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NONLOCAL EFFECTS ON RAYLEIGH-TYPE SURFACE WAVE IN A MICROPOLAR PIEZOELECTRIC MEDIUM

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Abstract. The properties of Rayleigh-type surface wave in a linear, homogeneous and transversely isotropic nonlocal micropolar piezoelectric solid half-space are explored. Dispersion relations for Rayleigh-type surface wave are derived for both charge free and electrically shorted cases. Using an algorithm of iteration method in MATLAB software, the wave speed of Rayleigh wave is computed for relevant material constants. The effects of nonlocality, angular frequency, micropolarity and piezoelectricity are illustrated graphically on the propagation speed of Rayleigh wave.

Keywords: nonlocality, microrotation, piezoelectricity, Rayleigh wave, dispersion.

1. INTRODUCTION

In 1885, Lord Rayleigh [1] investigated the existence of waves propagating along the boundary surface of an elastic solid half-space. In existing literature, these waves are known as Rayleigh waves and are widely applicable in the fields of seismology, acoustics, geophysics, telecommunications industry and material science. These waves are also very useful in a variety of transducers which process radar, television and radio signals. Rayleigh-type surface waves were studied extensively by many researchers in different models of elastic half-space with additional fields and parameters. Notable among them are Sveklo [2], Gold [3], Johnson [4], Royer and Dieulesaint [5], Destrade [6], Vinh and Ogden [7], Rehman et al. [8], Abd-Alla et al. [9], Sudheer et al. [10], Kundu et al. [11], Singh and Kaur [12, 13] and Kaur and Singh [14] .

In nonlocal elasticity theory, stress at a point is determined by both stress at that point and spatial derivatives of it. Nonlocal elasticity theory has been widely applied to the bending, vibration, and buckling behaviour of one-dimensional nanostructures including nanobeams, nanorods, and carbon nanotubes. Nonlocal elasticity models have received considerable attention by the researches intending to analyze or design micro/-nano structures. These models extend the main concepts in the classical theory of elasticity to approximate the behaviour of particles, as small as molecules or atoms. Eringen [15] observed that the lack of an internal characteristic length in the classical theory limits the application of this theory in the modeling of physical problems with significant micro-structural effects. Edelen and Laws [16], Edelen et al. [17], Eringen [18, 19], Eringen and Edelen [20] and Eringen [21] developed the theories of nonlocal elasticity characterized by the presence of nonlocality residuals fields like body force, mass, entropy and internal energy. Some dynamical problems based on nonlocal elasticity were studied by various prominent researchers including Chirita [22], Iesan [23], Eringen [24], Altan [25], Wang and Dhaliwal [26] and Eringen [27].

Recently, various researchers explored some wave propagation problems in context of nonlocal elasticity theories. Khurana and Tomar [28–31] studied the nonlocal effects on the properties of plane and surface waves in micropolar and microstretch half-spaces. Roy et al. [32] studied the effect of nonlocality on the Rayleigh-type surface wave in a rotating magneto-elastic half-space. Kaur et al. [33, 34] discussed the nonlocal effects on the properties of Rayleigh and Love waves in an elastic solid with voids. Tung [35] investigated the properties of Rayleigh surface wave in a nonlocal piezoelastic half-space. Singh [36] explored the nonlocal effects on the propagation of Rayleigh waves in a generalized thermoelastic solid half-space with voids. Biswas [37] studied the surface waves in an orthotropic porous medium in context of nonlocal thermoelastcity. Kaur and Singh [38] explored the existence of the Rayleigh-type surface waves along the stress-free surface of an isotropic nonlocal diffusive elastic half-space. Tung [39] studied the nonlocal effects on reflection and transmission of plane waves at an imperfect interface between two orthotropic micropolar half-spaces.

The micropolar theory of elasticity is an extension of classical elasticity with extra independent degrees of freedom for local rotation. Eringen [40–42] introduced the linear theory of micropolar elasticity where the motions of the particles are expressed in terms of displacement and micro-rotation vectors. The theory of micropolar elasticity was further applied by various researchers in piezoelectric materials including Cracium [43], Ciumasu and Vieru [44], Vieru and Ciumasu [45], Zhilin and Kolpakov [46], Aouadi [47] and Gales [48]. Recently, Singh and Sindhu [49, 50], Sangwan et al. [51], Singh et al. [52] and Bijarnia et al. [53] studied the properties of plane and surface waves in micropolar piezoelectric medium. However to the best of author's knowledge, the effect of nonlocality on the properties of Rayleigh-type surface waves in micropolar piezoelectric half-space is not explored yet. The present paper is organized as follows: In context of the nonlocal elasticity theories developed by Eringen [18, 19] and Eringen and Edelen [20] and the theory of micropolar piezoelectricity presented by Aouadi [47], the governing equations for a nonlocal transversely isotropic micropolar piezoelectric medium are specialized in a plane. In Section 3, the two-dimensional governing equations of motion are solved to obtain the general Rayleigh surface wave solutions which decay with depth. In Section 4, the relevant boundary conditions for charge free as well as electrically shorted cases are applied to derive the dispersion relations. In Section 5, some particular cases are discussed which are found in agreement with the earlier published works. In Section 6, the wave speed for one of the modes of Rayleigh wave is computed with MATLAB software for relevant material parameters used in earlier investigations. The effects of nonlocality, angular frequency, piezoelectricity and micropolarity on the wave speed of Rayleigh wave are illustrated graphically. The last section summarizes the theoretical and numerical findings.

2. FUNDAMENTAL EQUATIONS

Following Eringen [18, 19], Eringen and Edelen [20] and Aouadi [47], the fundamental system of field equations for linear theory of nonlocal micropolar piezoelectric material in the absence of body forces and body couples consists of:

- The equations of the motion

$$\sigma_{ji,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{1}$$

$$m_{ik,i} + \varepsilon_{ijk}\sigma_{ij} = \rho j \frac{\partial^2 \varphi_k}{\partial t^2}. \tag{2}$$

- The equations of the electric fields

$$D_{j,j} = q_e, \quad E_k = -\psi_{,k}. \tag{3}$$

- The constitutive equations

$$\left(1 - \tau^2\nabla^2\right) \sigma_{ij} = c_{ijkl}e_{kl} + b_{ijkl}\kappa_{kl} + \lambda_{ijk}E_k, \tag{4}$$

$$\left(1 - \tau^2\nabla^2\right) m_{ij} = b_{klij}e_{kl} + a_{ijkl}\kappa_{kl} + \beta_{ijk}E_k, \tag{5}$$

$$D_k = -\lambda_{ijk}e_{ij} - \beta_{ijk}\kappa_{ij} + \gamma_{kj}E_j. \tag{6}$$

- The geometrical equations

$$e_{ij} = u_{j,i} + \varepsilon_{ijk}\varphi_k, \quad \kappa_{ij} = \varphi_{j,i}, \tag{7}$$

where σ_{ij} is the stress tensor, τ is nonlocal parameter, $\vec{u}$ is the displacement vector, ρ is the mass density, $\vec{\varphi}$ is the microrotation vector, j is the micro-inertia, m_{ij} is the couple stress tensor, ε_{ijk} is the alternating symbol, D_k is the dielectric displacement vector, q_e is the volume charge density, E_j is the electric field vector, ψ is the electrostatic potential, e_{ij} and κ_{ij} are kinematic strain measures and $a_{ijkl}, b_{ijkl}, c_{ijkl}, \lambda_{ijk}, \beta_{ijk}$ and γ_{jk} are constitutive coefficients. The symbol ∇^2 is the Laplace operator. Superposed dot denote partial differentiation with respect to the time t. Subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinates. The constitutive coefficients satisfy the following symmetry relations

$$c_{ijkl} = c_{klij}, \quad a_{ijkl} = a_{klij}, \quad g_{ij} = g_{ji}. \tag{8}$$

We consider a half-space occupying linear, homogeneous, transversely isotropic nonlocal micropolar piezoelectric solid. Within the context of rectangular Cartesian coordinate system $Ox_1x_2x_3$, the boundary surface of the half-space is taken along x_1-x_2 plane and the positive x_3-axis is taken normal into the half-space. The half-space is assumed to be transversely isotropic in such a manner that the plane of isotropy is perpendicular to the x_3-axis. The x_1-axis is chosen in the direction of propagation of waves so that all particles on a line parallel to x_2-axis are equally displaced. Here, all the field quantities will be independent of x_2-coordinate. For a plane deformation parallel to x_1-x_3 plane, we take the components of the displacement and microrotation vector of the form

$$\vec{u} = (u_1, 0, u_3) \quad \text{and} \quad \vec{\varphi} = (0, \varphi_2, 0). \tag{9}$$

With the help of Eq. (9), Eqs. (1) to (8) are written in x_1-x_3 plane as

$$A_{11}\frac{\partial^2 u_1}{\partial x_1^2} + (A_{13}+A_{56})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + A_{55}\frac{\partial^2 u_1}{\partial x_3^2} + K_1\frac{\partial \varphi_2}{\partial x_3} - (\lambda_{15}+\lambda_{31})\frac{\partial^2 \psi}{\partial x_1 \partial x_3} = \rho\left(1-\tau^2\nabla^2\right)\frac{\partial^2 u_1}{\partial t^2}, \tag{10}$$

$$A_{66}\frac{\partial^2 u_3}{\partial x_1^2} + (A_{13}+A_{56})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + A_{33}\frac{\partial^2 u_3}{\partial x_3^2} + K_2\frac{\partial \varphi_2}{\partial x_1} - \lambda_{15}\frac{\partial^2 \psi}{\partial x_1^2} - \lambda_{33}\frac{\partial^2 \psi}{\partial x_3^2} = \rho\left(1-\tau^2\nabla^2\right)\frac{\partial^2 u_3}{\partial t^2}, \tag{11}$$

$$B_{77}\frac{\partial^2 \varphi_2}{\partial x_1^2} + B_{66}\frac{\partial^2 \varphi_2}{\partial x_3^2} - \chi\varphi_2 - K_1\frac{\partial u_1}{\partial x_3} - K_2\frac{\partial u_3}{\partial x_1} - \beta_{14}\frac{\partial^2 \psi}{\partial x_1^2} - \beta_{36}\frac{\partial^2 \psi}{\partial x_3^2} = \rho j\left(1-\tau^2\nabla^2\right)\frac{\partial^2 \varphi_2}{\partial t^2}, \tag{12}$$

$$\lambda_{15}\frac{\partial^2 u_3}{\partial x_1^2} + \lambda_{33}\frac{\partial^2 u_3}{\partial x_3^2} + (\lambda_{15}+\lambda_{31})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \beta_{14}\frac{\partial^2 \varphi_2}{\partial x_1^2} + \beta_{36}\frac{\partial^2 \varphi_2}{\partial x_3^2} + \gamma_{11}\frac{\partial^2 \psi}{\partial x_1^2} + \gamma_{33}\frac{\partial^2 \psi}{\partial x_3^2} = 0, \tag{13}$$

where $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, A_{11} = C_{1111}, A_{55} = C_{3131}, A_{13} = C_{1133} = C_{3311}, A_{56} = C_{3113} = C_{1331}, A_{66} = C_{1313}, A_{33} = C_{3333}, K_1 = A_{56} - A_{55} = C_{3113} - C_{3131}, K_2 = A_{66} - A_{56} = C_{1313} - C_{1331}, \chi = K_2 - K_1, B_{77} = a_{1212}, B_{66} = a_{3232}, \lambda_{31} = \lambda_{311}, \lambda_{33} = \lambda_{333}, \lambda_{15} = \lambda_{131} = \lambda_{113}, \lambda_{35} = \lambda_{313} = \lambda_{331}, \beta_{14} = \beta_{121}, \beta_{36} = \beta_{323}$.

3. RAYLEIGH WAVE SOLUTIONS

For Rayleigh wave propagation on surface along x_1-direction and diminishing quickly with x_3-direction, we consider the solutions of Eqs. (10) to (13) in the following form

$$[u_1, u_3, \varphi_2, \psi](x_1, x_3, t) = [\tilde{u}_1(x_3), \tilde{u}_3(x_3), \tilde{\varphi}_2(x_3), \tilde{\psi}(x_3)]\exp[ik(x_1 - ct)], \tag{14}$$

where $i = \sqrt{-1}$; k is the wave number and c is the phase speed.

Substituting these into Eqs. (10) to (13), we obtain a homogeneous system of four equations in $\tilde{u}_1(x_3), \tilde{u}_3(x_3), \tilde{\varphi}_2(x_3)$ and $\tilde{\psi}(x_3)$ as

$$(AD^2 - Lk^2)\tilde{u}_1 + ikGD\tilde{u}_3 + K_1 D\tilde{\varphi}_2 - ikFD\tilde{\psi} = 0, \tag{15}$$

$$ikGD\tilde{u}_1 + (BD^2 - Nk^2)\tilde{u}_3 + ikK_2\tilde{\varphi}_2 - \left(\lambda_{33}D^2 - \lambda_{15}k^2\right)\tilde{\psi} = 0, \tag{16}$$

$$-K_1 D\tilde{u}_1 - ikK_2\tilde{u}_3 + (CD^2 - Pk^2)\tilde{\varphi}_2 - \left(\beta_{36}D^2 - \beta_{14}k^2\right)\tilde{\psi} = 0, \tag{17}$$

$$ikFD\tilde{u}_1 + (\lambda_{33}D^2 - \lambda_{15}k^2)\tilde{u}_3 + (\beta_{36}D^2 - \beta_{14}k^2)\tilde{\varphi}_2 + (\gamma_{33}D^2 - \gamma_{11}k^2)\tilde{\psi} = 0, \tag{18}$$

where $D = \frac{d}{dx_3}$ and

$$A = A_{55} - \rho\tau^2\omega^2, \;\; B = A_{33} - \rho\tau^2\omega^2, \;\; C = B_{66} - \rho j\tau^2\omega^2,$$

$$L = A_{11} - \rho\tau^2\omega^2 - \rho c^2, \;\; N = A_{66} - \rho\tau^2\omega^2 - \rho c^2,$$

$$P = B_{77} + \frac{\chi}{k^2} - \rho j\tau^2\omega^2 - \rho j c^2, \;\; G = A_{13} + A_{56}, \;\; F = \lambda_{15} + \lambda_{31}.$$

From the above four equations, we obtain

$$[H_0D^8 - H_1D^6 + H_2D^4 - H_3D^2 + H_4](\tilde{u}_1, \tilde{u}_3, \tilde{\varphi}_2, \tilde{\psi}) = 0, \tag{19}$$

where the coefficients H_j $(j = 0, 1, 2, 3, 4)$ are given in Appendix A. A most general solution of Eq. (19) is

$$u_1\left(x_1, x_3, t\right) = \left(\sum_{j=1}^{4} A_j e^{-m_j x_3} + \sum_{j=1}^{4} A_j^* e^{m_j x_3}\right) \exp\left[ik(x_1 - ct)\right], \tag{20}$$

where A_j, A_j^* are constants and m_j are the roots of the equation

$$H_0m^8 - H_1m^6 + H_2m^4 - H_3m^2 + H_4 = 0. \tag{21}$$

Eq. (21) is bi-quadratic equation in m^2 and hence its roots are given by m_j^2 $(j = 1, \ldots, 4)$ such that

$$m_1^2 + m_2^2 + m_3^2 + m_4^2 = \frac{H_1}{H_0}, \quad m_1^2m_2^2 + m_2^2m_3^2 + m_3^2m_4^2 + m_4^2m_1^2 = \frac{H_2}{H_0},$$
$$m_1^2m_2^2m_3^2 + m_2^2m_3^2m_4^2 + m_3^2m_4^2m_1^2 = \frac{H_3}{H_0}, \quad m_1^2m_2^2m_3^2m_4^2 = \frac{H_4}{H_0}.$$

In general, the roots m_j $(j = 1, \ldots, 4)$ are complex. For the propagation of surface waves, we choose only those m_j $(j = 1, \ldots, 4)$ whose real parts are positive and which satisfies the decay conditions $u_1 \to 0, u_3 \to 0, \varphi_2 \to 0, \psi \to 0$ as $x_3 \to \infty$. The particular solutions satisfying the decay conditions in the half-space $(x_3 \geq 0)$ are obtained as

$$u_1\left(x_1, x_3, t\right) = \sum_{j=1}^{4} A_j e^{-m_j x_3} \exp\left[ik(x_1 - ct)\right], \tag{22}$$

$$u_3\left(x_1, x_3, t\right) = \sum_{j=1}^{4} \eta_j A_j e^{-m_j x_3} \exp\left[ik(x_1 - ct)\right], \tag{23}$$

$$\varphi_2\left(x_1, x_3, t\right) = \sum_{j=1}^{4} \xi_j A_j e^{-m_j x_3} \exp\left[ik(x_1 - ct)\right], \tag{24}$$

$$\psi\left(x_1, x_3, t\right) = \sum_{j=1}^{4} \zeta_j A_j e^{-m_j x_3} \exp\left[ik(x_1 - ct)\right], \tag{25}$$

where the coupling coefficients η_j, $\frac{\xi_j}{k}$ and ζ_j $(j = 1, 2, 3, 4)$ between displacement components, microrotation component and electric potential are obtained from Eqs. (15) to (18) and are given in Appendix B.

4. DERIVATION OF DISPERSION RELATIONS

Following Eringen [54], the required boundary conditions at stress-free surface $x_3 = 0$ of nonlocal micropolar piezoelectric half-space are the vanishing of nonlocal normal force stress component, nonlocal tangential force stress component, nonlocal tangential couple stress component, normal dielectric displacement component and electrostatic potential i.e.

$$\sigma_{33} = 0,\ \sigma_{31} = 0,\ m_{32} = 0, \tag{26}$$

$$D_3 = 0, \quad \text{(Charge Free Case)} \tag{27}$$

$$\psi = 0, \quad \text{(Electrically Shorted Case)} \tag{28}$$

where

$$\left(1 - \tau^2\nabla^2\right)\sigma_{33} = A_{13}\frac{\partial u_1}{\partial x_1} + A_{33}\frac{\partial u_3}{\partial x_3} - \lambda_{35}\frac{\partial \psi}{\partial x_1} - \lambda_{33}\frac{\partial \psi}{\partial x_3},$$

$$\left(1 - \tau^2\nabla^2\right)\sigma_{31} = A_{56}\frac{\partial u_3}{\partial x_1} + A_{55}\frac{\partial u_1}{\partial x_3} + K_1\varphi_2 - \lambda_{31}\frac{\partial \psi}{\partial x_1} - \lambda_{35}\frac{\partial \psi}{\partial x_3},$$

$$\left(1 - \tau^2\nabla^2\right)m_{32} = B_{66}\frac{\partial \varphi_2}{\partial x_3} - \beta_{36}\frac{\partial \psi}{\partial x_3},$$

$$D_3 = \lambda_{15}\frac{\partial u_1}{\partial x_1} + \lambda_{33}\frac{\partial u_3}{\partial x_3} + \beta_{36}\frac{\partial \varphi_2}{\partial x_3} + \gamma_{33}\frac{\partial \psi}{\partial x_3}.$$

The particular solutions (22) to (25) satisfy the boundary conditions given by (26) to (28) at stress-free surface $x_3 = 0$ and we obtain the following dispersion relation

$$\begin{vmatrix} \tilde{A}_1 & \tilde{A}_2 & \tilde{A}_3 & \tilde{A}_4 \\ \tilde{B}_1 & \tilde{B}_2 & \tilde{B}_3 & \tilde{B}_4 \\ \tilde{C}_1 & \tilde{C}_2 & \tilde{C}_3 & \tilde{C}_4 \\ \tilde{D}_1 & \tilde{D}_2 & \tilde{D}_3 & \tilde{D}_4 \end{vmatrix} = 0, \tag{29}$$

where

$$\tilde{A}_j = iA_{13} - A_{33}\eta_j\left(\frac{m_j}{k}\right) + \zeta_j\left(\lambda_{33}\frac{m_j}{k} - i\lambda_{35}\right),$$

$$\tilde{B}_j = i\eta_j A_{56} - A_{55}\frac{m_j}{k} + K_1\frac{\xi_j}{k} + \zeta_j\left(\lambda_{35}\zeta_j\frac{m_j}{k} - i\lambda_{31}\right),$$

$$\tilde{C}_j = \beta_{36}\zeta_j\left(\frac{m_j}{k}\right) - kB_{66}\left(\frac{\xi_j}{k}\right)\left(\frac{m_j}{k}\right),$$

$$\tilde{D}_j = i\lambda_{15} - \lambda_{33}\eta_j\left(\frac{m_j}{k}\right) - k\beta_{36}\left(\frac{\xi_j}{k}\right)\left(\frac{m_j}{k}\right) - \gamma_{33}\zeta_j\left(\frac{m_j}{k}\right), \quad \text{(Charge Free Case)}$$

$$\tilde{D}_j = \zeta_j, \quad \text{(Electrically Shorted Case)}$$

Eq. (29) is the frequency equation of Rayleigh waves in a nonlocal transversely isotropic micropolar piezoelectric half-space. The coefficients of frequency Eq. (29) are complex in nature and depend on angular frequency and nonlocal parameter.

5. SPECIAL CASES

1. In the absence of nonlocal parameter i.e. $\tau = 0$, Eq. (29) reduces to the dispersion relations for Rayleigh wave in micropolar piezoelectric medium which agree with Singh and Sindhu [49].

2. In absence of piezoelectric parameters i.e.

$$\lambda_{15} = 0,\ \lambda_{33} = 0,\ \lambda_{31} = 0,\ \lambda_{35} = 0,\ \beta_{14} = 0,\ \beta_{36} = 0,\ \gamma_{11} = 0,\ \gamma_{33} = 0,$$

the dispersion relations in Eq. (29) reduce for the case of Rayleigh wave in transversely isotropic nonlocal micropolar medium which agree with those derived by Khurana and Tomar [30] in absence of transverse isotropy.

3. In absence of micropolarity i.e.

$$B_{77} = 0,\ B_{66} = 0,\ K_1 = K_2 = 0,\ \chi = 0,\ \beta_{14} = 0,\ \beta_{36} = 0,$$
$$A_{11} = c_{11},\ A_{33} = c_{33},\ A_{55} = A_{56} = A_{66} = c_{44},\ A_{13} = c_{13},$$

Eq. (29) reduces to the dispersion relations for Rayleigh wave in nonlocal transversely isotropic piezoelectric medium which agree with Tung [35].

6. NUMERICAL RESULTS AND DISCUSSION

For the purpose of numerical computation of speed of Rayleigh wave, the following relevant material parameters are used (Singh and Sindhu [49,50], Sangwan et al. [51])

$$A_{11} = 17.8 \times 10^{10}\,\mathrm{Nm}^{-2},\ A_{33} = 18.43 \times 10^{10}\,\mathrm{Nm}^{-2},\ A_{13} = 7.59 \times 10^{10}\,\mathrm{Nm}^{-2},$$
$$A_{56} = 1.89 \times 10^{10}\,\mathrm{Nm}^{-2},\ A_{55} = 4.357 \times 10^{10}\,\mathrm{Nm}^{-2},\ A_{66} = 4.42 \times 10^{10}\,\mathrm{Nm}^{-2},$$
$$B_{77} = 0.278 \times 10^{9}\,\mathrm{N},\ B_{66} = 0.268 \times 10^{9}\,\mathrm{N},\ \lambda_{15} = 37\,\mathrm{Cm}^{-2},\ \lambda_{31} = 12\,\mathrm{Cm}^{-2},$$
$$\lambda_{33} = 1.33\,\mathrm{Cm}^{-2},\ \lambda_{35} = 0.23\,\mathrm{Cm}^{-2},\ \beta_{14} = 0.0001\,\mathrm{C},\ \beta_{36} = 0.0002\,\mathrm{C},$$
$$\gamma_{11} = 85.2\,\mathrm{C}^2\mathrm{N}^{-1}\mathrm{m}^{-2},\ \gamma_{33} = 28.7\,\mathrm{C}^2\mathrm{N}^{-1}\mathrm{m}^{-2},\ \rho = 1.74\ \times 10^3\,\mathrm{Kg\,m}^{-3},\ j = 0.196\,\mathrm{m}^2.$$

For above values of physical constants, Eq. (29) is numerically solved to compute the real part of Rayleigh wave speed. An iteration method in MATLAB software is used for numerical solutions of Eq. (29).

To illustrate the effect of frequency on wave speed of Rayleigh wave, the speeds c of Rayleigh wave are plotted against nonlocal parameter τ in Fig. 1 for charge free (CF) case when angular frequency $\omega = 2000$ (solid), 4000 (dotted), 8000 (dashed) respectively. For case $\omega = 2000$, 4000 and 8000, the value of wave speed c at $\tau = 0$ is 10823.12 $\mathrm{m\,s}^{-1}$ and it decreases very sharply to values 10460.15 $\mathrm{m\,s}^{-1}$ at $\tau = 1.225$, 10459.52 $\mathrm{m\,s}^{-1}$ at $\tau = 0.6131$ and 10459.62 $\mathrm{m\,s}^{-1}$ at $\varepsilon = 0.3065$, respectively. It is observed from this figure that the wave does not exist beyond a critical value of τ for a given angular frequency ω. The range of nonlocal parameter τ decreases as ω increases.

To illustrate effect of nonlocal parameter on speed of Rayleigh wave, the speeds c of Rayleigh wave are plotted against angular frequency ω in Fig. 2 and Fig. 3 for charge free (CF) and electrically shorted (ES) cases respectively. In Fig. 2 (for charge free case), the wave speed decreases monotonically against ω for $\tau = 0.1$ (dotted) and $\tau = 0.2$ (dashed). For local case ($\tau = 0$), it remains almost invariant. The variations of speed in Fig. 3 for ES case are found almost similar to those given in Fig. 2. From these figures, it is observed that the range of ω decreases as τ increases.

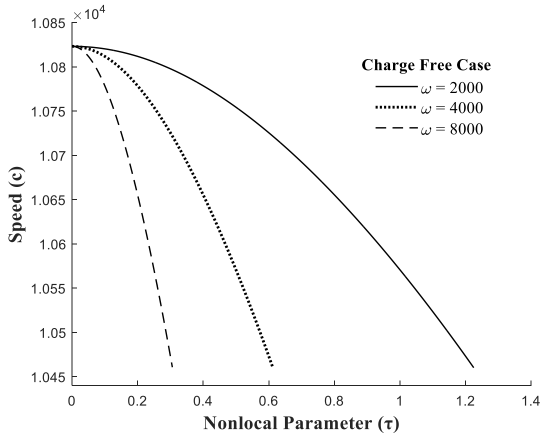

Fig. 1. Variations of wave speed c against nonlocal parameter τ for different values of angular frequency ω

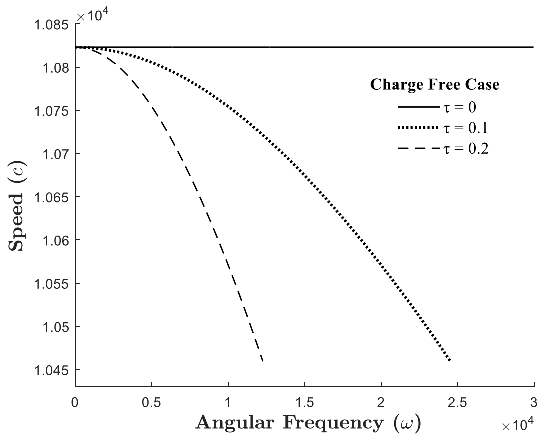

Fig. 2. Variations of wave speed c against angular frequency ω for different values of nonlocal parameter τ in charge free case

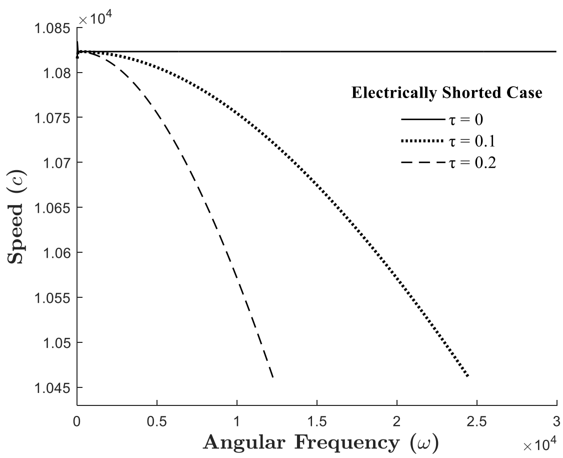

Fig. 3. Variations of wave speed c against angular frequency ω for different values of nonlocal parameter τ in electrically shorted case

The variations of wave speed shown in Fig. 4 and Fig. 5 for different τ are obtained after neglecting piezoelectric and microrotation in Fig. 2 or Fig. 3. The comparisons of speed variations in Fig. 4 with those given in Fig. 2 or Fig. 3 show the effect of piezoelectricity for different values of nonlocal parameter τ. The comparison of speed variations in Fig. 5 with those given in Fig. 2 or Fig. 3 show the effect of microrotation for different values of τ.

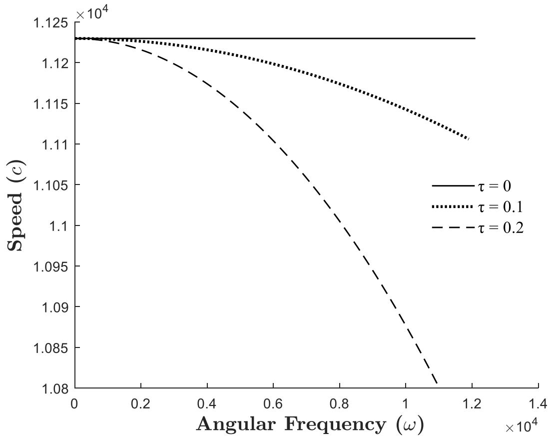

Fig. 4. Variations of wave speed c against angular frequency ω for different values of nonlocal parameter τ in absence of piezoelectric field

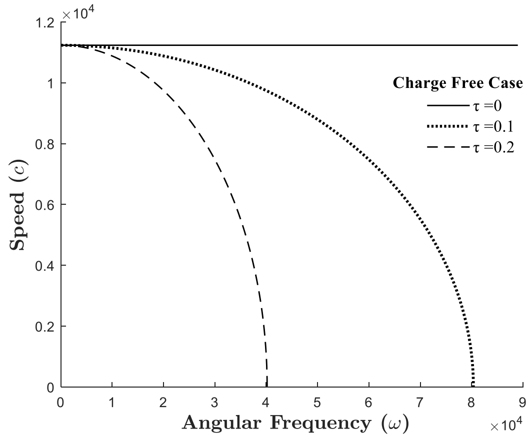

Fig. 5. Variations of wave speed c against angular frequency ω for different values of nonlocal parameter τ in absence of microrotation

7. CONCLUSIONS

A theoretical analysis of Rayleigh wave on a boundary of a nonlocal micropolar piezoelectric half-space is performed. The general surface wave solutions are obtained which decay with the depth. The dispersion relations are obtained for charge free as well as electrically shorted cases. The dispersion relations show the relations between wave speed, nonlocality, angular frequency, piezoelectric and micropolar parameters. For given angular frequency, nonlocal, piezoelectric and micropolar parameters, the wave speed is computed. From the numerical simulations and graphical illustrations, the effects of nonlocality, piezoelectricity and micropolarity are observed and some specific observations are made as follows:

- The speed variations of the Rayleigh wave are found similar for charge free as well as electrically cases.

- As the value of nonlocal parameter τ or angular frequency ω increases, the Rayleigh wave speed slows down at an increasing rate. The wave does not appear beyond a critical value of τ or ω. The critical value of τ depends on the value of angular frequency ω and it decreases with an increase in value of angular frequency. In a similar manner, the critical value of ω depends on the value of nonlocal parameter τ.

- In absence of piezoelectric or micropolar parameters, the Rayleigh wave speed becomes fast at each value of angular frequency ω for different nonlocal parameter.

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APPENDIX A

The expressions for $H_j\,(j=0,1,2,3,4)$ are

$$
\begin{aligned}
H_0 &= ABC\gamma_{33} + AB\beta_{36}^2 + AC\lambda_{33}^2,\\
H_1 &= [ABC\gamma_{11} + (BCL + ACN + ABP - CG^2)\gamma_{33} + 2AC\lambda_{15}\lambda_{33} + 2AB\beta_{14}\beta_{36} - 2CGF\lambda_{33}\\
&\quad + (BL + AN - G^2)\beta_{36}^2 + (CL + AP)\lambda_{33}^2 + BCF^2]\,k^2 - K_1^2\lambda_{33}^2 - BK_1^2\gamma_{33},\\
H_2 &= [(ABP + ACN + BCL - CG^2)\gamma_{11} + (ANP + BLP + CLN - PG^2)\gamma_{33} + 2(AP + CL)\lambda_{15}\lambda_{33}\\
&\quad + 2(AN + BL - G^2)\beta_{14}\beta_{36} - 2CGF\lambda_{15} - 2PGF\lambda_{33} + AB\beta_{14}^2 + LN\beta_{36}^2 + AC\lambda_{15}^2 + LP\lambda_{33}^2\\
&\quad + (BP + CN)F^2]k^4 + [2K_1K_2G\gamma_{33} + 2K_1K_2F\lambda_{33} - 2K_1^2\lambda_{15}\lambda_{33} - AK_2^2\gamma_{33} - BK_1^2\gamma_{11} - NK_1^2\gamma_{33}]k^2,\\
H_3 &= [(ANP + BLP + CLN - PG^2)\gamma_{11} + LNP\gamma_{33} + 2LP\lambda_{15}\lambda_{33} + 2LN\beta_{14}\beta_{36} - 2PGF\lambda_{15}\\
&\quad + (BL + AN - G^2)\beta_{14}^2 + (AP + CL)\lambda_{15}^2 + NPF^2]k^6 + [2K_1K_2F\lambda_{15} + 2K_1K_2G\gamma_{11}\\
&\quad - LK_2^2\gamma_{33} - (NK_1^2 + AK_2^2)\gamma_{11} - K_1^2\lambda_{15} - K_2^2F^2]k^4,\\
H_4 &= [LNP\gamma_{11} + LP\lambda_{15}^2 + LN\beta_{14}^2]k^8 - LK_2^2\gamma_{11}k^6.
\end{aligned}
$$

APPENDIX B

The expressions of the coupling coefficients η_h, $\dfrac{\xi_h}{k}$ and $\zeta_h\,(h=1,2,3,4)$ obtained from Eqs. (15) to (18) given as

$$
\eta_h = \frac{(\alpha_{1h}\alpha_{2h} - \alpha_{3h}\alpha_{6h})}{(\alpha_{2h}\alpha_{4h} + \alpha_{3h}\alpha_{5h})},\quad \zeta_h = \frac{(\alpha_{1h} - \alpha_{4h}\eta_h)}{\alpha_{3h}},\quad \frac{\xi_h}{k} = \frac{\left[\alpha_{7h} - i\left(G\eta_h - F\zeta_h\right)\left(\frac{m_h}{k}\right)\right]}{K_1\left(\frac{m_h}{k}\right)},
$$

where

$$
\begin{aligned}
\alpha_{1h} &= i\left(AK_2 - GK_1\right)\left(\frac{m_h}{k}\right)^2 - iK_2L,\\
\alpha_{2h} &= \left\{\beta_{36}\left(\frac{m_h}{k}\right)^2 - \beta_{14}\right\}^2 + \left\{\gamma_{33}\left(\frac{m_h}{k}\right)^2 - \gamma_{11}\right\}\left\{C\left(\frac{m_h}{k}\right)^2 - P\right\},\\
\alpha_{3h} &= \left(FK_2 - \lambda_{15}K_1\right)\left(\frac{m_h}{k}\right) + \lambda_{33}K_1\left(\frac{m_h}{k}\right)^3,\\
\alpha_{4h} &= \left(NK_1 - MK_2\right)\left(\frac{m_h}{k}\right) - BK_1\left(\frac{m_h}{k}\right)^3,\\
\alpha_{5h} &= -i\left(\frac{K_2}{k}\right)\left\{\beta_{36}\left(\frac{m_h}{k}\right)^2 - \beta_{14}\right\} - \left\{\lambda_{33}\left(\frac{m_h}{k}\right)^2 - \lambda_{15}\right\}\left\{C\left(\frac{m_h}{k}\right)^2 - P\right\},\\
\alpha_{6h} &= \left(\frac{K_1}{k}\right)\left(\frac{m_h}{k}\right)\left\{\beta_{36}\left(\frac{m_h}{k}\right)^2 - \beta_{14}\right\} + iF\left(\frac{m_h}{k}\right)\left\{C\left(\frac{m_h}{k}\right)^2 - P\right\},\\
\alpha_{7h} &= A\left(\frac{m_h}{k}\right)^2 - L.
\end{aligned}
$$