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DYNAMIC MODELLING AND SINGULARITY-FREE SIMULATION OF CLOSED LOOP MULTIBODY SYSTEM DRIVEN BY ELECTRIC MOTORS

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Abstract. This paper presents the dynamic model and singularity-free simulation of electromechanical systems including closed loop multibody systems, massless gear transmission and electric motors. The dynamic model of these systems is established in matrix form and written in a Differential-Algebraic Equations form by applying the Lagrangian equation with multipliers and substructure method. Moreover, this paper deals with two difficult issues in the simulation of closed-loop multibody systems which are to overcome smoothly the singular configurations and to stabilize the constrained equations due to accumulated errors. The singularity-free simulation is solved by using null-space of Jacobian matrix to eliminate the constraint forces – Lagrangian multipliers in equations of motion. The drift in the constraint equation during simulation is restricted by a combination of Baumgarte's stabilization and post-adjusting technique. Some numerical experiments are carried out to the planar 3RRR parallel manipulator driven by electric motors. Simulation results confirm the effectiveness of the proposed approach in overcoming the singular configurations and in stabilization of the constraint.

Keywords: closed loop multibody system, electromechanical system, singularity-free, constrained stabilization, post-adjusting technique.

1. INTRODUCTION

Most of the robots and machines used in industries are electromechanical systems which consist of the purely mechanical parts, considered as multibody system (MBS), and the electric parts including motors and sensors. For this system, the motion of multibody system is the system output due to the influence of currents or voltage applied to motors as the input. Serial or parallel robots, pumps, or compressors can be listed as examples of these electromechanical systems.

The mechanical part of robots and machines, as a multibody system (MBS) is normally described by a set of rigid bodies interconnected by active and passive joints. They can create an open loop or a closed loop MBS. Modelling and simulation of a such system has attracted numerous researchers [1–10]. Several methods such as Newton-Euler equations, principle of virtual work, principle of d'Alembert-Lagrange, Lagrange equations, Jourdain principle, and Kane equation have been used widely to formulate dynamic equations of an MBS [11–20]. In most of these studies, dynamics of actuators and gear transmission are not taken into account in the dynamic model. Thus, the obtained equations of motion using only the MBS model described incompletely the system's features.

For an open loop MBS, the equations of motion are ordinary differential equations (ODEs) in minimal generalized coordinates, but they are algebraic-differential equations (DAEs) in redundant generalized coordinates for a closed loop MBS. Numerical simulation requires solving of ODEs or DAEs for MBS with an open loop or a with closed loop, respectively. The solving of DAEs is more difficult than solving ODEs since the integrated results should satisfy not only differential equations but also the constraint equations which are commonly at position and velocity levels. Many researches have been devoted to solving of DAEs as well as simulation of MBS with closed loop structures, e.g. [21–24]. In these studies, popular methods such as Lagrange multiplier partition or elimination, coordinate partitioning method, and velocity transformations are commonly used due to their simplicity. However, these methods can be applied only in cases when the Jacobian matrix of constraints is full rank, hence, the algorithms can fail if the system passes through singular configurations.

In recent years, the problem of singularity-free dynamic simulation has attracted the attention of many researchers. In literatures [25–27], the authors used a modified Lagrangian formulation or augmented Lagrangian formulation for the dynamic analysis of closed loop mechanical systems to get over singular positions. In this method, the Lagrangian was added by two terms: a fictitious potential and kinematic energy, these terms were considered as penalty factor. Moreover, dissipative forces were added to the system to improve the stability. The stability and the accuracy of this method depend on the parameters chosen by users. This leads to the fact that the dynamics of the system may simulate only approximately the response of real systems. Furthermore, in each time step of integration the iterative operation must be applied to determine the generalized acceleration and the Lagrange's multipliers.

Besides the smooth passing singular positions, the techniques for constraint stabilization are also important to suppress accumulated errors. The popular and commonly used method was proposed by Baumgarte [28,29]. Many researcher have already applied the method successfully in their studies, e.g. [30], however this method does not meet the requirements when the MBS system moves through singular configurations.

In this paper we will focus our interest into modelling and dynamic singularity free simulation of electromechanical system. This is a typical example of mixed closed loop MBS driven by electric motors. Hence the system consists of three substructures: electric motors, closed loop MBS and massless gear transmissions. For the MBS substructure, the redundant generalized coordinates are used and Lagrange equations with multipliers lead to the dynamic systems of equations in DAE form. This dynamic model is nonlinear and coupling. For the transmission substructure the dynamic equations are derived by the power balance and with the assumption that the gear transmission is massless and frictionless. For the electric motors substructure the dynamics equations are derived by using Kirchhoff's laws and the angular momentum law. In order to get the compact form for better dealing with singularity configuration we will then simplify the system dynamic equations with the assumption that the electrical time constant is much smaller than the mechanical one. Null space of the Jacobian matrix [31] and the pseudoinverse are exploited to obtain redundant generalized acceleration after eliminating Lagrangian multipliers. The dependent coordinates and velocities obtained from integration are adjusted to satisfy the constraints at position and velocity levels.

The main contributions in this paper are to:

(i) Give out the matrix form of the system of equations of motion for an electromechanical system, in which the mechanical part has a closed loop structure. By introducing a matrix presenting the relation between generalized and active coordinates, the nonlinear dynamic equations of mixed system are obtained easily from pure mechanical ones.

(ii) Successful exploit null space of Jacobian matrix and pseudoinverse for smooth overcoming singularities of the MBS with closed loop.

(iii) Propose a combination of Baumgarte and post-adjusting techniques for stabilization of constraint equations in solving DAEs. The presented simulation results confirm the effectiveness of the proposed approach.

The rest of this paper is organized as follows: In Section 2, the dynamic equations of closed loop MBS actuated by electric motors are presented; Section 3 shows the technique of Singularity-Free Simulation with null space of the Jacobian matrix. Stabilization technique in simulation is provided in Section 4. For illustration, in Section 5 we will provide numerical simulation examples with a 3RRR planar parallel robot are shown. The conclusion is given in Section 6.

2. DYNAMIC MODEL OF AN ELECTROMECHANICAL SYSTEM WITH KINEMATIC LOOPS

This section presents the dynamic modelling for a closed loop multibody system driven by electric motors such as mechanism and parallel robots. Consider a closed loop multibody system which has n degrees of freedom and is driven by n motors. The system dynamic model is derived by applying Lagrange equation with multipliers and substructure method. To write the equations of motion for this kind of system, the variables used in this section are listed in Table 1.

Table 1. Nomenclature

Symbols	Description
$\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_n]^T = \mathbf{q}_a$	Active joint variables
$\mathbf{x} = [x_1, x_2, ..., x_n]^T$.	Position of moving platform
$\boldsymbol{\theta}_m = [\theta_{m1}, \theta_{m2}, ..., \theta_{mn}]^T$	Angle of the motor shaft
$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_r]^T$	Lagrangian multipliers
$\boldsymbol{\tau}_2 = [\tau_{2,1}, \tau_{2,2}, ..., \tau_{2,n}]^T$	Torque /force at the output of transmission
$\boldsymbol{\tau}_1 = [\tau_{1,1}, \tau_{1,2}, ..., \tau_{1,n}]^T$	Torque/force at the input of transmission
$\boldsymbol{\tau}_0 = [\tau_{0,1}, \tau_{0,2}, ..., \tau_{0,n}]^T$	Torque/force of the DC motor
$\mathbf{R}_G = \text{diag}(r_1, r_2, ..., r_n), r_i = \theta_{m,i}/\theta_i$	Matrix of gear reduction ratio
$\mathbf{J}_m = \text{diag}(J_{m,1}, J_{m,2}, ..., J_{m,n})$	Moment of inertia of rotors
$\mathbf{L}_a = \text{diag}(L_{a,1}, L_{a,2}, ..., L_{a,n})$	Motor coil inductances
$\mathbf{R}_a = \text{diag}(R_{a,1}, R_{a,2}, ..., R_{a,n})$	Motor coil resistances
$\mathbf{K}_e = \text{diag}(K_{e,1}, K_{e,2}, ..., K_{e,n})$	Back-emf constants
$\mathbf{K}_m = \text{diag}(K_{m,1}, K_{m,2}, ..., K_{m,n})$	Torque constants
$\mathbf{u} = [U_1, U_2, ..., U_n]^T$	Motor input voltages
$\mathbf{i} = [i_1, i_2, ..., i_n]^T$	Currents in electric motors
$\mathbf{D}_m = \text{diag}(b_1, b_2, ..., b_n)$	Viscous coefficients of motor shafts
$\mathbf{q} = [\mathbf{q}_a^T, \mathbf{q}_d^T, \mathbf{x}^T]^T$	Vector of generalized coordinates including active and passive joint variables and platform position.
$\mathbf{M}(\mathbf{q})$	Mass matrix
$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$	Coriolis matrix
$\mathbf{g}(\mathbf{q})$	Force vector due to gravity
$\boldsymbol{\Phi}_q(\mathbf{q})$	Jacobian matrix of constraint equations
$\mathbf{R}$	Elimination matrix/Null space of Jacobian matrix

2.1. Dynamic model of closed loop MBS

There are several methods to establish equations of motion of mechanical systems presented in references [1, 2, 4, 5]. Let's consider a closed loop MBS having n degrees of freedom. In this paper we use the set of the redundant generalized coordinates, which are presented by a vector $\mathbf{q} = [q_1, q_2, ..., q_m]^T, m > n$. With the Lagrange multipliers, the equations of motion are derived in matrix form as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{\Phi}_q^T(\mathbf{q})\boldsymbol{\lambda} = \mathbf{B}\boldsymbol{\tau}_2, \tag{1}$$

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}, \tag{2}$$

where: $\mathbf{M}(\mathbf{q})$ - $m \times m$ mass matrix; $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ - matrix of Coriolis and centrifugal term; $\mathbf{D}$ - damping matrix; $\mathbf{g}(\mathbf{q})$ - generalized force of the gravitation; $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_r]^T$ - the $r \times 1$ vector of Lagrangian multipliers, $r = m - n$; $\boldsymbol{\phi}(\mathbf{q}) = 0$, with $\boldsymbol{\phi} = [\phi_1, \phi_2, ..., \phi_r]^T$ including $m - n$ constraint equations; $\mathbf{\Phi}_q(\mathbf{q}) = \partial\boldsymbol{\phi}/\partial\mathbf{q}$ - the $r \times m$ Jacobian matrix; $\mathbf{B}$ - the matrix related to the control input arrangement and $\boldsymbol{\tau}_2$ the vector force/torque in the actuated joints.

Matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ can be determined from the mass matrix $\mathbf{M}(\mathbf{q})$ using the Kronecker product [32] or the Christoffel formula [5] as follows

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \left\{c_{ij}(\mathbf{q}, \dot{\mathbf{q}})\right\}, \quad c_{ij}(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\sum_{k=1}^{m}\left(\frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i}\right)\dot{q}_k. \tag{3}$$

Note that the mass matrix $\mathbf{M}(\mathbf{q})$ is symmetric positive definite, and the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ determined by (3) guarantees the skew-symmetric property of the matrix $\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$.

2.2. Dynamic model of an electric motor and gear transmission

With the assumption of massless and frictionless gear transmission, the output and input relation of n reducer is written as

$$\mathbf{R}_G\dot{\mathbf{q}}_a \equiv \mathbf{R}_G\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{\theta}}_m, \quad \boldsymbol{\tau}_2 = \mathbf{R}_G\boldsymbol{\tau}_1. \tag{4}$$

Dynamics of electric motors is described by the mechanical and electrical equations [20, 33–35]. The dynamic equations for n motors are written in matrix form as following

$$\mathbf{J}_m\ddot{\boldsymbol{\theta}}_m + \mathbf{D}_m\dot{\boldsymbol{\theta}}_m = \boldsymbol{\tau}_0 - \boldsymbol{\tau}_1, \tag{5}$$

$$\mathbf{L}_a\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i} + \mathbf{R}_a\mathbf{i} = \mathbf{u} - \mathbf{u}_e. \tag{6}$$

The electrical and mechanical interaction of n motors is shown by the relationship between motor torque and current [18, 33–35] and between EMFs voltages and motor speed as follows

$$\boldsymbol{\tau}_0 = \mathbf{K}_m\mathbf{i}, \quad \mathbf{u}_e = \mathbf{K}_e\dot{\boldsymbol{\theta}}_m. \tag{7}$$

So, the dynamic model of closed loop MBS driven by an electric motor is described by a set of equations from (1) to (7) in DAE form. They show the dynamic relationship between inputs (voltage $\mathbf{u}$) and outputs (motion $\mathbf{q}(t)$).

2.3. Simplified dynamic model

In order to use easily the algorithm for dealing with singular configuration we will reduce the systems described above. By using the approximation $\mathbf{L}_a \mathrm{d}\mathbf{i}/\mathrm{d}t \approx \mathbf{0}$, the current can be solved from Eqs. (6) and (7)

$$\mathbf{i} = \mathbf{R}_a^{-1}(\mathbf{u} - \mathbf{u}_e) = \mathbf{R}_a^{-1}(\mathbf{u} - \mathbf{K}_e\dot{\boldsymbol{\theta}}_m). \tag{8}$$

By substituting it into Eq. (7) one gets the motor torque generated by the coils acting on the rotors

$$\boldsymbol{\tau}_0 = \mathbf{K}_m\mathbf{i} = \mathbf{K}_m\mathbf{R}_a^{-1}(\mathbf{u} - \mathbf{K}_e\dot{\boldsymbol{\theta}}_m). \tag{9}$$

Substituting (9) into (5) we get the differential equations of motion of the rotors as follows

$$\begin{aligned}\mathbf{J}_m\ddot{\boldsymbol{\theta}}_m + \mathbf{D}_m\dot{\boldsymbol{\theta}}_m &= \boldsymbol{\tau}_0 - \boldsymbol{\tau}_1 \\ &= \mathbf{K}_m\mathbf{R}_a^{-1}(\mathbf{u} - \mathbf{K}_e\dot{\boldsymbol{\theta}}_m) - \boldsymbol{\tau}_1.\end{aligned} \tag{10}$$

Considering Eq. (4), $\boldsymbol{\tau}_1 = \mathbf{R}_G^{-1}\boldsymbol{\tau}_2$, Eq. (10) becomes

$$\mathbf{J}_m\ddot{\boldsymbol{\theta}}_m + \mathbf{D}_m\dot{\boldsymbol{\theta}}_m = \mathbf{K}_m\mathbf{R}_a^{-1}(\mathbf{u} - \mathbf{K}_e\dot{\boldsymbol{\theta}}_m) - \mathbf{R}_G^{-1}\boldsymbol{\tau}_2,$$

or

$$\mathbf{J}_m\ddot{\boldsymbol{\theta}}_m + (\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)\dot{\boldsymbol{\theta}}_m = \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{u} - \mathbf{R}_G^{-1}\boldsymbol{\tau}_2.$$

Substituting $\dot{\boldsymbol{\theta}}_m = \mathbf{R}_G\dot{\mathbf{q}}_a = \mathbf{R}_G\dot{\boldsymbol{\theta}}$ into the above equation yields

$$\mathbf{R}_G\mathbf{J}_m\mathbf{R}_G\ddot{\mathbf{q}}_a + \mathbf{R}_G(\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)\mathbf{R}_G\dot{\mathbf{q}}_a = \mathbf{R}_G\mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{u} - \boldsymbol{\tau}_2. \tag{11}$$

By defining the matrix $\mathbf{B}$ with the form:

$$\mathbf{B} = \begin{bmatrix} \mathbf{E}_{n\times n} \\ \mathbf{0}_{m-n,n} \end{bmatrix},$$

and multiplying this to Eq. (11) from left one gets

$$\mathbf{B}\mathbf{R}_G\mathbf{J}_m\mathbf{R}_G\ddot{\mathbf{q}}_a + \mathbf{B}\mathbf{R}_G(\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)\mathbf{R}_G\dot{\mathbf{q}}_a = \mathbf{B}\mathbf{R}_G\mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{u} - \mathbf{B}\boldsymbol{\tau}_2. \tag{12}$$

To eliminate the vector $\boldsymbol{\tau}_2$ from Eqs. (1) and (12), the matrix $\mathbf{Z} = [\mathbf{E}_{n\times n} \quad \mathbf{0}_{m,m-n}]$ is used. Here, the following relations are satisfied

$$\dot{\mathbf{q}}_a = \mathbf{Z}\dot{\mathbf{q}}, \quad \dot{\mathbf{q}}_a = \mathbf{Z}\dot{\mathbf{q}}, \quad \ddot{\mathbf{q}}_a = \mathbf{Z}\ddot{\mathbf{q}}. \tag{13}$$

Eq. (12) is rewritten as

$$\mathbf{B}\mathbf{R}_G\mathbf{J}_m\mathbf{R}_G\mathbf{Z}\ddot{\mathbf{q}} + \mathbf{B}\mathbf{R}_G(\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)\mathbf{R}_G\mathbf{Z}\dot{\mathbf{q}} = \mathbf{B}\mathbf{R}_G\mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{u} - \mathbf{B}\boldsymbol{\tau}_2 \tag{14}$$

By addition two equations (1) and (14) one gets

$$\begin{aligned}(\mathbf{M}(\mathbf{q}) + \mathbf{B}\mathbf{R}_G\mathbf{J}_m\mathbf{R}_G\mathbf{Z})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \Big(\mathbf{D} + \mathbf{B}\mathbf{R}_G(\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)\mathbf{R}_G\mathbf{Z}\Big)\,\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) \\ = \mathbf{B}\mathbf{R}_G\mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{u} + \boldsymbol{\Phi}_q^T(\mathbf{q})\boldsymbol{\lambda}.\end{aligned} \tag{15}$$

By defining the following matrices

$$\begin{aligned}
&\mathbf{M}_s(\mathbf{q}) = \left(\mathbf{M}(\mathbf{q}) + \mathbf{B}\mathbf{R}_G\mathbf{J}_m\mathbf{R}_G\mathbf{Z}\right), \quad \mathbf{C}_s(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \\
&\mathbf{D}_s = \left(\mathbf{D} + \mathbf{B}\mathbf{R}_G(\mathbf{D}_m + \mathbf{K}_m\mathbf{R}_a^{-1}\mathbf{K}_e)\mathbf{R}_G\mathbf{Z}\right), \quad \mathbf{g}_s(\mathbf{q}) = \mathbf{g}(\mathbf{q}), \quad \mathbf{B}_s = \mathbf{B}\mathbf{R}_G\mathbf{K}_m\mathbf{R}_a^{-1},
\end{aligned} \tag{16}$$

Eqs. (15) is rewritten in compact form as

$$\mathbf{M}_s(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_s(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}_s\dot{\mathbf{q}} + \mathbf{g}_s(\mathbf{q}) = \mathbf{B}_s\mathbf{u} + \mathbf{\Phi}_q^T(\mathbf{q})\boldsymbol{\lambda}. \tag{17}$$

Once again, the constraint equations are combined

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}. \tag{18}$$

Thus, the dynamic model of a closed loop MBS driven by electric motors is described by a set of differential algebraic equations (17) and (18). Note that the torques $\boldsymbol{\tau}_0$, $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ do not appear explicitly in the system. These equations will be used for the inverse and forward dynamic problems of a closed loop MBS.

It is worth noting that $\mathbf{B}\mathbf{J}_m r^2\mathbf{Z}$ is the symmetric and constant matrix, so the Coriolis matrices $\mathbf{C}_s(\mathbf{q},\dot{\mathbf{q}})$ or $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ calculating from mass matrices $\mathbf{M}_s(\mathbf{q})$ or $\mathbf{M}(\mathbf{q})$ are the same, and skew-symmetric property of matrix $\dot{\mathbf{M}}_s(\mathbf{q}) - 2\mathbf{C}_s(\mathbf{q},\dot{\mathbf{q}})$ is still remained [5].

3. SINGULARITY-FREE SIMULATION WITH NULL SPACE OF THE JACOBIAN MATRIX

The dynamic simulation of a closed loop MBS requires solving differential algebraic equations (17) and (18). This can be done by several ways. By using so-called implicit function, the system of equations can be firstly solved by using direct integration for independent generalized coordinates and then solving the nonlinear algebraic equations for dependent ones. In this method, it is not necessary to stabilize the constraint. However, including solving the nonlinear algebraic equations in the algorithm with implicit function is tedious task.

The common methods are differentiating constraint equations twice with respect to time and get constraint equations in acceleration level as

$$\mathbf{\Phi}_q(\mathbf{q})\ddot{\mathbf{q}} = -\dot{\mathbf{\Phi}}_q(\mathbf{q})\dot{\mathbf{q}}. \tag{19}$$

It can be inferred in the equations (17) and (19), we can easily obtain the vector of generalized acceleration and the vector of Lagrange multipliers $(\ddot{\mathbf{q}}, \boldsymbol{\lambda})$ if the Jacobian matrix of constraint equations has a full rank. It means the system is not in singular configurations. In this case, methods such as Lagrange multiplier partition [13, 21–23, 36], the transformed to independent coordinates, or Lagrange multiplier elimination are used. In the case of nonsingular configuration, elimination matrix $\mathbf{R}$ (with size $m \times n$) is usually defined from the Jacobi matrix by using the following formula

$$\mathbf{R} = \alpha \Big[\mathbf{E}, -[\mathbf{\Phi}_{q_d}^{-1}(\mathbf{q})\mathbf{\Phi}_{q_i}(\mathbf{q})]^T \Big]^T, \quad \alpha \neq 0, \tag{20}$$

where $\mathbf{q}_i \equiv \mathbf{q}_a, \mathbf{q}_d$ are the independent and dependent coordinates respectively; and $\mathbf{\Phi}_{q_i} = \partial \boldsymbol{\phi}/\partial \mathbf{q}_i$, $\mathbf{\Phi}_{q_d} = \partial \boldsymbol{\phi}/\partial \mathbf{q}_d$ are the Jacobian matrices with respect to generalized coordinates independent and dependent resp. It is clear the elimination matrix $\mathbf{R}$ satisfies the property

$$\mathbf{\Phi}_q(\mathbf{q})\mathbf{R} = \mathbf{0}.$$

It means that $\mathbf{R}$ creates the null space matrix of Jacobian matrix. Note that in this case the Jacobian matrix is full rank.

Alternatively, even for the case when the Jacobian matrix is not full rank, the elimination matrix can be calculated by using the pseudo inverse matrix. Based on the property of pseudo inverse matrix $\mathbf{\Phi}_q(\mathbf{q})\mathbf{\Phi}_q^+(\mathbf{q})\mathbf{\Phi}_q(\mathbf{q}) = \mathbf{\Phi}_q(\mathbf{q})$ [37], the elimination matrix can be chosen as

$$\mathbf{R} = \alpha \left[\mathbf{E}_m - [\mathbf{\Phi}_q^+(\mathbf{q})\mathbf{\Phi}_q(\mathbf{q})] \right], \tag{21}$$

where $\alpha \neq 0$ and $\mathbf{\Phi}_q^+$ is a pseudo inverse of $\mathbf{\Phi}_q$ satisfying $\mathbf{\Phi}_q\mathbf{\Phi}_q^+\mathbf{\Phi}_q = \mathbf{\Phi}_q$.

It can be clearly seen that, with an elimination matrix $\mathbf{R}$ from (21) we have

$$\mathbf{\Phi}_q(\mathbf{q})\mathbf{R} = \alpha\mathbf{\Phi}_q(\mathbf{q})[\mathbf{E}_m - \mathbf{\Phi}_q^+(\mathbf{q})\mathbf{\Phi}_q(\mathbf{q})] = \alpha[\mathbf{\Phi}_q(\mathbf{q}) - \mathbf{\Phi}_q(\mathbf{q})] = \mathbf{0}.$$

Hence, the matrix $\mathbf{R}$ in this case satisfies also the definition of the null space of Jacobian matrix. However, the size of $\mathbf{R}$ in this case is $m \times m$.

By using the elimination matrices defined in (20) or (21), the dynamics of a closed loop MBS in combination with the constraint equations at acceleration level becomes

$$\mathbf{H}\ddot{\mathbf{q}} = \mathbf{h}(\mathbf{u}, \dot{\mathbf{q}}, \mathbf{q}), \tag{22}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{R}^T\mathbf{M}_s(\mathbf{q}) \\ \mathbf{\Phi}_q(\mathbf{q}) \end{bmatrix} \text{ and } \mathbf{h} = \begin{bmatrix} \mathbf{R}^T\mathbf{B}_s\mathbf{u} - \mathbf{R}^T[\mathbf{C}_s(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}_s\dot{\mathbf{q}} + \mathbf{g}_s(\mathbf{q})] \\ -\dot{\mathbf{\Phi}}_q(\mathbf{q})\dot{\mathbf{q}} \end{bmatrix}.$$

Apparently, the Lagrangian multiplier $\boldsymbol{\lambda}$ has been eliminated.

Note that in case the Jacobian matrix $\mathbf{\Phi}_q(\mathbf{q})$ has a full rank, $rank[\mathbf{\Phi}_q(\mathbf{q})] = r$, $\mathbf{R}$ defined in (20), the matrix $\mathbf{H}$ is a square matrix and the vector $\ddot{\mathbf{q}}$ is obtained by multiplying (22) with $\mathbf{H}^{-1}$ from left. However, with $\mathbf{R}$ defined in (21) the matrix $\mathbf{H}$ has a size of $(m + r) \times m$ and a full rank and the vector $\ddot{\mathbf{q}}$ is obtained from (22) by using a pseudo inverse matrix $\mathbf{H}^+$ of $\mathbf{H}$.

The methods such as Lagrangian multiplier elimination and partition are only used when Jacobian matrix $\mathbf{\Phi}_q(\mathbf{q})$ has the full rank, it means that $rank[\mathbf{\Phi}_q(\mathbf{q})] = r$ equals the number of constraint functions. However, the simulation can break down or get stuck at singular positions where the Jacobian matrix reduces its rank, $rank[\mathbf{\Phi}_q(\mathbf{q})] < r$. The singular configuration and time at which singularity occurs depend on the structure and the parameters of the system. To ensure continuous and smooth simulation, we need to find a solution to deal with these singular points. To overcome the singular configuration of the system, the elimination matrix is not computed according to (20) but according to (21) by using SVD.

The advantage of this technique is the fact that the matrix $\mathbf{H}$ has a full rank, $rank[\mathbf{H}] = m$. According to the theorem of rank plus nullity, it is valid for $rank[\mathbf{\Phi}_q(\mathbf{q})] + rank[\mathbf{R}] = m$ [38]. Therefore, the generalized acceleration vector $\ddot{\mathbf{q}}$ is always determined from Eq. (22). After solving the equations (22) one gets

$$\ddot{\mathbf{q}} = \begin{bmatrix} \mathbf{R}^T\mathbf{M}_s(\mathbf{q}) \\ \mathbf{\Phi}_q(\mathbf{q}) \end{bmatrix}^+ \begin{bmatrix} \mathbf{R}^T\mathbf{B}_s\mathbf{u} - \mathbf{R}^T[\mathbf{C}_s(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}_s\dot{\mathbf{q}} + \mathbf{g}_s(\mathbf{q})] \\ -\dot{\mathbf{\Phi}}_q(\mathbf{q})\dot{\mathbf{q}} \end{bmatrix}. \tag{23}$$

Therefore, the solution $\ddot{\mathbf{q}}$ directly from (23) is always determined and independent of the fact that the system is at singular positions or not. Noting that at regular configuration of the system, the pseudoinverse matrix $\mathbf{H}^+$ is identical with inverse matrix $\mathbf{H}^{-1}$. The pseudoinverse matrix is used only at the singular configuration, because the number of rows in the matrix $\mathbf{H}$ increases at this configuration. In fact, some rows of the Jacobian matrix $\mathbf{\Phi}_q(\mathbf{q})$ can be deleted to get a square matrix $\mathbf{H}$. However, to do this it is necessary to search the singular configuration during a simulation process. This requires a duration checking the configuration at each time step of integration. To avoid doing this, we use the pseudoinverse matrix for both regular and singular configurations. This is a remarkable advantage when using the null space matrix instead of elimination matrix calculated according to (20). Note that, the total number of equations (22) are m, because it is always hold for rank $rank[\mathbf{\Phi}_q] + rank[\mathbf{R}^T\mathbf{M}] = m$ in all positions of workspace: singular and nonsingular.

4. STABILIZATION TECHNIQUE IN SIMULATION

In the mentioned methods for solving of DAEs, the constraint equations are used at acceleration level. At this level it is easy to get the generalized acceleration. However, the velocity and position level constraints are not completely satisfied due to integration errors the so-called drift error [39]. Hence, it is necessary to stabilize the constraints. The technique introduced by Baumgarte is widely applied in the dynamic simulation of constrained mechanical systems [28, 30, 40]. The main idea of this technique is that the acceleration constraint $\ddot{\boldsymbol{\phi}}(\mathbf{q}) = \mathbf{0}$ is replaced by the differential equation as following

$$\ddot{\boldsymbol{\phi}}(\mathbf{q}) + 2\delta\omega_0\dot{\boldsymbol{\phi}}(\mathbf{q}) + \omega_0^2\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}, \tag{24}$$

or

$$\mathbf{\Phi}_q(\mathbf{q})\ddot{\mathbf{q}} = -\dot{\mathbf{\Phi}}_q(\mathbf{q})\dot{\mathbf{q}} - 2\delta\omega_0\mathbf{\Phi}_q(\mathbf{q})\dot{\mathbf{q}} - \omega_0^2\boldsymbol{\phi}(\mathbf{q}). \tag{25}$$

It is clear that if parameters are selected $\delta > 0, \omega_0 > 0$, the solutions of (24) converge to zero. So, the constraints are not drift. However, this method is not stable when the system moves through singular configurations.

The other method for stabilization of constraints were used in [25, 26, 41–46], the so-called post-adjusting. In these works, after each or some integral steps, the integrated values are adjusted to satisfy the constraints at position and velocity levels. The authors have adjusted all generalized coordinates including dependent and independent ones. Based on the idea of Mass-Orthogonal Projection methods for constrained multibody dynamics [25], in this paper, only dependent coordinates are adjusted so that the constraints are satisfied. Comparison to the algorithm in [25], the number of adjusted variables is smaller. This leads to reduce time consume for adjusting. This method is described as follows.

Assuming that after getting the integral results $\mathbf{q}^*, \dot{\mathbf{q}}^*$, these results are adjusted to satisfy the constraints. Adjusting is performed at two levels: position and velocity manifolds.

4.1. Position modification (projection on the manifold of constraint equations)

From the results $\mathbf{q}^{*T} = [\mathbf{q}_i^T, \mathbf{q}_d^{*T}]$, we keep the independent variables $\mathbf{q}_i$ and adjust dependent variables $\mathbf{q}_d^*$ becoming $\mathbf{q}_d$ so that they satisfy the constraint equations, it means $\boldsymbol{\phi}(\mathbf{q}_i, \mathbf{q}_d) = 0$. The values $\mathbf{q}_d$ need to satisfy not only the constraint equations but also are closest to the values $\mathbf{q}_d^*$ with a weighting matrix. This issue leads to an optimal problem. By using V function, it needs to reach a minimum

$$V = \frac{1}{2}(\mathbf{q}_d - \mathbf{q}_d^*)^T\mathbf{M}(\mathbf{q}_d - \mathbf{q}_d^*) \to \min \text{ with constraint equations } \boldsymbol{\phi}(\mathbf{q}_i, \mathbf{q}_d) = 0, \tag{26}$$

where weighting matrix $\mathbf{M}$ is a positive one. It is well known that the solution $\mathbf{q}_d$ of problem (26) is on the manifold determined by constraint equations and closest to $\mathbf{q}^*$ with a weighting matrix $\mathbf{M}$.

By using the penalty function method, the Lagrange function is chosen as follows

$$L = V + \boldsymbol{\phi}^T(\mathbf{q})\mathbf{A}\boldsymbol{\phi}(\mathbf{q}) \to \min, \tag{27}$$

where $\mathbf{A}$ is diagonal and positive matrix containing penalty factors. Differentiating (27) w.r.t. $\mathbf{q}_d$ and set equal to zero one gets

$$\mathbf{h}(\mathbf{q}_d) = \frac{\partial L}{\partial \mathbf{q}_d} = \mathbf{M}(\mathbf{q}_d - \mathbf{q}_d^*) + \mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\boldsymbol{\phi}(\mathbf{q}) = 0. \tag{28}$$

There are $r = m - n$ nonlinear algebraic equations and they can be solved by using Newton–Raphson method. Expanding Taylor series of $\mathbf{h}(\mathbf{q}_d)$ nearby $\mathbf{q}_{d,0}$ yields

$$\mathbf{h}(\mathbf{q}_{d,0} + \Delta\mathbf{q}_d) = \mathbf{h}(\mathbf{q}_{d,0}) + \mathbf{H}(\mathbf{q}_{d,0})\Delta\mathbf{q}_d + ...$$

where

$$\begin{aligned}\mathbf{H}(\mathbf{q}_{d,0}) &= \frac{\partial}{\partial\mathbf{q}_d}[\mathbf{M}(\mathbf{q}_d - \mathbf{q}_d^*) + \mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\boldsymbol{\phi}(\mathbf{q})]_{\mathbf{q}_{\mathrm{d},0}} = \mathbf{M} + [\mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\boldsymbol{\phi}(\mathbf{q})]_{q_d},\\ \mathbf{H}(\mathbf{q}_o) &= \mathbf{M} + [\mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\boldsymbol{\phi}(\mathbf{q})]_{q_d} \simeq \mathbf{M} + [\mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\mathbf{\Phi}_{q_d}(\mathbf{q})].\end{aligned} \tag{29}$$

It is worth noting that in (29) the terms related to the matrix $\mathbf{\Phi}_{q_d,q_d}(\mathbf{q})$ are neglected. From the above analysis, the algorithm to modify $\mathbf{q}_d$ is shown:

1) Assigning $k = 0$, choosing the number of iterations N, $\mathbf{q}_d^{(k)} = \mathbf{q}_d^*$;

2) Calculate $\mathbf{h}(\mathbf{q}_d^{(k)})$;

3) If $\| \mathbf{h}(\mathbf{q}_d^{(k)}) \| < \varepsilon$ or $k \geq N$ then stop, else go to step 4;

4) Calculate the matrix $\mathbf{H}(\mathbf{q}_d^{(k)})$, solve the equations $\mathbf{h}(\mathbf{q}_d^{(k)}) + \mathbf{H}(\mathbf{q}_d^{(k)})\Delta\mathbf{q}_d = 0$ to find $\Delta\mathbf{q}_d$;

5) Update $\mathbf{q}_d^{(k+1)} = \mathbf{q}_d^{(k)} + \Delta\mathbf{q}_d$;

6) Increase $k, k = k + 1$; go to step 2.

4.2. Velocity modification (projection on the manifold of constraint equations at velocity level)

From the results $\dot{\mathbf{q}}^{*T} = [\dot{\mathbf{q}}_i^T, \dot{\mathbf{q}}_d^{*T}]$, we keep the independent velocities $\dot{\mathbf{q}}_i$ and adjust dependent velocities $\dot{\mathbf{q}}_d^*$ becoming $\dot{\mathbf{q}}_d$ so that they satisfy the constraint equations at velocity level, it means $\dot{\boldsymbol{\phi}}(\mathbf{q}_i, \mathbf{q}_d) = 0$. The values $\dot{\mathbf{q}}_d$ need to satisfy not only the velocity constraint equations but also are closest to the values $\dot{\mathbf{q}}_d^*$ with a weighting matrix. This issue leads to an optimal problem. By using V function, it needs to reach a minimum

$$V = \frac{1}{2}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}_d^*)^T\mathbf{M}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}_d^*) \to \min, \tag{30}$$

with constraint functions $\dot{\boldsymbol{\phi}}(\mathbf{q}) = \mathbf{\Phi}_{q_i}(\mathbf{q})\dot{\mathbf{q}}_i + \mathbf{\Phi}_{q_d}(\mathbf{q})\dot{\mathbf{q}}_d = 0$ and $\mathbf{M}$ the positive matrix is weight matrix.

By using the penalty function method, the Lagrange function is chosen as follows

$$L = \frac{1}{2}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}_d^*)^T\mathbf{M}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}_d^*) + \frac{1}{2}[\mathbf{\Phi}_{q_i}(\mathbf{q})\dot{\mathbf{q}}_i + \mathbf{\Phi}_{q_d}(\mathbf{q})\dot{\mathbf{q}}_d]^T\mathbf{A}[\mathbf{\Phi}_{q_i}(\mathbf{q})\dot{\mathbf{q}}_i + \mathbf{\Phi}_{q_d}(\mathbf{q})\dot{\mathbf{q}}_d] \to \min \tag{31}$$

where $\mathbf{A}$ is a positive diagonal matrix.

Differentiating (32) w.r.t. $\dot{\mathbf{q}}_d$ and set equal to zero one gets

$$\frac{\partial L}{\partial \dot{\mathbf{q}}_d} = \mathbf{M}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}_d^*) + [\mathbf{\Phi}_{q_d}(\mathbf{q})]^T \mathbf{A} [\mathbf{\Phi}_{q_i}(\mathbf{q})\dot{\mathbf{q}}_i + \mathbf{\Phi}_{q_d}(\mathbf{q})\dot{\mathbf{q}}_d] = 0, \tag{32}$$

or

$$[\mathbf{M} + \mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\mathbf{\Phi}_{q_d}(\mathbf{q})]\dot{\mathbf{q}}_d = \mathbf{M}\dot{\mathbf{q}}_d^* - \mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\mathbf{\Phi}_{q_i}(\mathbf{q})\dot{\mathbf{q}}_i. \tag{33}$$

From (33) one gets

$$\dot{\mathbf{q}}_d = [\mathbf{M} + \mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\mathbf{\Phi}_{q_d}(\mathbf{q})]^{-1}[\mathbf{M}\dot{\mathbf{q}}_d^* - \mathbf{\Phi}_{q_d}^T(\mathbf{q})\mathbf{A}\mathbf{\Phi}_{q_i}(\mathbf{q})\dot{\mathbf{q}}_i]. \tag{34}$$

Thus, in the calibration process we have found the state of the system that satisfies the constraint equation of the system, these results will be used for the next step.

5. NUMERICAL EXPERIMENTS

In this section, some numerical simulations for closed loop MBS driven by electric motors are implemented to illustrate the proposed approach. A 3RRR planar parallel robot is chosen for simulation.

5.1. Equations of motion

The considered 3RRR planar parallel robot is shown Fig. 1. The fixed base $O_1O_2O_3$ and the moving platform $B_1B_2B_3$ are the two equilateral triangles having edge length of L_0 and L_1, respectively. This robot has three same legs including two links $O_iA_i = l_1$, $A_iB_i = l_2$. The robot is actuated by three electric motors through gear systems. The system has three degree of freedom and the redundant generalized coordinates are defined as $\mathbf{q} = [\boldsymbol{\theta}^T, \mathbf{x}^T]^T = [\theta_1, \theta_2, \theta_3, x_C, y_C, \varphi]^T$, so $n = 3$, and $m = 6$.

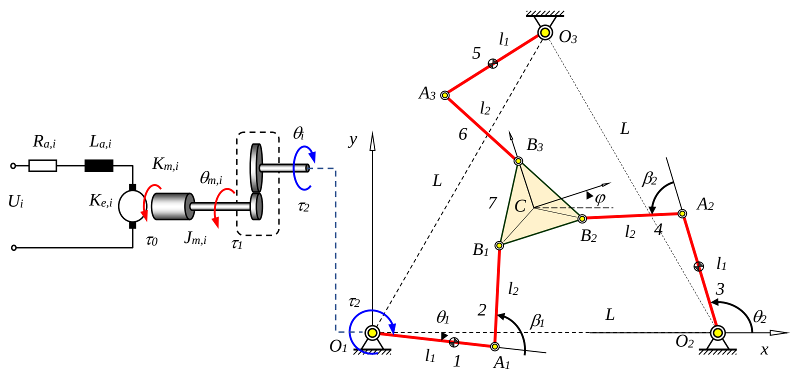


Fig. 1. Model of a 3RRR parallel planar robot driven by electric motors with gearbox

To get a simple model, the connecting link A_iB_i having masses of m_2 is considered as two particles with masses of $0.5m_2$ at two ends of links. The kinetic and potential energy of the mechanical part are given as following

$$T = \frac{1}{2}\sum_{k=1}^{3}(J_{C1} + \frac{1}{4}m_1 l_1^2 + \frac{1}{2}m_2 l_1^2)\dot{\theta}_k^2 + \frac{1}{2}(m_7 + 3\cdot\frac{1}{2}m_2)(\dot{x}_C^2 + \dot{y}_C^2) + \frac{1}{2}\left(J_{C7} + 3\cdot\frac{1}{2}m_2 b^2\right)\dot{\varphi}^2,$$
$$\Pi = 0. \tag{35}$$

By choosing $\mathbf{q}_a = [\theta_1, \theta_2, \theta_3]^T$ and $\mathbf{q} = [\theta_1, \theta_2, \theta_3, x_C, y_C, \varphi]^T$, so the matrices $\mathbf{B}$ and $\mathbf{Z}$ are obtained as

$$\mathbf{B} = \begin{bmatrix} \mathbf{E}_{3\times3} \\ \mathbf{0}_{3\times3} \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} \mathbf{E}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix}.$$

By applying the Lagrangian formulation, the equation of motion written in matrix form of (17) is given as

$$\mathbf{M}_s = \mathrm{diag}([(J_m r^2 + J_{C1} + \frac{1}{4}m_1 l_1^2 + \frac{1}{2}m_2 l_1^2)[1,1,1], (m_7 + 3\cdot\frac{1}{2}m_2)[1,1], J_{C7} + 3\cdot\frac{1}{2}m_2 b^2]),$$

$$\mathbf{C}_s(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}_{6\times6}, \quad \mathbf{D}_s = \mathrm{diag}([c, c, c, 0, 0, 0]), \quad \mathbf{g}_s(\mathbf{q}) = \mathbf{0}_{6\times1},$$

$$\mathbf{B}_s = \begin{bmatrix} k\mathbf{E}_{3\times3} \\ \mathbf{0}_{3\times3} \end{bmatrix}, \quad \mathbf{u} = [u_1, u_2, u_3]^T, \quad \mathbf{\Phi}_q^T(\mathbf{q}) = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{q}}\right)^T, \quad \boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^T.$$

where $k = rK_m R_a^{-1}, c = r^2 K_m R_a^{-1} K_e + d$.

Three constraint equations are given based on the constant length of the second link of each leg as

$$f_i = (\mathbf{r}_{B_i} - \mathbf{r}_{A_i})^T(\mathbf{r}_{B_i} - \mathbf{r}_{A_i}) - l_2^2 = 0, \quad i = 1, 2, 3, \tag{36}$$

where $\mathbf{r}_{A_i} = \begin{bmatrix} x_{O_i} + l_1\cos\theta_i \\ y_{O_i} + l_1\sin\theta_i \end{bmatrix}, \mathbf{r}_{B_i} = \begin{bmatrix} x_C + b\cos(\varphi + \alpha_i) \\ y_C + b\sin(\varphi + \alpha_i) \end{bmatrix}, \alpha_{1,2,3} = \left[\frac{7}{6}\pi, -\frac{1}{6}\pi, \frac{1}{2}\pi\right].$

The constraint equations (36) are rewriten as follows

$$\mathbf{f}(\mathbf{q}) = \mathbf{f}(\boldsymbol{\theta}, \mathbf{x}) = 0, \quad \mathbf{f} \in \mathbb{R}^3,$$

then the Jacobian matrices can be determined as

$$\mathbf{\Phi}_q(\mathbf{q}) = \frac{\partial \mathbf{f}}{\partial \mathbf{q}}, \quad \mathbf{\Phi}_i(\mathbf{q}) = \mathbf{\Phi}_\theta(\mathbf{q}) = \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}}, \quad \mathbf{\Phi}_d(\mathbf{q}) = \mathbf{\Phi}_x(\mathbf{q}) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}.$$

The singularities of this parallel manipulator are all configurations at which a rank of the Jacobian matrices decreases, it means $\det \mathbf{\Phi}_\theta(\mathbf{q}) = 0$ or/and $\det \mathbf{\Phi}_x(\mathbf{q}) = 0$. Geometrically, the singularity is observed whenever the three connecting links are parallel or concurrent, and one of three connecting links is fully extended or fold [38]. More about singularity of this kind of parallel manipulator can be found in [47].

5.2. Simulation results

The parameters of the robot are given as follows [48,49]. The edge length of the base is L_0 = 1.2 m. Mass and length of the legs are:

$$l_{i,1} = 0.581 \text{ m}, m_{i,1} = 2.072 \text{ kg}, J_{C1} = 0.13 \text{ kg.m}^2, l_{i,2} = 0.620 \text{ m}, m_{i,2} = 0.750 \text{ kg}.$$

Moving platform: $L_1 = a = 0.2$ m, $m_7 = 0.978$ kg, $J_{C7} = 0.007$ kg.m^2, $\alpha_{1,2,3} = \left[\frac{7}{6}\pi, -\frac{1}{6}\pi, \frac{1}{2}\pi\right]$.

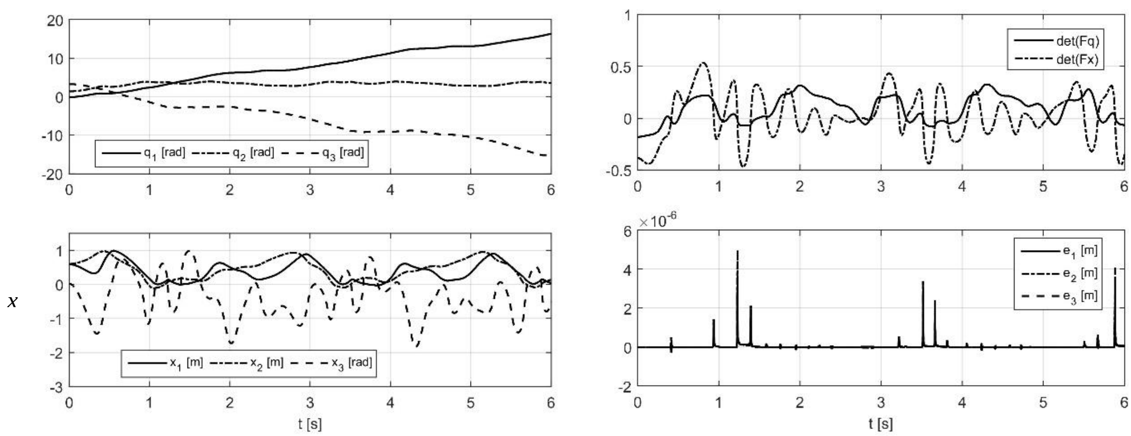
Transmission ratio of the reducer: r = 10.

DC motor's parameters: J_m = 0.01 kg.m^2, K_m = 3.00 Nm/A, K_e = 0.10 Vs/rad, R_a = 3.00 Ohm.

Initial position:

$$\mathbf{q}(0) = [-0.1259, 1.3727, 3.2675, 0.59363, 0.60145, 0.020707]^T.$$

Two simulations are performed, the first case is performed for the forward dynamic problem with the voltage applied to the motors: $u_1 = u_3 = 5V, u_2 = -5V$. In the second simulation, the PD control law with K_p = 150 and K_d = 50 is used to transfer independent coordinates to desired positions $\mathbf{q}_a^d = [0.4, 0.8, 1.0]^T$ [rad]. In these simulations, a combination of Baumgarte's and post-adjusting technique are exploited for stabilization of the constraint equations.

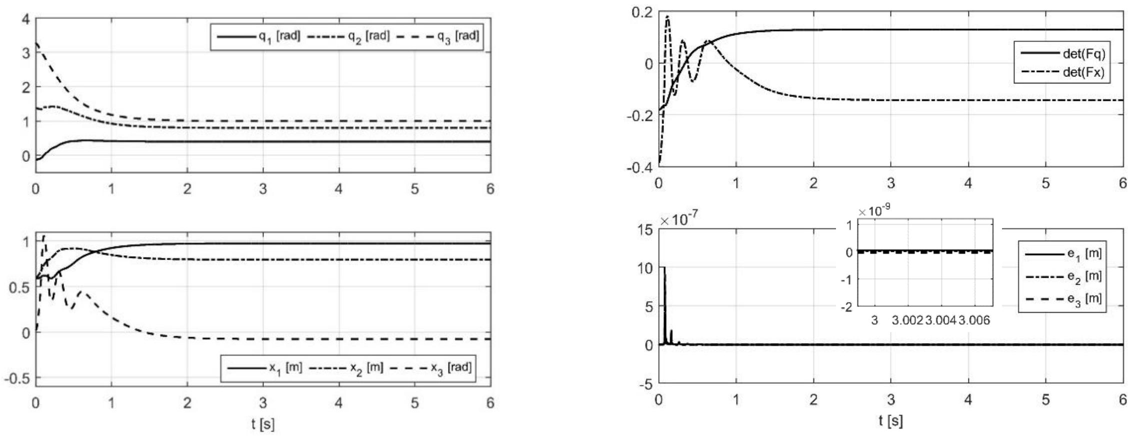


(a) Time history of generalized coordinates $\mathbf{q}$(t)

(b) Time history of $\det(\mathbf{\Phi}_\theta), \det(\mathbf{\Phi}_x)$ and error of constraint equations

Fig. 2. Simulation results of forward dynamic problem with $u_1 = u_3 = 5V, u_2 = -5V$

The parameters δ, ω_0 has a rolle as the PD controller with $k_p = \omega_0^2, k_d = 2\delta\omega_0$. These parameters are also similar to the stiffness and damping coefficients in the damped free vibration system, the value $\delta = 1$ is the critical damping ratio. The investigation of the influence of these parameters on the stability of the method is shown in [50–52]. In this paper, the parameters are selected: $\delta = 1$ and $\omega_0 = 100\sqrt{2}$. Two matrices of the method post-adjusting are chosen as $\mathbf{M} = 0.1\mathbf{I}$ and $\mathbf{A} = 100\mathbf{I}$, with unit matrix $\mathbf{I}$. The optimal parameter selection is not considered in this paper. The simulation results are shown in Figs. 2 and 3.



(a) Time history of generalized coordinates $\mathbf{q}(t)$

(b) Time history of $\det(\mathbf{\Phi}_\theta)$, $\det(\mathbf{\Phi}_x)$ and error of constraint equations

Fig. 3. Simulation results of PD control law with $K_P = 150$ and $K_D = 50$

The time history of generalized coordinates including active joints and position of the mobile platform are smooth. The center of the mobile platform x_1 and x_2 changed in the range $[-0.1, 1]$ m. In this simulation, the manipulator moves several times through singular configurations (Fig. 2(b)), at which the lines of time history of $\det(\mathbf{\Phi}_\theta)$ and $\det(\mathbf{\Phi}_x)$ intersects the abscissa. Fig. 2(b) shows that the errors of the constraint equations increase only at the singular configurations, but they are still small, about 10^{-6}. These errors decrease when the manipulator pass out of the singular configurations.

Fig. 3(a) shows that desired position of three active joints are reached after about 2.5 seconds with a PD control law. Fig. 3(b) shows that only in the first second the manipulator pass seven times through singular configurations [one time $\det(\mathbf{\Phi}_\theta) = 0$ and six times $\det(\mathbf{\Phi}_x) = 0$]. The values of the constraint equations are kept very small, around 10^{-9} m. These values increase only at the singular configurations, but they are also still small, around 10^{-7} m. These two simulations with the planar 3RRR parallel manipulators confirm also the stability of the integration process with the combination of Baumgarte's and post-adjusting techniques.

The Baumgarte's stabilization technique is simple and suitable for real time simulation. The method parameters have affect on error of constraint equations. However, it is not easy to choose the parameters for a given errors. Contrariwise, with the post-adjusting technique the error of constraint equations can be controlled by iterative computing. Because of this iterative computing, the post-adjusting technique is not suitable for real-time simulation.

6. CONCLUSION

In this paper, a singular problem in numerical simulation of a closed loop MBS has been successfully solved by exploiting null space of a Jacobian matrix. Along with the stabilization method Baumgarte's, post-adjusting method is added to ensure that constraints are not broken during the simulation. The limitation of the Baumgarte's technique in stabilization the constraint when the system passes through singularity is treated with post-adjusting method. The solving of the modified problem is based on the objective function optimization along with the penalty factor. The important advantage of using null space is that numerical simulations can be made to continually overcome singular positions without detection of singularity. Furthermore, the electric motors were also integrated in the whole dynamic models. The dynamic model in this study described not only mechanical parts but also electrical one. A 3RRR planar parallel manipulator which has singular configurations in their workspace is chosen for numerical simulation. The simulation results demonstrated the effectiveness of the proposed approach in overcoming the singular configurations and in stabilization of the constraint. The results obtained will be the basis for the development of control laws overcome singular positions in later studies.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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