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DYNAMIC RESPONSE OF ARBITRARY DOUBLE-CURVED SHELLS BY MERIDIAN CURVE DIGITALIZATION

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Abstract. For the resulting equation of double-curved shells, which is formed by revolution of an arbitrary in-plane meridian curve and cannot be represented analytically, there exists no analytical approach to problem setting and solution. This paper presents the digitalization of the meridian curve in the polar coordinate system, which forms double number series. The double number series then can be approximated by an interpolation function so that calculations can be performed in a similar methodology for an explicit function. Digitalization enables the input parameters in the form of interpolation functions. Procedures for the proposed selection of solution forms, formation of the kinetic equation, and computation of coefficients for the kinetic equation from on the interpolation and explicit functions are presented in the paper. The final solution is obtained by using the program Mathematica 7.0 to solve the system of nonlinear differential equations. Assessment of the dynamic response of the double-curved shell, especially responses with chaotic motion, is also presented in the paper.

Keywords: closed loop multibody system, electromechanical system, singularity-free, constrained stabilization, post-adjusting technique.

1. INTRODUCTION

Combined or joined shells often assume the double-curved shape. The shells are widely used in engineering, defense and space industries for application such as tanks, aircrafts, submarines, spacecraft, rockets,... Shapes of the combined shells are widely varied, and can be from conical, truncated conical, cylindrical, semi sphere depending on the usage requirement.

In the Ho Chi Minh City, Vietnam [1], a system of eight water towers with the shape of trumpet shell was built. The shells were formed by combining two different inverted truncated conical shells. The shells were subjected to hydrostatic and hydrodynamic pressures. Empirical formulae were used in the calculation of these shells.

The natural vibration of joined triple conical containing fluid by using continuous element method analyzed by Hien et al. [2]. Three separate equations of motion were developed and solved for the shells which had different semi-vertex angles.

Shakouri and Kouchakzadeh [3] presented natural frequencies and mode shapes of two joined isotropic conical shells by analytical and empirical methods and Xie et al. [4] presented an analytic solution for free and forced vibration problem for stepwise linear conical shells with general boundary conditions

The free vibration characteristics of laminated composite joined from conical and cylindrical shells published by Patel et al. [5]. Kinetic equations are formulated for the conical and cylindrical shells separately.

The variational method for the solution of the free vibration problem for joined cylindrical-conical shells used by Qu et al. [6]. Kinetic equations are formulated for the conical and cylindrical shells separately. Meanwhile, Kerboua and Lakis [7] used numerical method to analyze the aerodynamic behavior of a combined conical-cylindrical shell. Kinetic equations are formulated for the conical and cylindrical shells separately. Also these authors, Kerboua and Lakis [8] presented the dynamic behavior of a rocket filled with liquid. The rocket shell was formed by a combination of conical, cylindrical and semi sphere shells with different radii. The kinetic equations are formulated in the form of three separate equations for the conical, cylindrical and semi sphere shells.

Chronopoulosa et al. [9] published the solution on broadband response of a layered conical-cylindrical-conical shell. The equations of motion are three equations for conical-cylindrical-conical shells with different semi-vertex angles. An analytical substructure method for the vibration analysis of conical-cylindrical-spherical combined shells in vacuum condition used by Chen et al. [10]. The kinetic equations are formulated in the form of three separate equations for the conical, cylindrical and semi sphere shells.

Xie et al. [11] analyzed the free and forced vibration of ring-stiffened conical-cylindrical-spherical shells through a semi analytic method. The kinetic equations are formulated in the form of three separate equations for the conical, cylindrical and sphere shells. At the same time, Moonesun et al. [12] presented an experimental analysis on the bare hull resistance coefficient of submarine at snorkel depth. The equations of motion are four equations for the conical-truncated conical-cylindrical-spherical shell.

The approach adopted in this paper is to conceptualize the above mentioned combined or joined shells to double-curved shells formed by an arbitrary in-plane meridian curve, and find a solution from digitalization of input parameters. For this purpose, it is necessary to establish the input characteristic quantities of an equation of motion, based on the digitization of the meridian curve.

2. THE FGM ARBITRARY DOUBLE-CURVED SHELLS

2.1. Functionally graded materials (FGM)

Functionally graded material, which is often called FGM, has elastic modulus and variable density according to the law:

$$\begin{gathered} E(z) = E_m V_m + E_c V_c = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k \\ \rho(z) = \rho_m V_m + \rho_c V_c = \rho_m + (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^k \\ \nu(z) = \nu = \text{const}, \quad k \geq 0 \end{gathered} \tag{1}$$

The FGM normally consists of ceramic and metal materials for which each respective fraction volume (k) is selected reasonably and continuously from side to side. Structures using FGM do not often experience hot spot stresses at the interface between material layers, which may cause splitting and cracking in the material microstructure. FGM therefore usually has high hardness, high ductility and high thermal resistance. In practice, FGM can be used in rocket shells and space structures. It can also be used for civil structures.

2.2. Structural model

Consider an FGM arbitrary double curved shell with thickness h, which is formed by rotating an arbitrary in-plane curve about its axis.

According to classical shell theory and Von Karman geometric nonlinearity, the relationships between the strains, bending, twisting curvatures and displacements at the middle surface are as follows

$$\begin{cases} \varepsilon_\xi^0 = \dfrac{1}{R}\dfrac{\partial U}{\partial \xi} - \dfrac{W}{R} + \dfrac{1}{2}\left(\dfrac{1}{R}\dfrac{\partial W}{\partial \xi}\right)^2, \\ \varepsilon_\theta^0 = \dfrac{1}{R_0}\dfrac{\partial V}{\partial \theta} + \dfrac{U}{RR_0}\dfrac{\partial R_0}{\partial \xi} - \dfrac{W}{R_1} + \dfrac{1}{2}\left(\dfrac{1}{R_0}\dfrac{\partial W}{\partial \theta}\right)^2, \\ \gamma_{\xi\theta}^0 = \dfrac{1}{R_0}\dfrac{\partial U}{\partial \theta} + \dfrac{1}{R}\dfrac{\partial V}{\partial \xi} - \dfrac{V}{RR_0}\dfrac{\partial R_0}{\partial \xi} + \dfrac{1}{RR_0}\dfrac{\partial W}{\partial \xi}\dfrac{\partial W}{\partial \theta}. \end{cases} \tag{2}$$

where $\varepsilon_{\xi}^{0}, \varepsilon_{\theta}^{0}$ are the normal strains, $\gamma_{\xi\theta}^{0}$ is the shear strain at the middle surface of the shell, respectively.

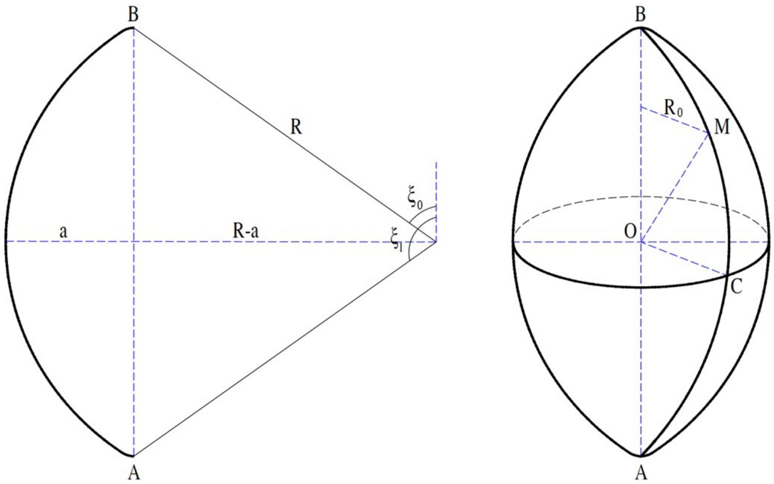

Fig. 1. The arbitrary double-curved shells model in the polar coordinate system

$$\begin{cases} \chi_{\xi} = \dfrac{1}{R^2}\dfrac{\partial^2 W}{\partial \xi^2}, \\ \chi_{\theta} = \dfrac{1}{R_0^2}\dfrac{\partial^2 W}{\partial \theta^2} + \dfrac{1}{R^2 R_0}\dfrac{\partial R_0}{\partial \xi}\dfrac{\partial W}{\partial \xi}, \\ \chi_{\xi\theta} = \dfrac{1}{RR_0}\dfrac{\partial^2 W}{\partial \xi \partial \theta} - \dfrac{1}{RR_0^2}\dfrac{\partial R_0}{\partial \xi}\dfrac{\partial W}{\partial \theta}, \end{cases} \tag{3}$$
$$R_0 = R\sin\xi.$$

where $\chi_{\xi}, \chi_{\theta}$ and $\chi_{\xi\theta}$ are bending and twisting curvatures at the middle surface of the shell; U is displacement in the meridian direction; V is displacement in the circumferential direction; W is displacement in the radial direction.

Assume that the in-plane curve is represented in the (ξ, R) polar coordinate system with origin O, the distance from the origin to an arbitrary point on an in-plane curve is called the curve radius, denoted R, the angle between R and the vertical axis passing through O is denoted ξ, the distance from this point on the in-plane curve to the rotational axis is denoted R_0, the distance from the arbitrary point on the in-plane curve to the point where it intersects with the rotational axis of the line perpendicular to the tangent of the in-plane curve at the above mentioned arbitrary point, is denoted R_1.

When $R = R(\xi)$ is the function of ξ, then R_0, R_1 are calculated according to the formula

$$R_1 = R_0\sqrt{R^2 + R'^2} / \left(R\sin(\xi) - R'\cos(\xi)\right), \tag{4}$$

in which $R' = \mathrm{d}R/\mathrm{d}\xi, 0 \leq \xi \leq \pi$.

When $R = R(\xi) = \text{const}$, then $R' = \mathrm{d}R/\mathrm{d}\xi = 0$ and $R_1 = R_0/\sin(\xi)$.

$R = R(\xi), R_0 = R_0(\xi), R_1 = R_1(\xi)$ are called the characteristic parameters.

When R is known, then R_0 and R_1 are also known, according to (4). In cases where the arbitrary double-curved shell is a combined shell, for which the in-plane meridian curve cannot be represented analytically formula, the following procedure is proposed for solving kinetic equations involving these shells.

Let us consider a arbitrary double-curved shell, formed by a in-plane curve ACB, rotates AB axis, go passing the points $(0, 5.908); C(\pi/2, 5.907); M(\xi, R); B(\pi, 10.976)$.

$R(\xi) \neq 0, R_0(\xi) = 0, R_1(\xi) = 0$ when $\xi = 0, \xi = \pi$.

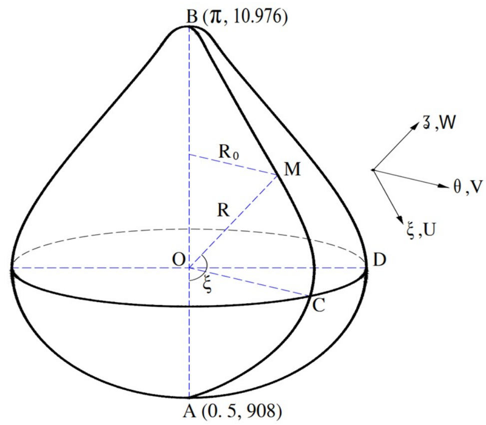

Fig. 2. The double-curved shells (as the teardrop shape)

In the (ξ, θ, z) polar coordinate system with origin O in the middle surface of the shell, ξ, θ, z are displacement in the meridian, circumferential, radial direction, $-h/2 \leq z \leq h/2$.

2.3. Determination of characteristic parameters by digitilization of the meridian curve

In the case the ACB meridian curve is arbitrary, and cannot be represented by an analytical formula, the following procedure is propose for the determination of the characteristic parameters:

- Digitize the ACB meridian curve into geometrical coordinates, forming a double number series.

- Evenly approximate the double number series into an interpolation function, so that calculations on the interpolation function can be performed as for an explicit function.

a) For the ACB meridian curve, it is possible to set up the double number series of geometrical coordinates (ξ, R).

```
list={{0,5.90897},{0.0649777,5.90958},{0.132254,5.91059},{0.195582,5.91047},
{0.280464,5.90973},{0.371283,5.90906},{0.433353,5.90886},{0.497255,5.90881},
{0.587523,5.90891},{0.675951,5.90898},{0.766849,5.90898},{0.851104,5.90894},
{0.949757,5.90891},{1.0494,5.90897},{1.1252,5.90897},{1.19354,5.90899},
```

```
{1.24661,5.90899},{1.3153,5.90888},{1.37959,5.90876},{1.45888,5.90882},
{1.54026,5.90889},{1.5708,5.90786},{1.61311,5.904},{1.67338,5.89488},
{1.72718,5.88516},{1.77877,5.87619},{1.84242,5.86781},{1.9039,5.86494},
{1.94151,5.86678},{2.00821,5.87876},{2.07057,5.90228},{2.15489,5.95585},
{2.23341,6.03095},{2.31172,6.13351},{2.38915,6.26621},{2.44201,6.37738},
{2.52024,6.57742},{2.58733,6.78778},{2.65771,7.05398},{2.70281,7.25337},
{2.74161,7.44578},{2.78567,7.6912},{2.8419,8.05273},{2.89255,8.4335},
{2.93024,8.75725},{2.98845,9.34354},{3.03023,9.86392},{3.04787,10.1238},
{3.05755,10.2781},{3.06938,10.4686},{3.08002,10.6275},{3.08789,10.7259},
{3.09755,10.8212},{3.10809,10.8972},{3.11789,10.9436},{3.12903,10.9688},
{ξ,10.9767}}.
```

Evenly approximate the "`list`" double number series, set up interpolation function by command in Mathematica 7.0:

$$\begin{aligned} &\mathtt{R}(\xi) = \mathtt{Interpolation}\,[\mathtt{list},\xi] \\ &\mathtt{R}(\xi) = \mathtt{InterpolatingFunction}[\{\{0,3.14159\}\},<>][\xi] \end{aligned} \tag{5}$$

b) For $R_0 = R(\xi)\sin(\xi)$, it is possible to set up the double number series of geometrical coordinates (ξ, R_0).

```
data={{0,0},{0.0649777,0.383721},{0.132254,0.779422},{0.195582,1.14863},
{0.280464,1.63582},{0.371283,2.14388},{0.433353,2.48122},{0.497255,2.81859},
{0.587523,3.27531},{0.675951,3.69689},{0.766849,4.10006},{0.851104,4.44358},
{0.949757,4.80556},{1.0494,5.12381},{1.1252,5.33199},{1.19354,5.49346},
{1.24661,5.60119},{1.3153,5.71708},{1.37959,5.80108},{1.45888,5.87186},
{1.54026,5.90613},{1.5708,5.90786},{1.61311,5.89871},{1.67338,5.86389},
{1.72718,5.81334},{1.77877,5.74956},{1.84242,5.65268},{1.9039,5.54256},
{1.94151,5.46824},{2.00821,5.32527},{2.07057,5.18037},{2.15489,4.96844},
{2.23341,4.75474},{2.31172,4.52559},{2.38915,4.28249},{2.44201,4.10637},
{2.52024,3.82896},{2.58733,3.57252},{2.65771,3.28165},{2.70281,3.08147},
{2.74161,2.8994},{2.78567,2.68005},{2.8419,2.3774},{2.89255,2.07862},
{2.93024,1.83712},{2.98845,1.42534},{3.03023,1.09616},{3.04787,0.947415},
{3.05755,0.862824},{3.06938,0.755321},{3.08002,0.653971},{3.08789,0.57572},
{3.09755,0.476487},{3.10809,0.365009},{3.11789,0.259418},{3.12903,0.13774},
{ξ,0}}.
```

Evenly approximate the "`data`" double number series set up interpolation function by command in Mathematica 7.0

$$\begin{aligned} &\mathtt{R_0}(\xi) = \mathtt{Interpolation}\,[\mathtt{data},\xi] \\ &\mathtt{R_0}(\xi) = \mathtt{InterpolatingFunction}[\{\{0,3.14159\}\},<>][\xi] \end{aligned} \tag{6}$$

c) For $R_1 = R_0\sqrt{R^2 + R'^2}/\left(R\sin(\xi) - R'\cos(\xi)\right)$, it is possible to set up the double number series of geometrical coordinates (ξ, R_1).

```
table={{0,1.34482×10^14,{0.0649777,6.16569},{0.132254,5.98688},
{0.195582,5.87599},{0.280464,5.87589},{0.371283,5.89534},{0.433353,5.90457},
{0.497255,5.90928},{0.587523,5.91108},{0.675951,5.90952},{0.766849,5.90867},
{0.851104,5.90856},{0.949757,5.90887},{1.0494,5.90927},{1.1252,5.90898},
{1.19354,5.90906},{1.24661,5.90885},{1.3153,5.90837},{1.37959,5.90853},
{1.45888,5.90897},{1.54026,5.90841},{1.5708,5.90811},{1.61311,5.9103},
{1.67338,5.91537},{1.72718,5.91693},{1.77877,5.91253},{1.84242,5.89525},
{1.9039,5.862},{1.94151,5.83209},{2.00821,5.75929},{2.07057,5.66947},
{2.15489,5.5213},{2.23341,5.35493},{2.31172,5.15818},{2.38915,4.93052},
{2.44201,4.75469},{2.52024,4.46402},{2.58733,4.18547},{2.65771,3.85976},
{2.70281,3.63032},{2.74161,3.41827},{2.78567,3.15992},{2.8419,2.80173},
{2.89255,2.44744},{2.93024,2.15962},{2.98845,1.65912},{3.03023,1.25092},
{3.04787,1.06786},{3.05755,0.974476},{3.06938,0.868067},{3.08002,0.790277},
{3.08789,0.75436},{3.09755,0.729609},{3.10809,0.723564},{3.11789,0.779872},
{3.12903,1.06505},{ξ,0.}}.
```

Evenly approximate the "`table`" double number series set up interpolation function by command in Mathematica 7.0

$$\begin{aligned} &\mathtt{R_1}(\xi) = \mathtt{Interpolation}\left[\mathtt{table}, \xi\right] \\ &\mathtt{R_1}(\xi) = \mathtt{InterpolatingFunction}[\{\{0, 3.14159\}\}, <>][\xi] \end{aligned} \tag{7}$$

The input characteristic quantities $R = R(\xi), R_0 = R_0(\xi), R_1 = R_1(\xi)$, when participating in the calculation are given as follows:

With $R(\xi)$, for the "`list`" double number series and the formula (5).

With $R_0(\xi)$, for the "`data`" double number series and the formula 6).

With $R_1(\xi)$, for the "`table`" double number series and the formula (7).

2.4. Observations

a) In the (ξ, R) polar coordinate system, according to [13]

$$ds^2 = dR^2 + R^2 d\xi^2, \quad \tan\mu = R/\frac{dR}{d\xi},$$

where ds is differential length, μ is angle between tangent to curve $R = R(\xi)$.

Formula (4) is established based on these formulas at the [13].

b) To computation the "`list`" double number series, follow these two steps:

- Use an exclusive command in AutoCAD to automatically display the geometrical coordinates on an in-plane curve in the in-plane coordinates.

- Use an exclusive command in Mathematica 7.0 for the double number series in the in-plane coordinates to convert to the double number series in polar coordinate system, which is named "`list`".

c) To computation the "`data`" double number series and "`table`" double number series, based on "`list`" double number series and the formulas for determining R_0, R_1.

3. THE EQUATION OF MOTION

In this study, the classical shell theory and Von Karman geometric nonlinearity were used to obtain the equation of motion and the nonlinear dynamic response of the FGM double-curved shells (as the teardrop shape).

The stress-strain relations of the FGM teardrop-shaped double-curved shells including the thermal effect are defined by the Hooke law

$$\begin{aligned} \sigma_\xi &= \frac{E(z)}{1-\nu^2}\left[\varepsilon_\xi^0 + \nu\varepsilon_\theta^0 - z\left(\chi_\xi + \nu\chi_\theta\right)\right] - \frac{E(z)\,\alpha(z)\,\Delta T(z)}{1-\nu},\\ \sigma_\theta &= \frac{E(z)}{1-\nu^2}\left[\varepsilon_\theta^0 + \nu\varepsilon_\xi^0 - z\left(\chi_\theta + \nu\chi_\xi\right)\right] - \frac{E(z)\,\alpha(z)\,\Delta T(z)}{1-\nu},\\ \sigma_{\xi\theta} &= \frac{E(z)}{1+\nu}\left(\gamma_{\xi\theta}^0 - z\chi_{\xi\theta}\right). \end{aligned} \tag{8}$$

Environment temperature is assumed to be uniformly raised from initial value T_i, at which the shell is thermal stress free, to final one T_j and temperature change $\Delta T = T_j - T_i$ is independent to thickness variable.

Suppose the shell has thickness varying on the meridional direction, i.e. $h = h(\xi)$, the force and moment resultants of the FGM teardrop-shaped double-curved shells are expressed in term of the stress components through the thickness as

$$\left(N_{ij}, M_{ij}\right) = \int_{-h/2}^{h/2} \sigma_{ij}\,[1, z]\,\mathrm{d}z, \quad (ij = \xi, \theta, \xi\theta). \tag{9}$$

Introduction of Eq. (8) into Eq. (9) gives the constitutive relations as

$$\begin{cases} N_\xi = \dfrac{E_1 h}{1-\nu^2}\left(\varepsilon_\xi^0 + \nu\varepsilon_\theta^0\right) - \dfrac{E_2 h^2}{1-\nu^2}\left(\chi_\xi + \nu\chi_\theta\right) - \dfrac{\Phi_0}{1-\nu},\\ N_\theta = \dfrac{E_1 h}{1-\nu^2}\left(\varepsilon_\theta^0 + \nu\varepsilon_\xi^0\right) - \dfrac{E_2 h^2}{1-\nu^2}\left(\chi_\theta + \nu\chi_\xi\right) - \dfrac{\Phi_0}{1-\nu},\\ N_{\xi\theta} = \dfrac{E_1 h}{2(1+\nu)}\gamma_{\xi_\theta}^0 - \dfrac{E_2 h^2}{1+\nu}\chi_{\xi_\theta}. \end{cases} \tag{10}$$

$$
\begin{cases}
M_\xi = \dfrac{E_2h^2}{1-\nu^2}\left(\varepsilon_\xi^0+\nu\varepsilon_\theta^0\right) - \dfrac{E_3h^3}{1-\nu^2}\left(\chi_\xi+\nu\chi_\theta\right) - \dfrac{\Phi_1}{1-\nu}, \\
M_\theta = \dfrac{E_2h^2}{1-\nu^2}\left(\varepsilon_\theta^0+\nu\varepsilon_\xi^0\right) - \dfrac{E_3h^3}{1-\nu^2}\left(\chi_\theta+\nu\chi_\xi\right) - \dfrac{\Phi_1}{1-\nu}, \\
M_{\xi\theta} = \dfrac{E_2h^2}{2\left(1+\nu\right)}\gamma_{\xi_\theta}^0 - \dfrac{E_3h^3}{1+\nu}\chi_{\xi_\theta}.
\end{cases}
\tag{11}
$$

where $\left(E_1h, E_2h^2, E_3h^3\right) = \int\limits_{-h/2}^{h/2} E\left(z\right)\left(1,z,z^2\right)\mathrm{d}z$, in which

$$
\begin{gathered}
E_1 = E_m + \frac{E_c-E_m}{\kappa+1}, \quad E_2 = \frac{\left(E_c-E_m\right)\kappa}{2\left(\kappa+1\right)\left(\kappa+2\right)}, \\
E_3 = \frac{E_m}{12} + \left(E_c-E_m\right)\left(\frac{1}{\kappa+3} - \frac{1}{\kappa+2} + \frac{1}{4\kappa+4}\right),
\end{gathered}
\tag{12}
$$

and

$$
\left(\Phi_0,\Phi_1\right) = \int\limits_{-h/2}^{h/2} E\left(z\right)\alpha\left(z\right)\Delta T\left(z\right)\left[1,z\right]\mathrm{d}z.
\tag{13}
$$

The nonlinear equation of motion of the teardrop-shaped double-curved shells based on classical shell theory is given by

$$
R_0\frac{\partial N_\xi}{\partial \xi} + R\frac{\partial N_{\xi\theta}}{\partial \theta} + \frac{\partial R_0}{\partial \xi}\left(N_\xi - N_\theta\right) - \rho_1 R R_0\frac{\partial^2 U}{\partial t^2} = 0,
\tag{14}
$$

$$
R\frac{\partial N_\theta}{\partial \theta} + R_0\frac{\partial N_{\xi\theta}}{\partial \xi} + 2\frac{\partial R_0}{\partial \xi}N_{\xi\theta} - \rho_1 R R_0\frac{\partial^2 V}{\partial t^2} = 0,
\tag{15}
$$

$$
\begin{cases}
\dfrac{1}{R^2}\dfrac{\partial^2 M_\xi}{\partial \xi^2} + \dfrac{2}{RR_0}\dfrac{\partial^2 M_{\xi\theta}}{\partial \xi\partial \theta} + \dfrac{1}{R_0^2}\dfrac{\partial^2 M_\theta}{\partial \theta^2} + \dfrac{2}{R^2R_0}\dfrac{\partial R_0}{\partial \xi}\dfrac{\partial M_\xi}{\partial \xi} \\
-\dfrac{1}{R^2R_0}\dfrac{\partial R_0}{\partial \xi}\dfrac{\partial M_\theta}{\partial \xi} + \dfrac{1}{RR_0^2}\dfrac{\partial R_0}{\partial \xi}\dfrac{\partial M_{\xi\theta}}{\partial \theta} + \dfrac{1}{R^2R_0}\dfrac{\partial^2 R_0}{\partial \xi^2}\left(M_\xi - M_\theta\right) \\
+\dfrac{N_\theta}{R_1} + \dfrac{N_\xi}{R} + N_\xi\chi_\xi + 2N_{\xi\theta}\chi_{\xi\theta} + N_\theta\chi_\theta + q - \rho_1\dfrac{\partial^2 W}{\partial t^2} - 2\rho_1\varepsilon\dfrac{\partial W}{\partial t} = 0.
\end{cases}
\tag{16}
$$

in which q is an external pressure load uniformly distributed on the outer surface of the shell,

$$
\rho_1 = \int\limits_{-h/2}^{h/2} \rho\left(z\right)\mathrm{d}z,
$$

$\rho\left(z\right)$ is calculated according to the formula (1), $\rho_1 = (\rho_m + \dfrac{\rho_c-\rho_m}{\kappa+1})h$, ε is the damping coefficient.

The FGM teardrop-shaped double-curved shells are assumed to be fixed along the meridian direction, for which the boundary conditions are

$$U = V = W = 0, \quad \frac{\partial W}{\partial \xi} = 0 \quad \text{when} \quad \xi = 0, \quad \xi = \pi. \tag{17}$$

4. THERMAL LOADS

Environment temperature is assumed to be uniformly raised from initial value T_i to final one T_j and temperature variation $\Delta T = T_j - T_i$ is independent to thickness. In this case, the thermal parameters Φ_0, Φ_1 can be expressed

$$\Phi_0 = \phi_0 \Delta T h, \quad \Phi_1 = \phi_1 \Delta T h^2, \tag{18}$$

where

$$\begin{aligned} \phi_0 &= E_m\alpha_m + \frac{1}{\kappa+1}\left(E_m\alpha_{cm} + E_{cm}\alpha_m\right) + \frac{1}{2\kappa+1}E_{cm}\alpha_{cm}, \\ \phi_1 &= \left(\frac{1}{\kappa+2} - \frac{1}{2\kappa+2}\right)\left(E_m\alpha_{cm} + E_{cm}\alpha_m\right) + \left(\frac{1}{2\kappa+2} - \frac{1}{4\kappa+2}\right)E_{cm}\alpha_{cm}. \end{aligned} \tag{19}$$

In this case, the temperature through the thickness is governed by the one dimentional Fourier equation of steady-state heat conduction established in curvilinear coordinate system whose origin is the center of the teardrop shell (in the Fig. 2).

5. THE RESULTING EQUATIONS BASED DISPLACEMENT

5.1. The preselected solutions

The FGM teardrop-shaped double-curved shells has two singularities, which are points with $R_0(\xi) = 0$, at $\xi = \xi_0 = 0, \xi = \xi_1 = \pi$, therefore, singular integrals may appear in the coefficient calculation for the kinetic equations. The preselected solutions must satisfy the boundary conditions, so that the singular integrals will converge and become computable. In addition, the preselected solutions must contain hidden function dependencies t, which can be determined during the solution process.

$$\left.\begin{aligned} U &= u \sin\frac{m\pi(\xi-\xi_0)}{\alpha_0}\sin\frac{n\theta}{2} \\ V &= v \sin\frac{m\pi(\xi-\xi_0)}{\alpha_0}\cos\frac{n\theta}{2} \\ W &= w \sin^2\frac{m\pi(\xi-\xi_0)}{\alpha_0}\sin\frac{n\theta}{2} \end{aligned}\right\} \tag{20}$$

in which $\alpha_0 = \xi_1 - \xi_0$; u, v, w are hidden funtions, with dependency t.

Using the preselected solutions (20), satisfied the boundary conditions, we have the integrals to calculate the coefficients of the solution equations being converge and singular as

$$U = V = W = 0 \quad \text{when} \quad \xi = \xi_0 = 0, \xi = \xi_1 = \pi,$$
$$\frac{\partial W}{\partial \xi} = 0 \quad \text{when} \quad \xi = \xi_0 = 0, \quad \xi = \xi_1 = \pi.$$

5.2. The resulting equations

To set up the resulting solutions based displacements, replacing Eq. (20) in Eqs. (2), (3), substituting the found strains into the corresponding expressions, we have the force and the moment resultants. The force and moment resultants are substituted into Eqs. (14), (15), (16) to get three equations containing u, v, w, ξ, θ, t. Applying the Galerkin method to convert the differential equation of motion in the form of partial differential to the ordinary differential, we get

$$\begin{aligned}
&\alpha_{11}u + \alpha_{12}v + \alpha_{13}w + \alpha_{14}w^2 - au'' = 0, \\
&\beta_{11}u + \beta_{12}v + \beta_{13}w + \beta_{14}w^2 - bv'' = 0, \\
&\lambda_{11}u + \lambda_{12}v + \lambda_{13}w + \lambda_{14}w^2 + \lambda_{21}uw + \lambda_{22}vw + \lambda_{23}w^3 + m_1\Delta T\phi_0 w \\
&\qquad + m_2\Delta T\phi_0 + m_3 q - m_4 w' - cw'' = 0,
\end{aligned} \tag{21}$$

where: $\alpha_{ij}, \beta_{ij}, \lambda_{ij}, m_i, a, b, c$ are constants; ϕ_0 is the thermal parameter, considered here as in the case of uniform thermal transfer and in the case of thermal transfer through the thickness; $q = q\,(t)$ is a harmonic load.

6. EXAMPLE

Investigate the dynamic response of the FGM teardrop-shaped double-curved shell of revolution with constant thickness as Fig. 2, subject to uniform thermal load and distributed harmonic load, with $\kappa = 3, 0 \leq \xi \leq \pi, 0 \leq \theta \leq 2\pi$. The characteristic parameters of the shell: $R = R(\xi), R_0 = R(\xi)\sin(\xi), R_1 = R_1 = R_0\sqrt{R^2 + R'^2}/\left(R\sin(\xi) - R'\cos(\xi)\right)$, according to Section 2.3. The input data given in the SI are as follows:

$$\begin{aligned}
&E_m = 70.10^9\ \text{N/m}^2, E_c = 380.10^9\ \text{N/m}^2, \rho_m = 2702 kg/m^3, \rho_c = 3800\ \text{kg/m}^3, \kappa = 3, \\
&\nu = 0.3, h = 0.018\ \text{m}, m = 3, n = 1, \rho_1 = \left[\rho_m + (\rho_c - \rho_m)\,/\,(\kappa + 1)\right] h, q = Q\cos\left[\omega t\right], \\
&\alpha_m = 23.10^{-6}\,{}^0C^{-1}, \alpha_c = 7.4.10^{-6}\,{}^0C^{-1}, K_m = 204\ \text{W/mK}, K_c = 10.4\ \text{W/mK}, \\
&\phi_0 = E_m\alpha_m + \frac{1}{\kappa+1}\left(E_m\alpha_{cm} + E_{cm}\alpha_m\right) + \frac{1}{2\kappa+1}E_{cm}\alpha_{cm}, \\
&\phi_1 = \left(\frac{1}{\kappa+2} - \frac{1}{2\kappa+2}\right)\left(E_m\alpha_{cm} + E_{cm}\alpha_m\right) + \left(\frac{1}{2\kappa+2} - \frac{1}{4\kappa+2}\right)E_{cm}\alpha_{cm}.
\end{aligned}$$

With the above input data, after applying the program automatically calculate the coefficients of the resulting equation (15). Eq. (15) has the form:

-3.669660120341877$\times 10^{10}$ u[t]+2.923862676648991$\times 10^{8}$ w[t]+9.468902703961835 $\times 10^{7}$ w[t]^{2}-188.08106641091916 ro1*u$''$[t]==0
-5.463981860570105$\times 10^{10}$ v[t]+6.281564792003205$\times 10^{10}$ w[t]-1.161251563614549 $\times 10^{9}$ w[t]^{2}-188.08106641091916 ro1*v$''$[t]== 0
2.67171698248182 q-1.494969450673264$\times 10^{7}$ u[t]+2.134768974881463$\times 10^{9}$ v[t] -5.919802087392517$\times 10^{9}$ w[t]+7407511.373038583 u[t]*w[t]-3.080572966132525 $\times 10^{8}$ v[t]*w[t]+7.564727344231719$\times 10^{8}$ w[t]^{2}-9.816228016791746$\times 10^{8}$ w[t]^{3} -0.022843735714297597ΔT*ϕ 0+0.053460156519092084 w[t] * ΔT * ϕ 0 -36.23386685849541 ε*ro1*w$'$[t]- 18.116933429247705 ro1*w$''$[t]==0

Case 1:

$$\phi_0 = 4.81357.10^6, ro1 = 53.577, \varepsilon = 12.4, \Delta T = T_c - T_m = -10, Q = 10^6, \omega = 2.4.$$

With the initial condition: $u(0) = v(0) = w(0) = 0, u'(0) = v'(0) = w'(0) = 0$.

Apply Mathematica 7.0 program to solve the nonlinear differential equation system, we have.

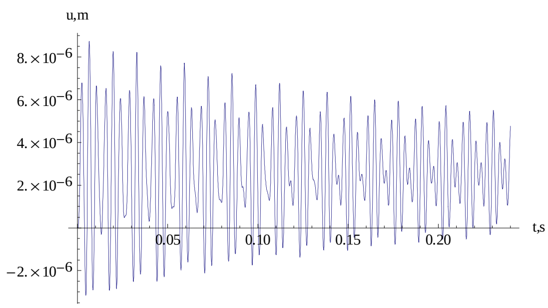

Fig. 3. Amplitude $u(t)$ varies at time $t(0, 0.24)$

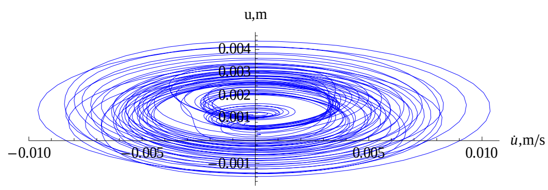

Fig. 4. Phase plan $u'(t) - 500u(t)$

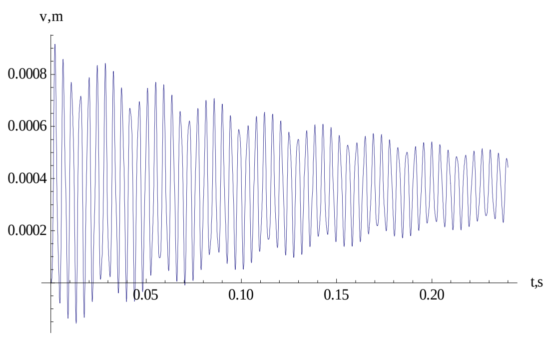

Fig. 5. Amplitude $v(t)$ varies at time $t(0, 0.24)$

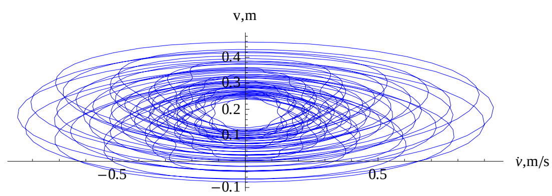

Fig. 6. Phase plan $v'(t) - 500v(t)$

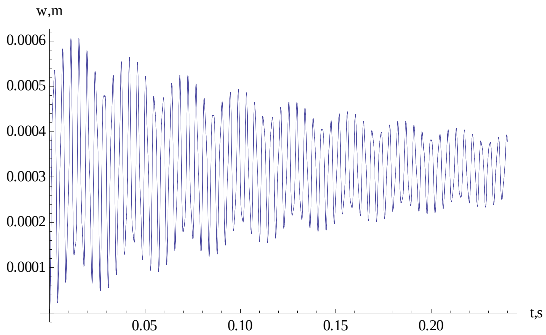

Fig. 7. Amplitude $w(t)$ varies at time $t(0, 0.24)$

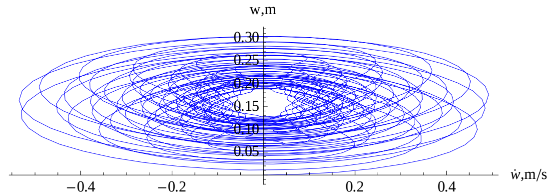

Fig. 8. Phase plan $w'(t) - 500w(t)$

The maximum value of displacements varies at the $t(0, 0.64)$.

$$u = 8.72641.10^{-6}\text{ m}, v = 0.000916314\text{ m}, w = 0.000606869\text{ m}.$$

Value of displacement bounded over time t; u have signs of chaotic motion, the phase curves intersected complex, v, w vibrate with small amplitudes in groups, not harmonic vibration, vibrate with decreasing amplitudes.

Case 2:

$$\phi_0 = 4.81357.10^6, ro1 = 53.577, \varepsilon = -0.8, \Delta T = T_c - T_m = 85.6, Q = 10^6, \omega = 2.4.$$

With the initial condition: $u(0) = v(0) = w(0) = 0, u'(0) = v'(0) = w'(0) = 0$.

Apply Mathematica 7.0 program to solve the nonlinear differential equation system, we have.

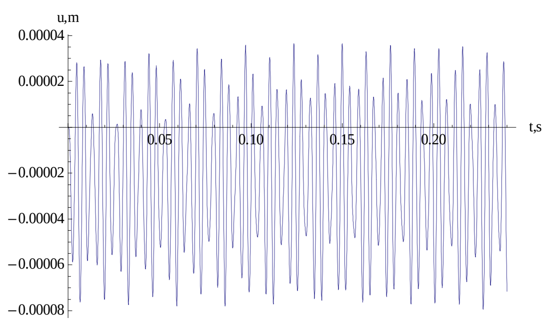

Fig. 9. Amplitude $u(t)$ varies at time $t(0, 0.064)$

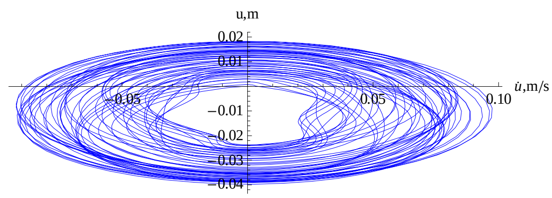

Fig. 10. Phase plan $u'(t) - 5000u(t)$

The maximum value of displacements varies at the $t(0, 0.64)$.

$$u = 0.000036486\text{ m}, v = 0.00215424\text{ m}, w = 0.000214712\text{ m}.$$

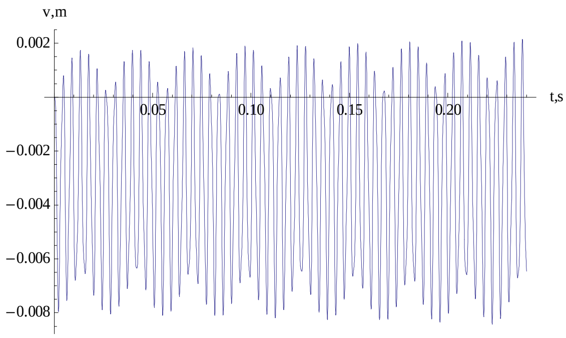

Fig. 11. Amplitude $v(t)$ varies at time $t(0, 0.064)$

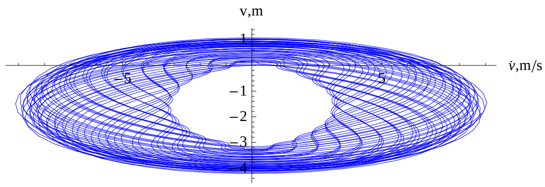

Fig. 12. Phase plan $v'(t) - 500v(t)$

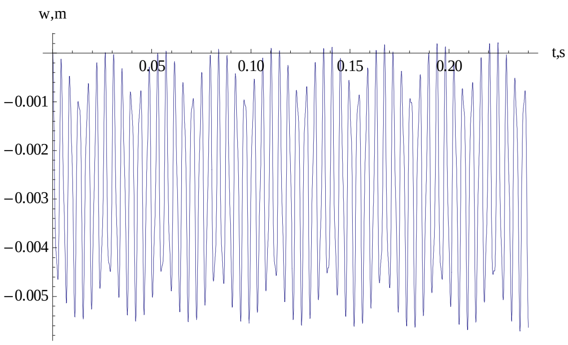

Fig. 13. Amplitude $w(t)$ varies at time $t(0, 0.064)$

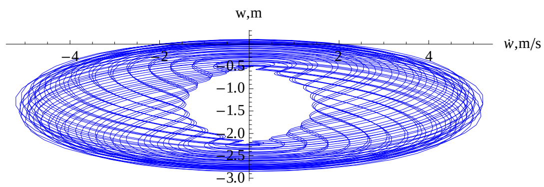

Fig. 14. Phase plan $w'(t) - 500w(t)$

The value of the displacement increases with time t; u, v, w show signs of chaotic motion, the phase curves intersected, arranged in the cross loops, not harmonic vibration, vibrated with bounded amplitudes.

Discussion: Because the corresponding published works could not be found for comparison. The reliability of the algorithm and the calculation program can be illustrated by choosing a simple structure that is the spherical shell, constant thickness, subjected to mechanical - thermal loads, and solved by two methods: meridian curve digitalization and analytical methods. The results of Eq. (21) by these two methods are completely coincidental, the method of meridian curve digitalization solves the problem of double-curved shells with arbitrary meridian curve, and the analytical method is are not. That is the reason proposed method of meridian curve digitalization presented in this paper.

7. CONCLUSIONS

In this paper, the followings are presented and discussed:

- Seting up the resulting equation, solving the problem of nonlinear dynamics of the FGM teardrop-shaped double-curved shell with constant thickness, subject to thermal load and harmonic load.

- Proposing a method for setting the characteristic parameters for the double-curved shell with arbitrary meridian curve for using in the calculation of the coefficients of the solution equation.

- Proposing preselected solutions, such that the fixed boundary conditions can be satisfied, and the singular integrals converge and are computable.

- Found the dynamic response, including the dynamic response at singular points, shown observation on special signs of the dynamic response.

- The method of setting characteristic parameters by digitizing the meridian curve can be applied to solve a wider range of problems for combined shells in general.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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