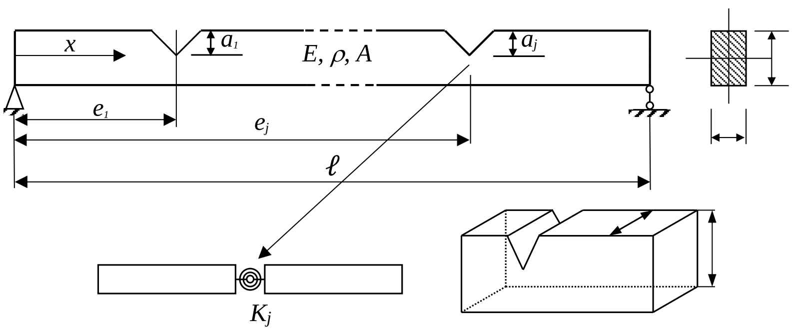


Fig. 1. Model of multiple cracked beam

2.1. Vibration shape of multiple cracked beam

As well known, vibration shape of an EB-beam is defined as solution of equation

$$\frac{d^4\phi(x)}{dx^4} - \lambda^4\phi(x) = 0, \quad \lambda^4 = \rho A\omega^2/EI, \tag{1}$$

that is solved together with boundary conditions. Moreover, in the recently published review [3] general solution for vibration shape of multiple cracked EB beam has been obtained in the form

$$\phi_0(x,\omega) = C_1\Phi_1(\lambda x) + C_2\Phi_2(\lambda x) + C_3\Phi_3(\lambda x) + C_4\Phi_3(\lambda x), \tag{2}$$

where C_1, C_2, C_3, C_4 are constants, $\lambda = \left(\rho A\omega^2/EI\right)^{1/4}$ and $\Phi_k(\lambda x), k = 1,2,3,4$ are

$$\begin{aligned}
\Phi_1(\lambda x) &= L_{01}(\lambda x) + \sum_{j=1}^{n}\mu_{j1}K\left(x-e_j\right), \quad \Phi_2(\lambda x) = L_{02}(\lambda x) + \sum_{j=1}^{n}\mu_{j2}K\left(x-e_j\right),\\
\Phi_3(\lambda x) &= L_{03}(\lambda x) + \sum_{j=1}^{n}\mu_{j3}K\left(x-e_j\right), \quad \Phi_4(\lambda x) = L_{04}(\lambda x) + \sum_{j=1}^{n}\mu_{j4}K\left(x-e_j\right),
\end{aligned} \tag{3}$$

$$K(x) = \begin{cases} 0 & \text{for } x<0 \\ S(x) & \text{for } x\geq 0 \end{cases}, \quad S(x) = (1/2\lambda)\left(\sinh\lambda x + \sin\lambda x\right), \tag{4}$$

$$\begin{aligned}
\mu_{j1} &= \gamma_j\left[L''_{01}\left(\lambda e_j\right) + \sum_{k=1}^{j-1}\mu_{k1}S''\left(e_j-e_k\right)\right],\\
\mu_{j2} &= \gamma_j\left[L''_{02}\left(\lambda e_j\right) + \sum_{k=1}^{j-1}\mu_{k2}S''\left(e_j-e_k\right)\right],\\
\mu_{j3} &= \gamma_j\left[L''_{03}\left(\lambda e_j\right) + \sum_{k=1}^{j-1}\mu_{k3}S''\left(e_j-e_k\right)\right],\\
\mu_{j4} &= \gamma_j\left[L''_{04}\left(\lambda e_j\right) + \sum_{k=1}^{j-1}\mu_{k4}S''\left(e_j-e_k\right)\right].
\end{aligned} \tag{5}$$

The functions $L_{01}(\lambda x), L_{02}(\lambda x), L_{03}(\lambda x), L_{04}(\lambda x)$ are four independent solutions of free vibration problem for uncracked beam and $(n\times 4)$ - matrix of so-called damage indexes

$$[\boldsymbol{\mu}] = \left\{\mu_{jk}, j = 1,\ldots,n; k = 1,2,3,4\right\},$$

given in Eq. (5) is calculated from crack parameters $\left(e_j, \gamma_j, j = 1,\ldots,n\right)$ by

$$\{\boldsymbol{\mu}\} = [\mathbf{G}]^{-1}[\mathbf{B}], \tag{6}$$

where $\mathbf{G} = \left[g_{ij}, i,j = 1,2,3,\ldots,n\right]$ is $n\times n$ - matrix with elements

$$g_{ij} = \{1 \text{ if } i=j; 0 \text{ for } i<j; -\gamma_i S''\left(e_i-e_j\right) \text{ for } i>j\},$$

and matrix $[\mathbf{B}] = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4]$ of 4 vectors of dimension n:

$$\begin{aligned}
\{\mathbf{b}_1\} &= \left\{\gamma_1 L''_{01}(\lambda e_1),\ldots,\gamma_n L''_{01}(\lambda e_n)\right\}^T, \ \{\mathbf{b}_2\} = \left\{\gamma_1 L''_{02}(\lambda e_1),\ldots,\gamma_n L''_{02}(\lambda e_n)\right\}^T,\\
\{\mathbf{b}_3\} &= \left\{\gamma_1 L''_{03}(\lambda e_1),\ldots,\gamma_n L''_{03}(\lambda e_n)\right\}^T, \ \{\mathbf{b}_4\} = \left\{\gamma_1 L''_{04}(\lambda e_1),\ldots,\gamma_n L''_{04}(\lambda e_n)\right\}^T,\\
\gamma_j &= 6\pi\left(1-\nu^2\right)(h/\ell)\, f_b\left(a_j/h\right),
\end{aligned} \tag{7}$$

$$f_b(z) = z^2\Big(0.6272 - 1.04533z + 4.5948z^2 - 9.9736z^3 + 20.2948z^4 - 33.0351z^5 + 47.1063z^6 - 40.7556z^7 + 19.6z^8\Big). \tag{8}$$

Substituting vibration modes (2) into boundary conditions for simply supported beam

$$\phi(0,\omega) = 0, \quad \phi''(0,\omega) = 0, \quad \phi(\ell,\omega) = 0, \quad \phi''(\ell,\omega) = 0, \tag{9}$$

leads to the equations allowing one to find natural frequencies and mode shapes of multiple cracked EB-beams. Namely, it is easily to verify that vibration shape (2) satisfying boundary conditions (9) at the left end of beam can be represented as

$$\phi_0(x,\omega) = AL_1(\lambda x) + BL_2(\lambda x), \tag{10}$$

with

$$L_1(\lambda x) = \sinh\lambda x + \sum_{j=1}^{n}\mu_{j1}K\left(x - e_j\right), \quad L_2(\lambda x) = \sin\lambda x + \sum_{j=1}^{n}\mu_{j2}K\left(x - e_j\right). \tag{11}$$

Then, remaining boundary conditions in (11) yield the equations

$$AL_1(\lambda\ell) + BL_2(\lambda\ell) = 0, \quad AL_1''(\lambda\ell) + BL_2''(\lambda\ell) = 0, \tag{12}$$

that result in the so-called frequency equation for multiple cracked beams in the form

$$L_1(\lambda\ell)\,L_2''(\lambda\ell) - L_1''(\lambda\ell)\,L_2(\lambda\ell) = 0. \tag{13}$$

In case of beam with single crack, Eq. (13) gets the form:

$$f_{ss}(\lambda) + \gamma_1 g_{ss}(\lambda, e_1) = 0, \tag{14}$$

where

$$f_{ss}(\lambda) = 2\lambda\sinh\lambda\ell\sin\lambda\ell,$$
$$g_{ss}(\lambda, e) = \sinh\lambda(\ell - e_1)\sinh\lambda e_1\sin\lambda\ell - \sinh\lambda\ell\sin\lambda(\ell - e_1)\sin\lambda e_1.$$

2.2. Frequency response function

We consider now forced vibration in multiple cracked beams under concentrated at x_e load described by the equations [20]

$$EI\frac{\partial^4}{\partial x^4}\left(1 + \mu_2\frac{\partial}{\partial t}\right)w(x,t) + \rho A\mu_1\frac{\partial w(x,t)}{\partial t} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = P_0 e^{i\omega t}\delta\left(x - x_p\right). \tag{15}$$

Seeking solution of Eq. (15) in the form

$$w(x,t) = \phi(x)\,e^{i\omega t},$$

one gets following equation for vibration shape $\phi(x)$

$$\frac{d^4\phi(x)}{dx^4} - \hat{\lambda}^4\phi(x) = \hat{P}_0\delta\left(x - x_p\right), \tag{16}$$

where

$$\hat{\lambda}^4 = \frac{\rho A\left(\omega^2 - i\omega\mu_1\right)}{EI\left(1 + i\omega\mu_2\right)}, \quad \hat{P}_0 = \frac{P_0}{EI\left(1 + i\omega\mu_2\right)}.$$

First, let's consider complex frequency parameter

$$\hat{\lambda}^4 = \frac{\rho A\left(\omega^2 - i\omega\mu_1\right)}{EI\left(1 + i\omega\mu_2\right)} = \frac{\rho A\omega^2\left(\alpha - i\beta\right)}{EI} = \frac{\rho A\hat{\omega}^2}{EI}, \tag{17}$$

with

$$\begin{gathered}\hat{\omega}^2 = \omega^2\left(\alpha + i\beta\right), \\ \alpha = \left(1 + \mu_1\mu_2\right) / \left(1 + \omega^2\mu_2^2\right), \quad \beta = \left(\mu_1 + \omega^2\mu_2\right) / \omega\left(1 + \omega^2\mu_2^2\right).\end{gathered} \tag{18}$$

It was well-known that general solution of Eq. (15) is

$$\phi\left(x\right) = \phi_0\left(x\right) + \hat{P}_0\int_0^x h\left(x - s\right)\delta\left(s - x_p\right)ds = \phi_0\left(x\right) + \hat{P}_0 h\left(x - x_p\right), \tag{19}$$

where functions

$$h\left(x\right) = \left\{0\text{: for } x \le x_p;\ \left(\sinh\hat{\lambda}x - \sin\hat{\lambda}x\right)/2\hat{\lambda}^3\text{: for } x > x_p\right\}, \tag{20}$$

and $\phi_0\left(x\right)$ given in Eq. (10). Since both the functions $\phi_0\left(x\right)$ and $h\left(x\right)$ satisfy the first two conditions in (9), the solution (19) also satisfies the boundary conditions. Therefore, putting (19) into remaining conditions (9) at the beam's right end yields

$$AL_1\left(\hat{\lambda}\ell\right) + BL_2\left(\hat{\lambda}\ell\right) = -\hat{P}_0 h\left(\ell - x_0\right), \quad AL_1''\left(\hat{\lambda}\ell\right) + BL_2''\left(\hat{\lambda}\ell\right) = -\hat{P}_0 h''\left(\ell - x_0\right),$$

that give the constants A, B to be calculated as

$$A = \left(\hat{P}_0/\Delta\right)\hat{A}, \quad B = \left(\hat{P}_0/\Delta\right)\hat{B},$$

with

$$\begin{aligned}\hat{A} &= \left[h''\left(\ell - x_p\right)L_2\left(\hat{\lambda}\ell\right) + h\left(\ell - x_0\right)L_2''\left(\hat{\lambda}\ell\right)\right], \\ \hat{B} &= \left[L_1''\left(\hat{\lambda}\ell\right)h\left(\ell - x_p\right) - L_1\left(\hat{\lambda}\ell\right)h''\left(\ell - x_p\right)\right],\end{aligned} \tag{21}$$

$$\Delta = L_1\left(\hat{\lambda}\ell\right)L_2''\left(\hat{\lambda}\ell\right) - L_1''\left(\hat{\lambda}\ell\right)L_2\left(\hat{\lambda}\ell\right), \tag{22}$$

Finally, solution (19) gets the form

$$\phi\left(x, x_p, \omega\right) = \left(\hat{P}_0/\Delta\right)\left[h\left(x - x_p\right)\Delta + \hat{A}L_1\left(\hat{\lambda}x\right) + \hat{B}L_2\left(\hat{\lambda}x\right)\right], \tag{23}$$

and therefore, the so-called frequency response function of the beam measured at x_q is

$$FRF\left(x_p, x_q, \omega\right) = \phi\left(x_p, x_q, \omega\right)/\hat{P}_0 = h\left(x_q - x_p\right) + \frac{\hat{A}L_1\left(\hat{\lambda}x_q\right) + \hat{B}L_2\left(\hat{\lambda}x_q\right)}{L_1\left(\hat{\lambda}\ell\right)L_2''\left(\hat{\lambda}\ell\right) - L_1''\left(\hat{\lambda}\ell\right)L_2\left(\hat{\lambda}\ell\right)}. \tag{24}$$

Denote the module of the frequency response function by $H_{pq}\left(\omega\right) = \left|FRF\left(x_p, x_q, \omega\right)\right|$ that is examined below as a frequency domain signal in dependence upon measuring response location x_q and excitation position x_p. In case if both measuring response and excitation locations are duplicated $x_p = x_q \equiv x_m$, the location is called driving point and the function $H_{pq}(\omega)$ is reduced to

$$H_m(\omega) = |FRF(x_m, x_m, \omega)| = \left| \frac{\hat{A}L_1(\hat{\lambda}x_m) + \hat{B}L_2(\hat{\lambda}x_m)}{L_1(\hat{\lambda}\ell)L_2''(\hat{\lambda}\ell) - L_1''(\hat{\lambda}\ell)L_2(\hat{\lambda}\ell)} \right|. \tag{25}$$

3. SPECTRAL ASSURANCE CRITERION FOR CRACKED BEAM

In this section the so-called frequency domain assurance criterion is developed for detecting cracks in beam structures by measurement of frequency response functions. First, let's consider a correlation coefficient defined for two vectors $\boldsymbol{S} = \{S_j, j = 1, \ldots, N\}$ and $\boldsymbol{Q} = \{Q_j, j = 1, \ldots, N\}$ as

$$\Phi(\boldsymbol{S}, \boldsymbol{Q}) = \left[\left(\sum_{k=1}^{N} S_k Q_k \right)^2 / \left(\sum_{k=1}^{N} S_k^2 \times \sum_{k=1}^{N} Q_k^2 \right) \right]^{1/2}.$$

Obviously, the coefficient ranges from 0 to 1 depending on the correlation between the vectors and represents the degree of their consistency. Particularly, the vectors are acknowledged as entirely consistent if the coefficient equals 1. Originally, the coefficient was utilized for checking similarity of two mode shapes for a structure as the so-called Modal Assurance Criterion (MAC) [13] and then it had got to be employed for mode shape-based damage detection as Mode Shape Damage Index (MSDI) [18] in case the vectors are mode shapes of damaged and undamaged structures. Then, Messina and Williams developed MAC for structural damage detection by measurements of natural frequencies and introduced the so-called Multiple Location Assurance Criterion (MDAC) [15, 16]. It is worth noting here the case when the correlation coefficient is examined for frequency response functions, and it is called Frequency Domain Assurance Criterion (FDAC). FDAC was used first for model correlation and updating [11, 12] and then for damage detection [17].

If the compared signals are frequency response functions of cracked and intact beams, the coherence coefficient describes the change of the functions due to crack. Therefore, the coefficient calculated for a frequency response function at cracked $H_{pq}^c(\omega) = H_{pq}(\omega, e, a)$ and intact $H_{pq}^0(\omega) = H_{pq}(\omega)$ respectively conditions, as

$$\mathrm{SDI}(e, a, p, q) = \left[\left(\sum_{k=1}^{N} H_{pq}^0(\omega_j) H_{pq}^c(\omega_j) \right)^2 / \left(\sum_{k=1}^{N} H_{pq}^{02}(\omega_j) \times \sum_{k=1}^{N} H_{pq}^{c2}(\omega_j) \right) \right]^{1/2}, \tag{26}$$

is termed by spectral damage index (SDI) of the given frequency response function. This coefficient characterizes the sensitivity of frequency response function to a crack, and it will be employed below for examining the crack-induced change in a frequency response function.

However, the above-introduced SDI is still difficult to employ for crack detection from a measured frequency response of a structure because of the following reasons. First, it is seldom to have frequency responses of both intact and damaged structures simultaneously measured. Second, every measured frequency response gives only one value of SDI that is insufficient for detecting even a single crack represented by two parameters, its location and depth. Therefore, one should have at least two frequency response functions of the structure under consideration that might be measured at different locations for various loads. Thus, let's introduce the coefficient

$$\mathrm{SAC}\left(e,a,p,q,p',q'\right)=\left[\left(\sum_{k=1}^{N}H_{pq}^{c}\left(\omega_j\right)H_{p'q'}^{c}\left(\omega_j\right)\right)^2\Big/\left(\sum_{k=1}^{N}H_{pq}^{c2}\left(\omega_j\right)\times\sum_{k=1}^{N}H_{p'q'}^{c2}\left(\omega_j\right)\right)\right]^{\frac{1}{2}}, \tag{27}$$

that is called Spectral Assurance Criterion (SAC) calculated from two different frequency response functions measured for a structure in the same damage condition. The SAC determined by quotient (27) is ranged between 0 and and 1 and it equals to 1 for the same or fully consistent frequency response functions. The deviation of SAC from unique represents the effect of cracks and locations where the frequency response functions have been measured. Therefore, SAC could be efficiently used as a novel indicator for crack identification by frequency response functions if the response measurement and applied load locations are properly chosen.

Let's consider the problem of detecting a single crack represented by its location and depth (e,a), using the above-introduced SAC. Suppose that the indicator calculated from an established model of the cracked structure as a function of the crack parameters is represented by function $\Lambda_k=f_k\left(e,a\right)=\mathrm{SAC}\left(e,a,p_k,q_k,p_k^{'},q_k^{'}\right),k=1,2,\ldots,m$ and its values calculated from measured frequency response functions are denoted by $(\Lambda_1^*,\ldots,\Lambda_m^*)$. Hence, the crack location and depth would be positive roots of the equations

$$\Lambda_1^*=f_1\left(e,a\right),\ldots,\Lambda_m^*=f_m\left(e,a\right). \tag{28}$$

If the functions $f_k\left(e,a\right),k=1,2,\ldots$ are available only graphically and given in the form of the surfaces $z_k=f_k\left(e,a\right),k=1,2,\ldots m$, actual crack location and depth would be intersections of the contours $z_1=\Lambda_1^*,\ldots,z_m=\Lambda_m^*$ in the plane (e,a). This is the desired solution to the crack detection problem resolved by using the so-called contour method. In the case of measurement noise presence, the solution to the crack detection problem can be found as a solution to the problem

$$\mathcal{E}=\sum_{k=1}^{m}\left[\Lambda_k^*-f_k\left(e,a\right)\right]^2\underset{(e,a)}{\Rightarrow}\min. \tag{29}$$

4. NUMERICAL VALIDATION AND ANALYSIS

Numerical illustrations are accomplished in this section for, first, analysis of crack-induced changes in the spectral damage index that represent a new insight to sensitivity of FRFs to crack and, second, demonstration of the spectral assurance criterion applied for crack identification from measured frequency response functions. For shortness, we consider herein only functions (25) that are diagonal elements of matrix function $H_{pq}(\omega)$ in three typical points $x_p = x_q = x_m$, where $x_m = 0.25, 0.5, 0.75$, respectively.

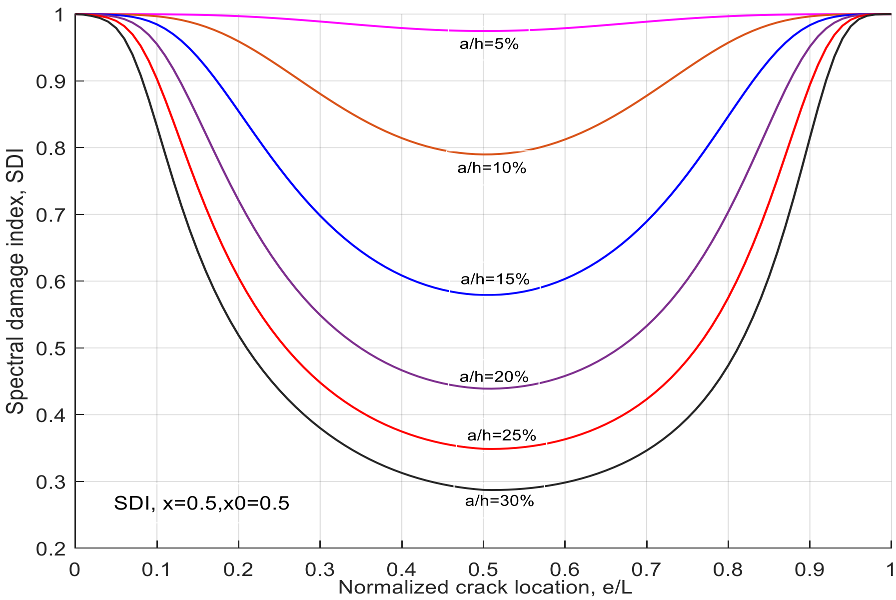


Fig. 2. Spectral damage index of FRF in dependence on crack location and depth in case of driving positions $x_m = 0.5$

4.1. Sensitivity of FRF to crack observed through spectral damage index (SDI)

SDI as functions of crack position in different relative crack depth are shown in Fig. 2 for $x_m = 0.5$ and Fig. 3 for $x_m = 0.25, 0.75$. Obviously, graphs displayed in the Figures, representing the crack-induced changes in SDI have the same form as that for the first natural frequency but with significantly increased magnitude. Namely, deviation of SDI from unique increasing with crack depth and it reaches maximum for crack at beam middle. Symmetric cracks have the same effect on the SDI, but FRFs defined at symmetrical driving points (0.25 and 0.75) have different sensitivities to crack, especially, for the crack of relative depth higher than 10%. The above remarks confirm the significant advantage of using FRFs for crack detection compared to the natural frequencies that are much less sensitive to cracks and do not allow to detection of symmetrical cracks in beams with symmetric boundary conditions. This will be substantiated below in the subsequent subsection.

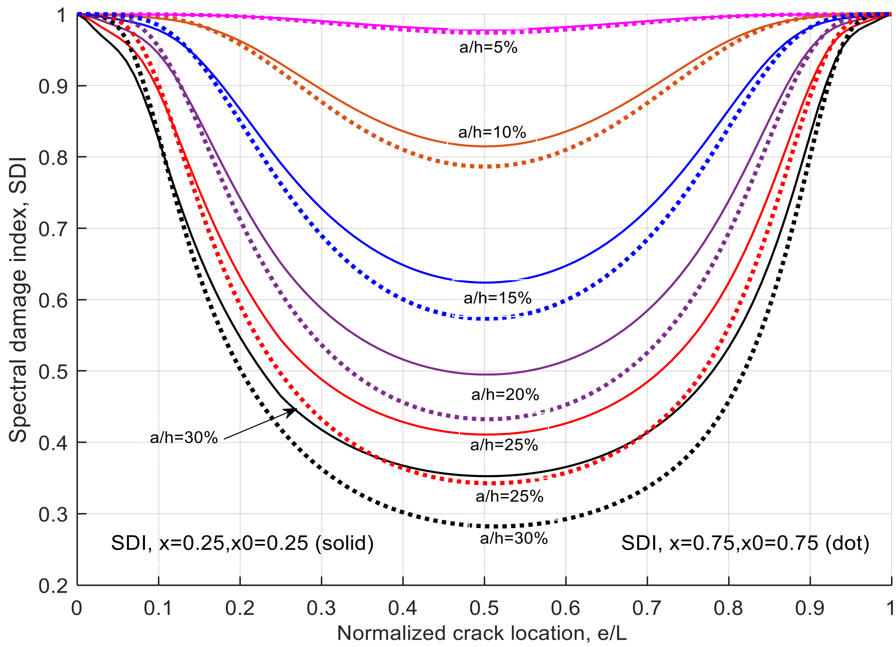


Fig. 3. Spectral damage index of FRF diagonal elements in dependence on crack location and depth in case of driving positions $x_m = 0.25; 0.75$

4.2. Crack detection by contours plots of SAC

In this subsection we consider three cases of crack location: (a) $e/L = 0.25$, (b) $e/L = 0.25$, (c) $e/L = 0.75$ with relative crack depth: $a/h = 5 - 10 - 20 - 30\%$. So, we have 12 scenarios given in Table 1. Furthermore, propose that three FRFs have been measured for the 12 crack scenarios as

$$H_1^{(k)}(\omega) = |FRF(0.25, 0.25, \omega)|, \quad H_1^{(k)}(\omega) = |FRF(0.5, 0.5, \omega)|,$$
$$H_1^{(k)}(\omega) = |FRF(0.75, 0.75, \omega)|, \quad k = 1, 2, \ldots, 12,$$

and corresponding values of SAC are computed as follow

$$S_{12}^{(k)} = SAC[H_1(\omega), H_2(\omega)], \quad S_{13}^{(k)} = SAC[H_1(\omega), H_3(\omega)],$$
$$S_{23}^{(k)} = SAC[H_2(\omega), H_3(\omega)].$$

Table 1. Scenarios of cracked beam denoted by SCJK

a/h	$e/L = 0.25$	$e/L = 0.5$	$e/L = 0.75$
5%	SC11	SC12	SC13
10%	SC21	SC22	SC23
20%	SC31	SC32	SC33
30%	SC41	SC42	SC43

Now, for every crack scenario three contours $S_{12}^{(k)} = S_{12}(e,a)$; $S_{13}^{(k)} = S_{13}(e,a)$; $S_{23}^{(k)} = S_{23}(e,a)$ are plotted and depicted in 12 boxes of Figs. 4, 5, 6 corresponding to three cases of crack locations (a), (b), (c) given above. Thus, results of crack identification are displayed in the boxes where coordinates of exact interaction of three plotted contours give detected actual crack location and depth.

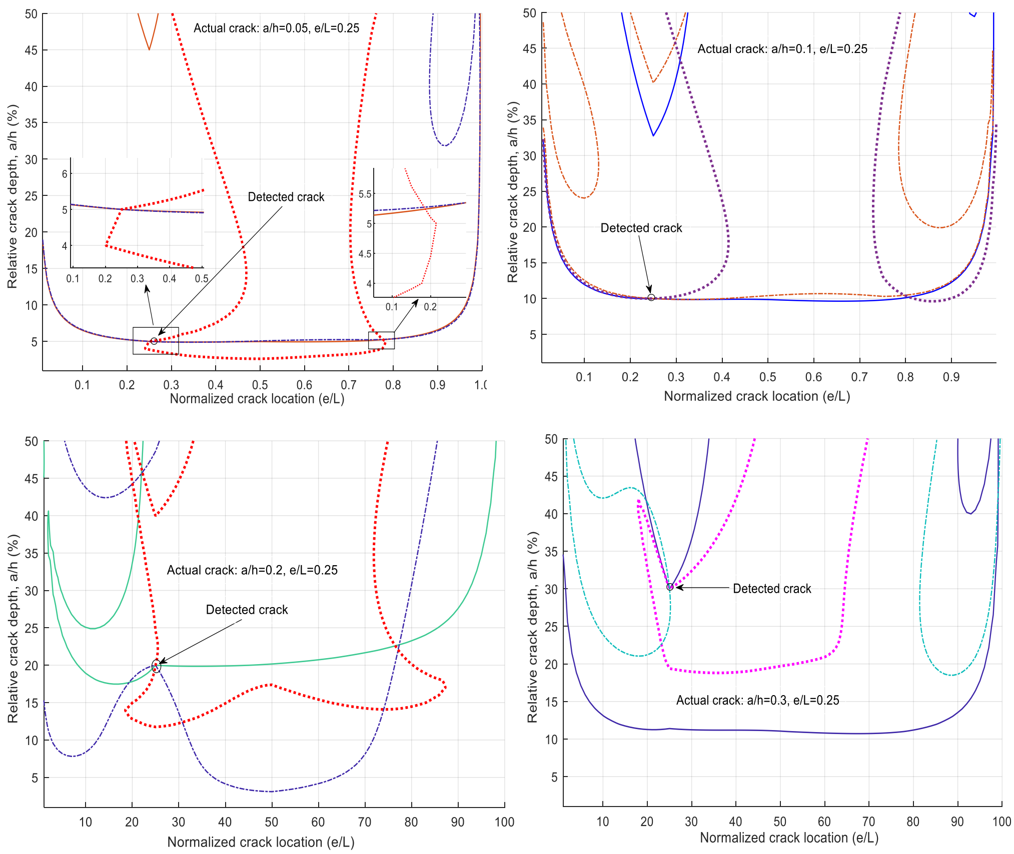


Fig. 4. Single crack detected by the contour method using spectral assurance criterion. Actual crack at position $e/L = 0.25$, in case of different crack depth $5 - 10 - 20 - 30\%$ (SC11 - SC41)

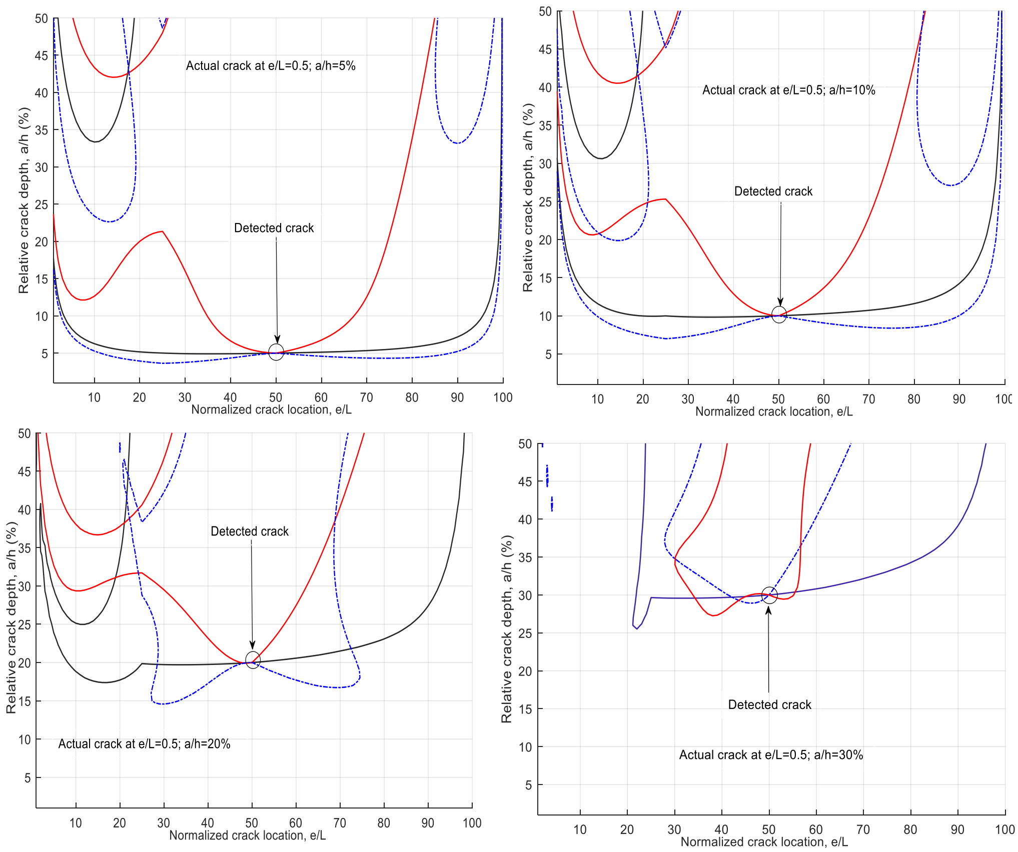
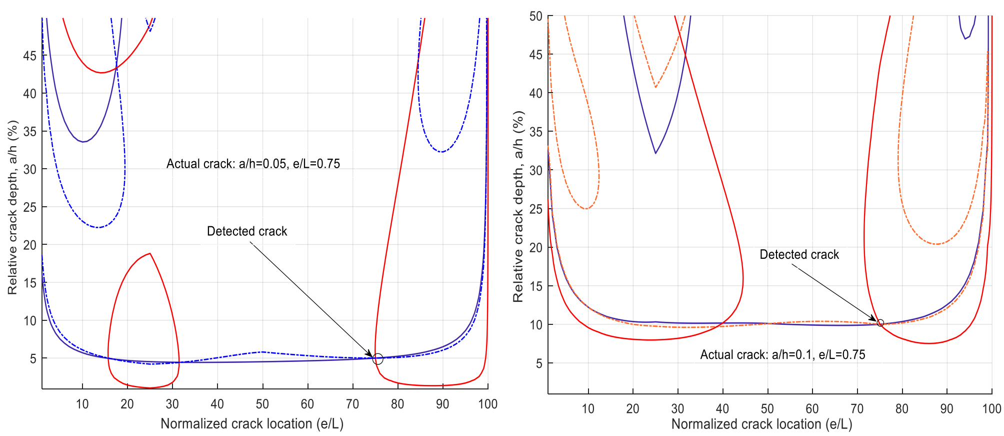


Fig. 5. Single crack detected by the contour method using spectral assurance criterion Actual crack at position $e/L = 0.5$, in case of different crack depth $5 - 10 - 20 - 30\%$ (SC12 - SC42)



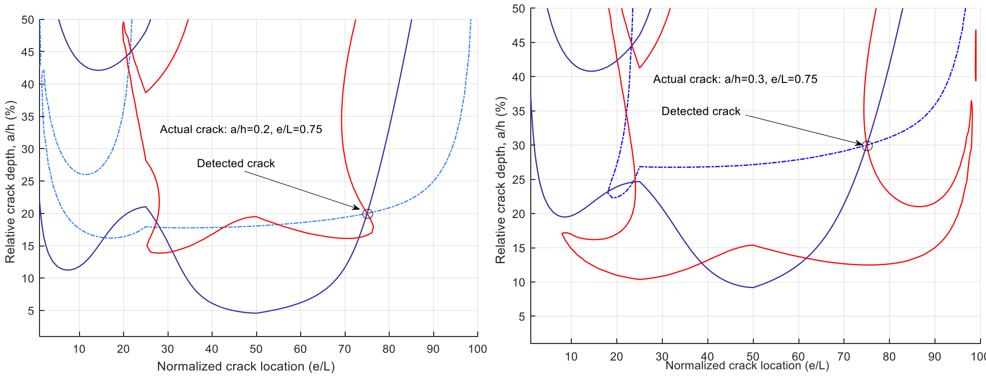


Fig. 6. Single crack detected by the contour method using spectral assurance criterion. Actual crack at position $e/L = 0.75$, in case of different crack depth $5 - 10 - 20 - 30\%$ (SC13 - SC43)

Observing all the boxes in the Figures allows us to make the following remarks: (1) All actual cracks can be exactly identified, even the small ones of depth 5%, for accurately measured FRFs, however, cracks of depth more than 10% are more clearly and easily detected; (2) Symmetrical cracks such as $e/L = 0.25$ and $e/L = 0.75$ are both uniquely detected by measurements of FRFs, but they are more difficult to be detected than the crack at the beam middle in case of small depth 5% and 10%.

5. CONCLUSION

Thus, a novel criterion has been developed for crack identification in beams from measured frequency response functions. First, a coherent coefficient between frequency response functions of intact and cracked beams called spectral damage index is determined and used for sensitivity analysis of FRFs to crack. Numerical analysis shows that frequency response functions of beam structures, in the viewpoint of the spectral damage index, are more significantly sensitive to cracks compared to the dynamical characteristics such as natural frequencies and mode shapes. Then, the above coefficient calculated for different FRFs of a cracked beam and called spectral assurance criterion is conducted as a novel indicator for crack identification of the beam structure. Using the spectral assurance criterion in combination with the contour method allows obtaining the exact and unique solution of the crack identification problem even for beams with symmetric boundary conditions.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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