A NEW ANALYTICAL APPROACH OF NONLINEAR THERMAL BUCKLING OF FG-GPLRC CIRCULAR PLATES AND SHALLOW SPHERICAL CAPS USING THE FSDT AND GALERKIN METHOD

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Abstract. A new analytical approach for nonlinear thermal buckling of Functionally Graded Graphene Platelet Reinforced Composite (FG-GPLRC) circular plates and shallow spherical caps using the first-order shear deformation theory (FSDT) is presented in this paper. The circular plates and shallow spherical caps are assumed to be subjected to uniformly distributed thermal loads. By applying the Galerkin method, the relations between thermal load-deflection are achieved to determine the postbuckling behavior and critical buckling loads of the considered structures. Special effects on the nonlinear thermal behavior of circular plates and shallow spherical caps with five different material distribution laws, different Graphene platelet (GPL) mass fractions, and geometrical dimensions are explored and discussed in numerical examples.

Keywords: nonlinear stability, first-order shear deformation theory, FG-GPLRC, thermal load, Galerkin method.

1. INTRODUCTION

The important portions in many engineering structures are constituted by shallow spherical caps, for example, the aircraft, missile, and aerospace components. As a result, the problems relating to mechanic behaviors in the design of this structure have attracted the attention of many researchers.

Functionally graded material (FGM) was proposed to be made from ceramic and metal. Creating from these advantageous properties of two constituent materials, FGM has been used widely in many engineering structures in high-temperature environments such as civil engineering, mechanical engineering, aerospace, nuclear plants, ... Najafizadeh and Hedayati [1] presented the analytical solutions for the thermo-elastic...
stability of FGM circular plates using the refined theory and adjacent equilibrium criterion. By using the isogeometric finite element formulation, the thermal buckling analysis of FGM circular plates was investigated by Loc et al. [2]. Applying the assumption of the shallowness of the caps, the complex spherical coordinate system was approximated to the polar coordinate system, and the nonlinear buckling and vibration of axisymmetric and un-axisymmetric functionally graded thin shallow spherical caps under uniform external pressure including temperature effects were analyzed by Bich et al. [3, 4]. The thermo-mechanical behavior of the FGM spherical caps was analyzed based on the classical shell theory (CST) and first shear deformation theory (FSDT), using different methods for nonlinear static buckling problem [5], and nonlinear vibration [6, 7].

Functionally Graded Graphene Platelet Reinforced Composite (FG-GPLRC) is an advanced composite material, which is created by reinforcing the graphene platelet (GPL) into the isotropic matrix, becoming popular in engineering design. Many authors focused on the static and dynamic behavior of many types of FG-GPLRC structures. However, a few reports of FG-GPLRC shallow spherical caps are obtained in the open literature. Huo et al. [8] studied the bending problem of circular/annular sector FG-GPLRC plates by applying a 3D-poroflexibility theory. The three-dimensional elasticity solutions for the free vibration and bending of the FG-GPLRC spherical caps were analyzed by Liu et al. [9].

This paper presents a new analytical approach to investigate the nonlinear thermal buckling of FG-GPLRC circular plates and shallow spherical caps with the clamped boundary condition at the edge by using the FSDT and Galerkin method. The effects of geometrical and material properties on the buckling behavior of circular plates and spherical caps with five different material distribution laws of GPL are investigated and discussed in numerical examples.

2. GEOMETRICAL AND MATERIAL PROPERTIES

Consider the FG-GPLRC shallow spherical caps/circular plates with radius of curvature \( R \), base radius \( a \), thickness \( h \), and the coordinate system \((r, \theta, z)\) as shown in Fig. 1. Five distribution laws of GPL are considered to be UD-GPLRC, X-GPLRC, O-GPLRC, A-GPLRC, and V-GPLRC. The shallow spherical caps/circular plates are subjected to uniformly distributed thermal loads with the clamped boundary condition at the edge.

The Young’s modulus of the spherical caps/circular plates is determined based on the extended Halpin–Tsai model as follows [10]

\[
E(z) = \frac{3}{8} \frac{1 + \zeta_1 \delta_1 V_{GPL}(z)}{1 - \delta_1 V_{GPL}(z)} E_m + \frac{5}{8} \frac{1 + \zeta_2 \delta_2 V_{GPL}(z)}{1 - \delta_2 V_{GPL}(z)} E_m, \tag{1}
\]

where

\[
\delta_1 = \frac{(E_{GPL}/E_m) - 1}{(E_{GPL}/E_m) + \zeta_1}, \quad \delta_2 = \frac{(E_{GPL}/E_m) - 1}{(E_{GPL}/E_m) + \zeta_2}, \quad \zeta_1 = 2 \left( \frac{a_{GPL}}{t_{GPL}} \right), \quad \zeta_2 = 2 \left( \frac{b_{GPL}}{t_{GPL}} \right), \tag{2}
\]

with \( E_m \) and \( E_{GPL} \) are respectively the elastic moduli of the matrix and GPL, \( a_{GPL}, b_{GPL} \) and \( t_{GPL} \) are the length, width and thickness of the GPL, respectively, and \( V_{GPL} \) is the
volume fraction of the GPL ($V_m + V_{GPL} = 1$) defined as

$$V_{GPL}(z) = \frac{W_{GPL}}{W_{GPL} + \left(\frac{\rho_{GPL}}{\rho_m}\right)\left(1 - W_{GPL}\right)},$$

where $\rho_m$ and $\rho_{GPL}$ are the densities of the matrix and the GPL, respectively, $W_{GPL}$ is the mass fraction of GPL which depends on five distribution laws of GPL of spherical caps/circular plates with the following functions

$$W_{GPL}(z) = \begin{cases} 
W_{GPL}^* & \text{for } -\frac{h}{2} \leq z \leq \frac{h}{2} \quad \text{UD - GPLRC,} \\
4\frac{|z|}{h}W_{GPL}^* & \text{for } -\frac{h}{2} \leq z \leq \frac{h}{2} \quad \text{X - GPLRC,} \\
2\left(1 - \frac{2|z|}{h}\right)W_{GPL}^* & \text{for } -\frac{h}{2} \leq z \leq \frac{h}{2} \quad \text{O - GPLRC,} \\
\left(1 - \frac{2z}{h}\right)W_{GPL}^* & \text{for } -\frac{h}{2} \leq z \leq \frac{h}{2} \quad \text{V - GPLRC,} \\
\left(1 + \frac{2z}{h}\right)W_{GPL}^* & \text{for } -\frac{h}{2} \leq z \leq \frac{h}{2} \quad \text{A - GPLRC,} 
\end{cases}$$

where $W_{GPL}^*$ is denote of total mass fraction of GPL.

According to the rule of mixture, the Poisson’s ratio and thermal expansion coefficient of FG-GPLRC shallow spherical caps/circular plates are determined as

$$\nu(z) = \nu_m (1 - V_{GPL}) + \nu_{GPL} V_{GPL},$$
$$\alpha(z) = \alpha_m (1 - V_{GPL}) + \alpha_{GPL} V_{GPL}.$$
3. ESTABLISHMENT PROCESS OF GOVERNING EQUATIONS

Due to the assumption that the shallow spherical caps/circular plates are axisymmetrically deformed, based on the FSDT, displacement components at any point at a distance \( z \) from the mid-surface are determined as

\[
\dot{u} (r, z) = u (r) + z \psi (r), \quad \dot{v} (r, z) = 0, \quad \ddot{w} (r, z) = \dot{w} (r) + w^* (r),
\]

where \( w^* (r) \) is the initial imperfect deflection of spherical caps.

Hooke’s law is applied to the FG-GPLRC shallow spherical caps/circular plates, taking into account the thermal strains, presented as

\[
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_{rz}
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{12} & Q_{22}
\end{bmatrix} \begin{bmatrix}
\varepsilon_r \\
\varepsilon_\theta \\
\varepsilon_{rz}
\end{bmatrix} - \begin{bmatrix}
\alpha \Delta T \\
\alpha \Delta T
\end{bmatrix}, \quad \sigma_{rz} = Q_{44} \varepsilon_{rz},
\]

where \( \Delta T \) is the uniformly distributed thermal load, and the reduced stiffnesses can be determined as

\[
Q_{11} = Q_{22} = \frac{E (z)}{1 - [v (z)]^2}, \quad Q_{12} = \frac{E (z) v (z)}{1 - [v (z)]^2}, \quad Q_{44} = \frac{E (z)}{2 (1 + v (z))}.
\]

The strain components of the shallow spherical caps are defined as

\[
\begin{bmatrix}
\varepsilon_r \\
\varepsilon_\theta \\
\varepsilon_{rz}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_r^0 + z \chi_r \\
\varepsilon_\theta^0 + z \chi_\theta \\
\psi + w_r
\end{bmatrix},
\]

where \( \varepsilon_{r}^0, \varepsilon_{\theta}^0 \) are the strains at the mid-surface, \( \chi_r, \chi_\theta \) are curvature constituents, expressed as

\[
\begin{bmatrix}
\varepsilon_{r}^0 \\
\varepsilon_{\theta}^0
\end{bmatrix} = \begin{bmatrix}
u_r - \frac{w}{R} + \frac{1}{2} w_r^2 + w_r w_r^* \\
r u - \frac{w}{R}
\end{bmatrix}, \quad \begin{bmatrix}
\chi_r \\
\chi_\theta
\end{bmatrix} = \begin{bmatrix}
\psi_r \\
\frac{\psi}{r}
\end{bmatrix}.
\]

The forces and moments are determined by

\[
\begin{align*}
(N_r, M_r) &= \int_{-h/2}^{h/2} (1, z) \sigma_r dz, \\
(N_\theta, M_\theta) &= \int_{-h/2}^{h/2} (1, z) \sigma_\theta dz, \\
Q_r &= \int_{-h/2}^{h/2} \sigma_{rz} dz.
\end{align*}
\]

Substituting Eq. (9) into Eq. (7), and then substituting the resultant equations into Eq. (11), the force and moment expressions are obtained

\[
\begin{bmatrix}
N_r \\
N_\theta \\
M_r \\
M_\theta
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & B_{11} & B_{12} \\
A_{12} & A_{22} & B_{12} & B_{22} \\
B_{11} & B_{12} & D_{11} & D_{12} \\
B_{12} & B_{22} & D_{12} & D_{22}
\end{bmatrix} \begin{bmatrix}
\varepsilon_r^0 \\
\varepsilon_\theta^0 \\
\chi_r \\
\chi_\theta
\end{bmatrix} - \begin{bmatrix}
\Phi_1 \\
\Phi_1 \\
\Phi_2 \\
\Phi_2
\end{bmatrix}, \quad Q_r = K_s H_{44} (\psi + w_r),
\]

where

\[
\begin{align*}
H_{44} &= \int_{-h/2}^{h/2} Q_{44} dz, \\
\Phi_1 &= \Delta T \Phi_1^*, \\
\Phi_1^* &= \int_{-h/2}^{h/2} (Q_{11} + Q_{12}) dz.
\end{align*}
\]
where $K_s = \frac{5}{6}$ is the shear correction factor.

The nonlinear equilibrium equations of shallow spherical caps/circular plates are

$$
(rN_r)_r - N_\theta = 0, \quad (rM_r)_r - M_\theta - rQ_r = 0,
$$

$$
(rQ_r)_r + \frac{r}{R} (N_r + N_\theta) + \left[ rN_r (w_r + w^*_r) \right]_r = 0.
$$

(14)

Substituting Eq. (10) into the forces and moments (12), then substituting the result into Eq. (14), the equilibrium equation system is rewritten by

$$
L_1 \equiv - \frac{B_{22} \psi}{r} + B_{11} \psi_r - \frac{rA_{11} w_r}{R} + rA_{11} w_r w_{rr} + rA_{11} w_r w^*_r + rA_{11} w_r w^*_{rr}
$$

$$
- \frac{rA_{12} w_r}{R} + A_{11} u_r + \frac{A_{11} w^2_r}{2} - \frac{A_{12} w^2_r}{2} - \frac{A_{11} w}{r} - \frac{A_{22} u}{r} + \frac{A_{22} w}{r}
$$

$$
+ rB_{11} \psi_{rr} - A_{12} w_r w^*_r + A_{11} w_r w^*_r + rA_{11} u_{rr} = 0,
$$

(15)

$$
L_2 \equiv B_{11} u_r + \frac{B_{11} w^2_r}{2} - \frac{B_{12} w^2_r}{2} - \frac{rB_{11} w_r}{R} + rB_{11} w_r w_{rr} + rB_{11} w_r w^*_r
$$

$$
+ rB_{11} w_r w^*_{rr} - \frac{rB_{12} w_r}{R} - K_s H_{44} r \psi - K_s H_{44} r w_r + rB_{11} u_{rr} + B_{11} w_r w^*_r
$$

$$
- \frac{B_{11} w}{r} - \frac{B_{22} u}{r} + \frac{B_{22} w}{r} + rD_{11} \psi_{rr} - B_{12} w_r w^*_r - \frac{D_{22} \psi}{r} + D_{11} \psi_r = 0,
$$

(16)

$$
L_3 \equiv \frac{A_{11} w^3_r}{2} - w_r \Phi^*_r \Delta T - w^*_r \Phi^*_r \Delta T + w^*_r A_{11} u_r + \frac{3A_{11} w^2_r w^*_r}{2} + w^*_r B_{11} \psi_r
$$

$$
- r w^*_r \Phi^*_r \Delta T + w_r A_{11} u_r - \frac{2 \Phi^*_r \Delta T}{R} + K_s H_{44} w_r + A_{11} w_r (w^*_r)^2 - r w_{rr} \Phi^*_r \Delta T
$$

$$
+ w_r B_{11} \psi_r + K_s H_{44} \psi + 3 r w_r A_{11} w_r w^*_r - \frac{r w_{rr} A_{11} w}{R} - \frac{r w_{rr} A_{12} w}{R}
$$

$$
+ 2 r w^*_r A_{11} w_r w^*_r - \frac{r w^*_r A_{11} w}{R} - \frac{r w^*_r A_{12} w}{R} + r B_{12} \psi_r + r B_{12} \psi_r + \frac{r A_{12} u_r}{R} - \frac{r A_{12} \omega_r}{2R}
$$

$$
+ \frac{A_{22} u}{R} - \frac{r A_{22} w}{R^2} + \frac{r B_{22} \psi_r}{R} + \frac{r B_{22} \psi_r}{R} + r w_r A_{11} u_{rr} + \frac{3 r w^2_r A_{11} w_{rr}}{2} + \frac{3 r w^2_r A_{11} w^*_{rr}}{2}
$$

$$
+ r w_r A_{12} u_r + r w_r B_{11} \psi_{rr} + r w_r B_{12} \psi_r - \frac{w_r A_{11} w}{R} - \frac{w_r A_{12} w}{R} + r w_{rr} A_{11} u_r + r w_{rr} A_{12} u_r
$$

$$
+ r w_{rr} B_{11} \psi_r + w_{rr} B_{12} \psi_r + r (w^*_r)^2 A_{11} w_{rr} + w^*_r A_{12} u_r + r w^*_r B_{11} \psi_r
$$

$$
+ w^*_r B_{12} \psi_r - \frac{w^*_r A_{11} w}{R} - \frac{w^*_r A_{12} w}{R} + r w^*_r A_{11} u_r + w^*_r A_{12} u_r + r w^*_r B_{11} \psi_r + w^*_r B_{12} \psi
$$

$$
+ K_s H_{44} r \psi_r + K_s H_{44} r w_{rr} + \frac{r A_{11} u_r}{R} - \frac{r A_{11} w^2_r}{2R} - \frac{A_{12} u}{R} - \frac{2 r A_{12} w}{R^2} = 0.
$$

(17)
4. BOUNDARY CONDITION AND SOLVING PROBLEM

In this paper, the FG-GPLRC shallow spherical caps/circular plates are assumed to be clamped and immovable at the edge, subjected to uniformly distributed thermal loads. The following approximate solutions for the displacement components and rotation are assumed to be in the forms

\[ u = U \sin \frac{\pi r}{a}, \quad \psi = \Psi \sin \frac{\pi r}{a}, \quad w = W \cos^2 \frac{\pi r}{2a}, \quad w^* = W^* \cos^2 \frac{\pi r}{2a}, \quad (18) \]

where \( U, W, \) and \( \Psi \) are maximal displacement constituents and rotation, \( W^* \) is maximal imperfection.

Substituting the solution forms (18) into equilibrium equations (15)–(17) and applying the Galerkin method, leads to

\[ X_{11} W^2 + X_{12} W + X_{13} U + X_{14} \Psi = 0, \quad (19) \]
\[ X_{21} W^2 + X_{22} W + X_{23} U + X_{24} \Psi = 0, \quad (20) \]
\[ X_{31} W^3 + X_{32} W^2 + \left( X_{33} U + X_{34} \Psi + X_{35} + \frac{\pi^2 \Phi^*_1 \Delta T}{16} \right) W + X_{36} U + X_{37} \Psi + X_{38} \Phi^*_1 \Delta T = 0, \quad (21) \]

The expressions of \( U \) and \( \Psi \) can be solved from Eqs. (19) and (20), and substituting them into Eq. (21), leads to

\[
\begin{pmatrix}
X_{31} - \frac{X_{33} (X_{11} X_{24} - X_{14} X_{21})}{X_{13} X_{24} - X_{14} X_{23}} \\
+ \frac{X_{34} (X_{11} X_{23} - X_{13} X_{21})}{X_{13} X_{24} - X_{14} X_{23}}
\end{pmatrix} W^3 + \left( \begin{pmatrix}
X_{32} & X_{33} (X_{12} X_{24} - X_{14} X_{22}) \\
& X_{13} X_{24} - X_{14} X_{23}
\end{pmatrix} W + \begin{pmatrix}
X_{34} (X_{12} X_{23} - X_{13} X_{22}) \\
& X_{13} X_{24} - X_{14} X_{23}
\end{pmatrix}
\right) W^2 + \begin{pmatrix}
X_{35} - \frac{X_{36} (X_{12} X_{24} - X_{14} X_{22})}{X_{13} X_{24} - X_{14} X_{23}} \\
+ \frac{X_{37} (X_{12} X_{23} - X_{13} X_{22})}{X_{13} X_{24} - X_{14} X_{23}}
\end{pmatrix} W = - \left( \frac{\pi^2 \Phi^*_1}{16} W + X_{38} \Phi^*_1 \right) \Delta T. \quad (22)
\]

In the case of perfect circular plates \((W^* = 0, R \rightarrow \infty)\), the thermal critical buckling loads \( \Delta T_{cr} \) (K) can be determined by applying \( W \rightarrow 0 \) in Eq. (22), as

\[
\Delta T_{cr} = - \frac{16}{\pi^2 \Phi^*_1} \left[ X_{35} - \frac{X_{36} (X_{12} X_{24} - X_{14} X_{22})}{X_{13} X_{24} - X_{14} X_{23}} + \frac{X_{37} (X_{12} X_{23} - X_{13} X_{22})}{X_{13} X_{24} - X_{14} X_{23}} \right]. \quad (23)
\]
5. NUMERICAL RESULTS AND DISCUSSIONS

5.1. Validation

Consider the clamped FGM circular plates subjected to uniformly distributed thermal loads. Table 1 compared the thermal critical buckling loads of present results with those of the adjacent equilibrium criterion solutions [1] and the isogeometric finite element solutions [2] using FSDT and HSDT, respectively. Clearly, good agreements can be observed in these comparisons.

Table 1. Comparison of thermal critical buckling loads (K) of FGM circular plates with previous results

<table>
<thead>
<tr>
<th>h/a</th>
<th>0.05</th>
<th>0.04</th>
<th>0.03</th>
<th>0.02</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSDT [1]</td>
<td>146.8150</td>
<td>94.0810</td>
<td>53.0290</td>
<td>23.6030</td>
<td>5.9060</td>
</tr>
<tr>
<td>HSDT [2]</td>
<td>144.9953</td>
<td>93.4005</td>
<td>52.8191</td>
<td>23.5719</td>
<td>5.9093</td>
</tr>
<tr>
<td>Present</td>
<td>147.0435</td>
<td>94.4065</td>
<td>53.2351</td>
<td>23.7019</td>
<td>5.9318</td>
</tr>
</tbody>
</table>

5.2. Numerical examples

In this sub-section, material properties and efficiency parameters of GPLs and copper matrix are chosen as the report of Wang et al. [10]. The effects of the mass fraction of GPL, geometric ratio a/h and distribution laws on the thermal critical buckling loads of circular plates are presented in Table 2. As can be seen, when the mass fraction of the GPL increases, the thermal critical buckling load of plates increases for the four distribution laws UD-GPLRC, X-GPLRC, V-GPLRC, and A-GPLRC, in which the largest critical buckling loads are obtained for X-GPLRC plates. Especially, an abnormal tendency can be observed for O-GPLRC plates. In addition, the thermal critical buckling load increases strongly as the geometric ratio a/h decreases in all investigated cases.

Effects of GPL distribution law on the thermal load-deflection postbuckling curves of FG-GPLRC circular plates and shallow spherical caps are presented in Figs. 2 and 3. The results show that with the same mass fraction of GPL and geometrical parameters, the thermal load-deflection postbuckling curves of X-GPLRC circular plates and shallow spherical caps are the highest, and those of the O-GPLRC are the lowest. The difference between the five thermal load-deflection postbuckling curves in the circular plates is more clearly observable than that of the shallow spherical caps. The bifurcation points can be observed for FG-GPLRC circular plates, oppositely, those can not be obtained for FG-GPLRC spherical caps.
Table 2. The effects of mass fraction of GPL, geometric ratio $a/h$ and GPL distribution laws on thermal critical buckling loads $\Delta T_{cr}$ (K) of circular plates

<table>
<thead>
<tr>
<th>$a/h$</th>
<th>$W_{GPL}^a$ (%)</th>
<th>$h = 0.01$ m, $R = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UD-GPLRC</td>
<td>X-GPLRC</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>33.61831326</td>
</tr>
<tr>
<td></td>
<td>35.91610556</td>
<td>41.76246442</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>59.63674508</td>
</tr>
<tr>
<td></td>
<td>63.71532393</td>
<td>74.06237924</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>133.3591719</td>
</tr>
<tr>
<td></td>
<td>165.4813810</td>
<td>120.4360026</td>
</tr>
</tbody>
</table>

Fig. 2. Effect of GPL distribution law on the thermal load-deflection postbuckling curves of FG-GPLRC circular plates

Figs. 4 and 5 show the large effects of the mass fraction of GPL on the thermal load-deflection postbuckling curves of UD-GPLRC circular plates and X-GPLRC shallow spherical caps, respectively. Obviously, the deflections of circular plates and shallow spherical caps decrease rapidly as the mass fraction of GPL increases for both cases.
A new analytical approach of nonlinear thermal buckling of FG-GPLRC circular plates ... postbuckling curves of FG-GPLRC shallow spherical caps.

**Fig. 3.** Effect of GPL distribution law on the thermal load-deflection postbuckling curves of FG-GPLRC shallow spherical caps

**Fig. 4.** Effect of mass fraction of GPL on the thermal load-deflection postbuckling curves of UD-GPLRC circular plates

**Fig. 5.** Effect of mass fraction of GPL on the thermal load-deflection postbuckling curves of X-GPLRC shallow spherical caps
Figs. 6–9 present the thermal load-deflection postbuckling curves of circular plates and shallow spherical caps with different geometrical ratios $a/R$ and $a/h$, applied to three distribution laws of GPL: X-GPLRC, O-GPLRC, and V-GPLRC. Clearly, the thermal load-deflection postbuckling curves of circular plates and shallow spherical caps are gradually lower as the geometrical ratio $a/R$ decreases for all three GPL distribution laws. Conversely, those are gradually upper when the geometrical ratio $a/h$ increases.

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**Fig. 6.** Effect of $a/R$ ratio on the thermal load-deflection postbuckling curves of X-GPLRC shallow spherical caps

**Fig. 7.** Effect of $a/R$ ratio on the thermal load-deflection postbuckling curves of O-GPLRC shallow spherical caps
This paper presents an analytical approach of nonlinear thermal buckling of axisymmetrical FG-GPLRC circular plates and shallow spherical caps. The governing equations are derived by using the FSDT with von Karman geometrical nonlinearity, and the equilibrium equations are solved by the Galerkin method. Numerical results show that:

- When the mass fraction of the GPL increases, the thermal critical buckling load of plates increases for the four distribution laws UD-GPLRC, X-GPLRC, V-GPLRC, and
A-GPLRC, in which the largest critical buckling loads are obtained for X-GPLRC plates. Especially, an abnormal tendency can be observed for O-GPLRC plates;
- The load-carrying capacities of the shallow spherical caps and circular plates both increase markedly with the increase in the mass fraction of the GPL;
- The large effects of geometrical ratios on the critical buckling loads and postbuckling curves of circular plates and spherical caps can be also obtained from investigated examples.

DECLARATION OF COMPETING INTEREST
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