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A UNIFIED FORMULATION OF VARIOUS SHELL THEORIES FOR THE ANALYSIS OF LAMINATED COMPOSITE SPHERICAL SHELLS

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Abstract. This study investigates the static and free vibration responses of orthotropic laminated composite spherical shells using various refined shear deformation theories. Displacement-based refined shear deformation theories are presented herein for the analysis of laminated composite spherical shells via unified mathematical formulations. Equations of motion associated with the present theory are derived within the framework of Hamilton's principle. Analytical solutions for the static and free vibration problems of laminated spherical shells are obtained using Navier's technique for the simply supported boundary conditions. Few higher order and classical theories are recovered from the present unified formulation; however, many other theories can be recovered from the present unified formulation. The numerical results are obtained for symmetric as well as anti-symmetric laminated shells. The present results are compared with previously published results and 3-D elasticity solution. From the numerical results, it is concluded that the present theories are in good agreement with other higher order theories and 3-D solutions.

Keywords: refined shell theories, shear deformation, laminated shells, static analysis, free vibration.

1. INTRODUCTION

The orthotropic laminated shells are used in many engineering structures due to their higher strength-to-weight and stiffness-to-weight ratios. Laminated shells are often subjected to static and dynamic loading. Therefore, static and free vibration analysis of laminated shells is an active area of research in the last few decades. Transverse shear deformation plays an important role in the accurate static and vibration analysis of orthotropic laminated shells. Laminated shells are analyzed using the classical theories [1, 2] and higher order refined theories. Since classical theories are inaccurate to predict the accurate structural response, refined theories are required for the analysis of laminated shells which take into account the effects of transverse shear deformations. Refer review articles of Sayyad and Ghugal [3], Sayyad and Ghugal [4], Qatu [5, 6], and Qatu et al. [7]. Three-dimensional elasticity solutions for isotropic and laminated shells are presented by Bhimaraddi [8] and, Bhimaraddi and Chandrashekhara [9]. Reddy [10] and, Reddy and Liu [11] have presented the static and free vibration analysis of laminated composite shells using third order shear deformation theory which satisfies the traction free boundary conditions. A unified formulation of refined theories for the static and free vibration analysis of laminated cylindrical shells is presented by Soldatos and Timarci [12], and Timarci and Soldatos [13]. Pradyumna and Bandyopadhyay [14] presented static and free vibration analysis of laminated composite shells using a finite element method based on higher-order shear deformation theory. Mantari and Soares [15,16] and Mantari et al. [17, 18] presented deflection and frequency analysis of laminated composite shells using non-polynomial shear deformation theories. Khare et al. [19] and Garg et al. [20] have presented higher order shear and normal deformation theory for the static and free vibration analysis of laminated composite shells. Sayyad and Ghugal [21–24] have developed a trigonometric shear deformation theory for the static and free vibration analysis of laminated composite plates which is recently extended by Sayyad and Ghugal [25] for the static and free vibration analysis of functionally graded sandwich curved beams. Carrera and Brischetto [26,27] and Carrera et al. [28,29] have applied Carrera's unified formulation (CUF) for the mechanical and thermal analysis of laminated composite and sandwich shells. Tornabene [30,31] have presented a vibration analysis of laminated composite shells resting on elastic foundation using the first-order shear deformation theory and the generalized differential quadrature (GDQ) method. Tornabene et al. [32, 33] have presented the static analysis of doubly curved anisotropic shell panels using Carrera's unified formulation and the differential quadrature method. Tornabene et al. [34], Viola et al. [35], Brischetto and Tornabene [36] have presented a new procedure to recover interlaminar stresses in laminated composite and sandwich shells and panels. Sayyad and Ghugal [37] have developed a unified formulation for the static and free vibration analysis of laminated composite plates and shells. Naik and Sayyad [38, 39] and, Shinde and Sayyad [40, 41] have developed a fifth-order shear and normal deformation theory for the static analysis of laminated composite plates/shells subjected to mechanical or thermal loadings. Monge et al. [42] have presented bending analysis of doubly curved laminated shells using refined hybrid models based on Navier's technique. The objectives of the present study are as follows.

The first objective of this study is to present mathematical formulations of various polynomial and non-polynomial type refined shell theories via unified formulation. The parabolic shell theory (PST), trigonometric shell theory (TST), hyperbolic shell theory (HST), exponential shell theory (EST), first-order shell theory (FST), and classical shell theory (CST) are recovered from the present unified formulation. The second objective of the present study is to present deflection, stresses and fundamental frequencies of orthotropic laminated shells. The third objective of this study is to recover transverse shear stresses in orthotropic laminated shells from equilibrium equations of the 3-D elasticity problem. Governing equations of motion are derived within the framework of Hamilton's principle. Navier's technique is employed to obtain analytical solutions for simply supported shells. The present results are compared with exact elasticity solutions and found in good agreement. Authors dedicate this paper to Professor J.N. Reddy on the Occasion of his 75th birthday and his outstanding contribution in the area of mechanics of laminated plates and shells [43–45].

2. THE MATHEMATICAL MODELING

Fig. 1 shows a laminated shell under consideration in the Cartesian coordinate system, x and y curves in the figure represent lines of principal curvature; R_1 and R_2 are the principal radii of curvature. A laminated shell is made up of fibrous composite material and composed of several layers. It is assumed that all layers of laminated shells are perfectly bonded. The top surface of the laminates shell is subjected to transverse load q (x, y).

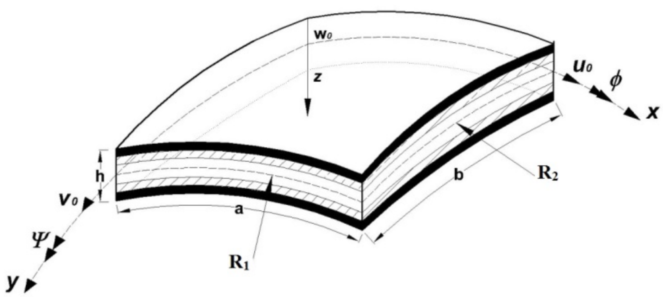

Fig. 1. Laminated shell under consideration

A mathematical formulation of various refined theories is presented in this study using a unified displacement field

$$\begin{aligned} u\left(x,y,z,t\right) &= \left(1+\frac{z}{R_1}\right)u_0\left(x,y,t\right) - z\frac{\partial w_0}{\partial x} + f\left(z\right)\phi\left(x,y,t\right), \\ v\left(x,y,z,t\right) &= \left(1+\frac{z}{R_2}\right)v_0\left(x,y,t\right) - z\frac{\partial w_0}{\partial y} + f\left(z\right)\psi\left(x,y,t\right), \\ w\left(x,y,t\right) &= w_0\left(x,y,t\right), \end{aligned} \tag{1}$$

where u, v and w are the displacements of any arbitrary point within the shell domain in the x-, y- and z-directions, respectively; u_0, v_0, w_0 are the displacements of a point on the mid-plane of the shell domain in the x-, y- and z-directions, respectively. $f(z)$ represents shape functions which ensures traction free boundary conditions on the top and bottom surfaces of the shell. Various higher order shear deformation theories (HSDTs) and classical theories are recovered from the present unified displacement field after selecting following shape functions

PST: $f\left(z\right) = z - \left(4/3\right)\left(z^3/h^2\right)$,

TST: $f\left(z\right) = \left(h/\pi\right)\sin\left(\pi z/h\right)$,

HST: $f\left(z\right) = z\cosh\left(1/2\right) - h\sinh\left(z/h\right)$,

EST: $f\left(z\right) = z\,e^{-2\left(z^2/h^2\right)}$,

FST: $f\left(z\right) = z$,

CST: $f\left(z\right) = 0$.

Using the strain-displacement relations from the theory of elasticity, the following nonzero strain quantities are obtained

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} + \frac{w}{R_1} = \frac{\partial u_0}{\partial x} + \frac{w_0}{R_1} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x},\\
\varepsilon_y &= \frac{\partial v}{\partial y} + \frac{w}{R_2} = \frac{\partial v_0}{\partial y} + \frac{w_0}{R_2} - z\frac{\partial^2 w_0}{\partial y^2} + f(z)\frac{\partial \psi}{\partial y},\\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} + f(z)\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x},\\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u_0}{R_1} = f'(z)\,\phi,\\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{v_0}{R_2} = f'(z)\,\psi.
\end{aligned} \tag{2}$$

The in-plane normal and shear stresses for the k^{th} lamina are obtained using the following constitutive relations

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^{(k)}, \tag{3}$$

where Q_{ij} are the stiffness coefficients

$$\begin{gathered}
Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{12} = \frac{\mu_{21}E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}},\\
Q_{66} = G_{12}, \quad Q_{55} = G_{13}, \quad Q_{44} = G_{23}.
\end{gathered} \tag{4}$$

Stress resultants associated with the present unified displacement field are obtained by integrating stresses over the thickness of the shell

$$\begin{aligned}
\left\{N_x \quad M_x^b \quad M_x^s\right\}^T &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left\{1 \quad z \quad f(z)\right\}^T \sigma_x^k \,\mathrm{d}z,\\
\left\{N_y \quad M_y^b \quad M_y^s\right\}^T &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left\{1 \quad z \quad f(z)\right\}^T \sigma_y^k \,\mathrm{d}z,\\
\left\{N_{xy} \quad M_{xy}^b \quad M_{xy}^s\right\}^T &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left\{1 \quad z \quad f(z)\right\}^T \tau_{xy}^k \,\mathrm{d}z,\\
\left\{Q_x \quad Q_y\right\}^T &= \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \left\{\tau_{xz} \quad \tau_{yz}\right\}^T f'(z)\,\mathrm{d}z,
\end{aligned} \tag{5}$$

where (N_x, N_y, N_{xy}) are the axial force, $\left(M_x^b, M_y^b, M_{xy}^b\right)$ and $\left(M_x^s, M_y^s, M_{xy}^s\right)$ are the bending moments, and (Q_x, Q_y) are the shear forces. A superscript 'b' is corresponding to classical bending and superscript 's' is the corresponding refinement due to shear deformation. The equations of motion are derived from Hamilton's principle

$$\int_{t_1}^{t_2} (\delta U - \delta V + \delta K)\,\mathrm{d}t = 0, \tag{6}$$

where δU is the strain energy, δV is the potential energy and δK is the kinetic energy. The symbol δ is a variational operator; t_1 and t_2 are the initial and final times. Substituting values of all energies, Eq. (6) leads to the following form

$$\int_{dv} (\sigma_x \delta \varepsilon_x + \tau_{zx} \delta \gamma_{xz})\,\mathrm{d}v - \int_{\Omega}^{q} (x,y)\,\delta w\,\mathrm{d}\Omega + \rho \int_{dv} \left(\frac{\partial^2 u}{\partial t^2}\,\delta u + \frac{\partial^2 v}{\partial t^2}\,\delta v + \frac{\partial^2 w}{\partial t^2}\,\delta w \right) \mathrm{d}v = 0, \tag{7}$$

The equations of motion of the present refined theories are obtained by integrating the Eq. (7) by parts and setting the coefficients of unknown variables equal to zero

$$\begin{aligned} &A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + (A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x \partial y} + \left(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2}\right)\frac{\partial w_0}{\partial x} - B_{11}\frac{\partial^3 w_0}{\partial x^3} \\ &\quad - (B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x \partial y^2} + As_{11}\frac{\partial^2 \phi}{\partial x^2} + As_{66}\frac{\partial^2 \phi}{\partial y^2} + (As_{12}+As_{66})\frac{\partial^2 \psi}{\partial x \partial y} \\ &\quad - \left(I_1 + 2\frac{I_2}{R_1} + \frac{I_3}{R_1^2}\right)\frac{\partial^2 u_0}{\partial t^2} + \left(I_2 + \frac{I_3}{R_1}\right)\frac{\partial^3 w_0}{\partial x \partial t^2} - \left(I_4 + \frac{I_5}{R_1}\right)\frac{\partial^2 \phi}{\partial t^2} = 0, \end{aligned} \tag{8}$$

$$\begin{aligned} &(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x \partial y} + A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} - B_{22}\frac{\partial^3 w_0}{\partial y^3} - (B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x^2 \partial y} \\ &\quad + \left(\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2}\right)\frac{\partial w_0}{\partial y} + (As_{12}+As_{66})\frac{\partial^2 \phi}{\partial x \partial y} + As_{22}\frac{\partial^2 \psi}{\partial y^2} + As_{66}\frac{\partial^2 \psi}{\partial x^2} \\ &\quad - \left(I_1 + 2\frac{I_2}{R_2} + \frac{I_3}{R_2^2}\right)\frac{\partial^2 v_0}{\partial t^2} + \left(I_2 + \frac{I_3}{R_2}\right)\frac{\partial^3 w_0}{\partial y \partial t^2} - \left(I_4 + \frac{I_5}{R_2}\right)\frac{\partial^2 \psi}{\partial t^2} = 0, \end{aligned} \tag{9}$$

$$
\begin{aligned}
& B_{11}\frac{\partial^3 u_0}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 u_0}{\partial x\partial y^2}-\left(\frac{A_{11}}{R_1}+\frac{A_{12}}{R_2}\right)\frac{\partial u_0}{\partial x}+B_{22}\frac{\partial^3 v_0}{\partial y^3}+(B_{12}+2B_{66})\frac{\partial^3 v_0}{\partial x^2\partial y}\\
& -\left(\frac{A_{12}}{R_1}+\frac{A_{22}}{R_2}\right)\frac{\partial v_0}{\partial y}+\left(\frac{2B_{11}}{R_1}+\frac{2B_{12}}{R_2}\right)\frac{\partial^2 w_0}{\partial x^2}-D_{11}\frac{\partial^4 w_0}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0}{\partial y^4}\\
& +\left(\frac{2B_{12}}{R_1}+\frac{2B_{22}}{R_2}\right)\frac{\partial^2 w_0}{\partial y^2}-\left(\frac{A_{11}}{R_1^2}+2\frac{A_{12}}{R_1R_2}+\frac{A_{22}}{R_2^2}\right)w_0+Bs_{11}\frac{\partial^3\phi}{\partial x^3}+(Bs_{12}+2Bs_{66})\frac{\partial^3\phi}{\partial x\partial y^2}\\
& -\left(\frac{As_{11}}{R_1}+\frac{As_{12}}{R_2}\right)\frac{\partial\phi}{\partial x}+Bs_{22}\frac{\partial^3\psi}{\partial y^3}+(Bs_{12}+2Bs_{66})\frac{\partial^3\psi}{\partial x^2\partial y}-\left(\frac{As_{12}}{R_1}+\frac{As_{22}}{R_2}\right)\frac{\partial\psi}{\partial y}-\left(I_2+\frac{I_3}{R_1}\right)\frac{\partial^3 u_0}{\partial x\partial t^2}\\
& +I_3\frac{\partial^4 w_0}{\partial x^2\partial t^2}-I_5\frac{\partial^3\phi}{\partial x\partial t^2}-\left(I_2+\frac{I_3}{R_2}\right)\frac{\partial^3 v_0}{\partial y\partial t^2}+I_3\frac{\partial^4 w_0}{\partial y^2\partial t^2}-I_5\frac{\partial^3\psi}{\partial y\partial t^2}-I_1\frac{\partial^2 w_0}{\partial t^2}-q=0,
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
& As_{11}\frac{\partial^2 u_0}{\partial x^2}+As_{66}\frac{\partial^2 u_0}{\partial y^2}+(As_{12}+As_{66})\frac{\partial^2 v_0}{\partial x\partial y}-Bs_{11}\frac{\partial^3 w_0}{\partial x^3}-(Bs_{12}+2Bs_{66})\frac{\partial^3 w_0}{\partial x\partial y^2}\\
& +\left(\frac{As_{11}}{R_1}+\frac{As_{12}}{R_2}\right)\frac{\partial w_0}{\partial x}+Ass_{11}\frac{\partial^2\phi}{\partial x^2}+Ass_{66}\frac{\partial^2\phi}{\partial y^2}-Acc_{55}\phi+(Ass_{12}+Ass_{66})\frac{\partial^2\psi}{\partial x\partial y}\\
& -\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial^2 u_0}{\partial t^2}+I_5\frac{\partial^3 w_0}{\partial x\partial t^2}-I_6\frac{\partial^2\phi}{\partial t^2}=0,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
& (As_{12}+As_{66})\frac{\partial^2 u_0}{\partial x\partial y}+As_{66}\frac{\partial^2 v_0}{\partial x^2}+As_{22}\frac{\partial^2 v_0}{\partial y^2}-Bs_{22}\frac{\partial^3 w_0}{\partial y^3}-(Bs_{12}+2Bs_{66})\frac{\partial^3 w_0}{\partial x^2\partial y}\\
& +\left(\frac{As_{12}}{R_1}+\frac{As_{22}}{R_2}\right)\frac{\partial w_0}{\partial y}+(Ass_{12}+Ass_{66})\frac{\partial^2\phi}{\partial x\partial y}+Ass_{22}\frac{\partial^2\psi}{\partial y^2}+Ass_{66}\frac{\partial^2\psi}{\partial x^2}-Acc_{55}\psi\\
& -\left(I_4+\frac{I_5}{R_2}\right)\frac{\partial^2 v_0}{\partial t^2}+I_5\frac{\partial^3 w_0}{\partial y\partial t^2}-I_6\frac{\partial^2\psi}{\partial t^2}=0,
\end{aligned}
\tag{12}
$$

where $A_{ij}, B_{ij}, D_{ij}, As_{ij}, Bs_{ij}, Ass_{ij}, Acc_{ij}$ are the shell stiffnesses

$$
\begin{aligned}
\left\{A_{ij}\quad B_{ij}\quad D_{ij}\right\} &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} Q_{ij}^k\left\{1\quad z\quad z^2\right\}\mathrm{d}z,\\
\left\{As_{ij}\quad Bs_{ij}\quad Ass_{ij}\right\} &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} Q_{ij}^k f(z)\left\{1\quad z\quad f(z)\right\}\mathrm{d}z,\\
Acc_{ij} &= \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} Q_{ij}^k\ \left[f'(z)\right]^2\,\mathrm{d}z.
\end{aligned}
\tag{13}
$$

3. NAVIER'S SOLUTIONS

Analytical solutions for the static and free vibration problems are obtained using Navier's technique for the simply supported boundary conditions stated in Eq. (14)

$$\left\{\begin{array}{c} N_x \\ v_0 \\ w \\ \psi \\ M_x^b \\ M_x^s \end{array}\right\}_{x=0,a} = 0 \text{ and } \left\{\begin{array}{c} N_y \\ u_0 \\ w \\ \phi \\ M_y^b \\ M_y^s \end{array}\right\}_{y=0,b} = 0. \tag{14}$$

It is assumed that the shell is subjected to transverse load on the top surface. This load is expressed using a double trigonometric series

$$q\left(x,y\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn} \sin\alpha x \sin\beta y, \tag{15}$$

where, $\alpha = m\pi/a, \beta = n\pi/b, q_{mn}$ is Fourier coefficient of transverse load, (m,n) are odd positive integers and q_0 is the maximum intensity of load. In the case of sinusoidally distributed loading $q_{mn} = q_0$ and $m = n = 1$. As per Navier's technique, the unknown variables are also presented in the double trigonometric forms which satisfy the simply supported boundary conditions exactly

$$\left\{\begin{array}{c} u_0 \\ v_0 \\ w_0 \\ \phi \\ \psi \end{array}\right\} = \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty} \left\{\begin{array}{l} u_{mn} \cos\alpha x \sin\beta y \\ v_{mn} \sin\alpha x \cos\beta y \\ w_{mn} \sin\alpha x \sin\beta y \\ \phi_{mn} \cos\alpha x \sin\beta y \\ \psi_{mn} \sin\alpha x \cos\beta y \end{array}\right\} \sin\omega t, \tag{16}$$

where ω represents the natural frequency, and $u_{mn}, v_{mn}, w_{mn}, \phi_{mn}, \psi_{mn}$ are the unknowns. Substitution of Eqs. (15) and (13) into Eqs. (8)–(12) leads to the following systems of equations.

Static analysis:

$$[K]\left\{\Delta\right\} = \left\{F\right\}. \tag{17}$$

Free vibration analysis:

$$\left\{[K] - \omega^2 [M]\right\}\left\{\Delta\right\} = \left\{0\right\}, \tag{18}$$

where $[K]$, $[M]$, $\{\Delta\}$ and $\{F\}$ are the stiffness matrix, mass matrix, a vector of unknowns and force vector, respectively.

Elements of stiffness matrix $[K]$ are as follows

$$\begin{aligned}
K_{11} &= A_{11}\alpha^2 + A_{66}\beta^2, \quad K_{12} = (A_{12} + A_{66})\,\alpha\beta, \\
K_{13} &= -\left[\frac{A_{11}}{R_1}\alpha + \frac{A_{12}}{R_2}\alpha + B_{11}\alpha^3 - (B_{12} + 2B_{66})\,\alpha\beta^2\right], \\
K_{14} &= As_{11}\alpha^2 + As_{66}\beta^2, \quad K_{15} = (As_{12} + As_{66})\,\alpha\beta, \\
K_{22} &= A_{66}\alpha^2 + A_{22}\beta^2, \\
K_{23} &= -\left[\frac{A_{12}}{R_1}\beta + \frac{A_{22}}{R_2}\beta + B_{22}\beta^3 - (B_{12} + 2B_{66})\,\alpha^2\beta\right], \\
K_{24} &= (As_{12} + As_{66})\,\alpha\beta, \quad K_{25} = As_{66}\alpha^2 + As_{22}\beta^2, \\
K_{33} &= D_{11}\alpha^4 + 2\,(D_{12} + 2D_{66})\,\alpha^2\beta^2 + D_{22}\beta^4 + \left(\frac{2B_{11}}{R_1} + \frac{2B_{12}}{R_2}\right)\alpha^2 \\
&\quad + \left(\frac{2B_{12}}{R_1} + \frac{2B_{22}}{R_2}\right)\beta^2 + \left(\frac{A_{11}}{R_1^2} + 2\frac{A_{12}}{R_1 R_2} + \frac{A_{22}}{R_2^2}\right) w_0, \\
K_{34} &= -\left[Bs_{11}\alpha^3 + (Bs_{12} + 2Bs_{66})\,\alpha\beta^2 + \left(\frac{As_{11}}{R_1} + \frac{As_{12}}{R_2}\right)\alpha\right], \\
K_{35} &= -\left[Bs_{22}\beta^3 + (Bs_{12} + 2Bs_{66})\,\alpha^2\beta + \left(\frac{As_{12}}{R_1} + \frac{As_{22}}{R_2}\right)\beta\right], \\
K_{44} &= Ass_{11}\alpha^2 + Ass_{66}\beta^2 + Acc_{55}, \\
K_{45} &= (Ass_{12} + Ass_{66})\,\alpha\beta, \quad K_{55} = Ass_{22}\beta^2 + Ass_{66}\alpha^2 + Acc_{55}.
\end{aligned} \tag{19}$$

The stiffness matrix $[K]$ is a symmetric matrix. Elements of mass matrix $[M]$ are

$$\begin{aligned}
&M_{11} = \left(I_1 + 2\frac{I_2}{R_1} + \frac{I_3}{R_1^2}\right), \quad M_{12} = 0, \quad M_{13} = -\left(I_2 + \frac{I_3}{R_1}\right)\alpha, \\
&M_{14} = \left(I_4 + \frac{I_5}{R_1}\right), \quad M_{15} = 0, \quad M_{22} = \left(I_1 + 2\frac{I_2}{R_2} + \frac{I_3}{R_2^2}\right), \\
&M_{23} = -\left(I_2 + \frac{I_3}{R_2}\right), \quad M_{24} = 0, \quad M_{25} = \left(I_4 + \frac{I_5}{R_2}\right), \quad M_{33} = I_3\alpha^2 + I_3\beta^2 + I_1, \\
&M_{34} = -I_5\alpha, \quad M_{35} = -I_5\beta, \quad M_{44} = I_6, \quad M_{45} = 0, \quad M_{55} = I_6,
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\{I_1 \quad I_2 \quad I_3\} &= \sum_{k=1}^{N} \rho^k \int_{h_k}^{h_{k+1}} \{1 \quad z \quad z^2\}\,\mathrm{d}z, \\
\{I_4 \quad I_5 \quad I_6\} &= \sum_{k=1}^{N} \rho^k \int_{h_k}^{h_{k+1}} f(z)\,\{1 \quad z \quad f(z)\}\,\mathrm{d}z,
\end{aligned} \tag{21}$$

$$\{\Delta\} = \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{mn} \\ \phi_{mn} \\ \psi_{mn} \end{Bmatrix} \quad \text{and} \quad \{F\} = \begin{Bmatrix} 0 \\ 0 \\ q_0 \\ 0 \\ 0 \end{Bmatrix}. \tag{22}$$

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the accuracy of the various refined shell theories has been checked for the static and free vibration responses of laminated spherical shells only. Deflection and stresses are determined for sinusoidally distributed loadings. Modulus of elasticity and Poisson's ratio for the isotropic shells are assumed as ($E =$) 210 GPa and ($\mu =$) 0.3, respectively. In the case of the orthotropic and laminated spherical shells, the following material properties are used

$$\frac{E_1}{E_2} = 25, \quad \frac{E_3}{E_2} = 1, \quad \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5, \quad \frac{G_{23}}{E_2} = 0.2, \quad \mu_{12} = 0.25. \tag{23}$$

To maintain the continuity of interlaminar stresses at the layer interface, transverse shear stresses of k^{th} lamina are evaluated from 3-D stress equilibrium equations of elasticity neglecting the body forces

$$\tau_{xz}^{(k)} = -\sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left(\frac{\partial \sigma_x^{(k)}}{\partial x} + \frac{\partial \tau_{xy}^{(k)}}{\partial y} \right) \mathrm{d}z + C_1 \text{ and } \tau_{yz}^{(k)} = -\sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left(\frac{\partial \sigma_y^{(k)}}{\partial y} + \frac{\partial \tau_{xy}^{(k)}}{\partial x} \right) \mathrm{d}z + C_2. \tag{24}$$

The constants of integrations can be determined by imposing the continuity and boundary conditions at the appropriate locations. The following non-dimensional forms are used to present the numerical results

$$\bar{w} = \frac{wE_2}{q_0}, \quad \bar{\sigma}_x = \frac{\sigma_x h^2}{q_0 a^2}, \quad \bar{\tau}_{xz} = \frac{\tau_{xz} h}{q_0 a}. \tag{25}$$

Table 1 shows the comparison of central deflection of isotropic and orthotropic spherical shells ($R_1 = R_2 = R$) subjected to sinusoidally distributed loadings, respectively. Central deflections of the middle surface of the shell ($x = a/2, y = b/2, z = 0$) are obtained for various h/a and R/a ratios. The numerical results obtained by using the present refined shell theories (PST, TST, HST, EST, FST, CST) are compared with the 3-D elasticity solution given by Bhimaraddi [8]. Examination of Table 1 reveals that the present results are in good agreement with those obtained by 3-D elasticity solution. It

Table 1. Comparison of central deflection ($\bar{w}$) for isotropic and orthotropic spherical shells with different h/a and R/a ratios ($a/b = 1, R_1 = R_2 = R$)

R/a	Theory	Isotropic			Orthotropic		
		$h/a = 0.01$	$h/a = 0.1$	$h/a = 0.15$	$h/a = 0.01$	$h/a = 0.1$	$h/a = 0.15$
1	3-D	100.59	8.7095	4.9497	75.397	4.7117	2.5641
	PST	99.652	7.4756	3.8931	74.120	3.4471	1.7229
	TST	99.652	7.4754	3.8928	74.206	3.4536	1.7204
	HST	99.652	7.4756	3.8931	74.206	3.4553	1.7231
	EST	99.652	7.4759	3.8935	74.206	3.4525	1.7143
	FST	99.652	7.4588	3.8626	74.205	3.3561	1.6322
	CST	99.644	7.3702	3.6979	74.200	2.7447	1.0191
2	3-D	396.45	18.451	7.7240	285.72	5.9693	2.6788
	PST	394.40	17.014	6.9265	282.32	5.2610	2.3180
	TST	394.40	17.013	6.9254	282.32	5.2571	2.3134
	HST	394.40	17.014	6.9265	282.32	5.2612	2.3183
	EST	394.40	17.016	6.9276	282.32	5.2547	2.3025
	FST	394.40	16.927	6.8306	282.30	5.0346	2.1567
	CST	394.37	16.480	6.3322	282.24	3.7736	1.2015
3	3-D	875.36	23.381	8.5912	593.43	6.2215	2.6635
	PST	872.07	22.278	8.0945	587.39	5.8248	2.4764
	TST	872.07	22.277	8.0930	587.39	5.8199	2.4711
	HST	872.07	22.278	8.0945	587.39	5.8249	2.4767
	EST	872.07	22.281	8.0960	587.39	5.8170	2.4587
	FST	872.07	22.129	7.9639	587.32	5.5485	2.2931
	CST	872.00	21.371	7.2945	587.00	4.0551	1.2427
4	3-D	1518.3	25.785	8.9235	953.25	6.3014	2.6494
	PST	1513.7	24.984	8.6021	944.66	6.0517	2.5371
	TST	1513.7	24.982	8.6005	944.66	6.0465	2.5315
	HST	1513.7	24.984	8.6021	944.66	6.0519	2.5374
	EST	1513.7	24.987	8.6039	944.66	6.0433	2.5185
	FST	1513.7	24.797	8.4548	944.49	5.7541	2.3451
	CST	1513.6	23.849	7.7043	943.66	4.1638	1.2578
5	3-D	2301.4	27.061	9.0755	1325.5	6.3332	2.6393
	PST	2295.5	26.472	8.8593	1314.8	6.1629	2.5662
	TST	2295.5	26.470	8.8576	1314.8	6.1575	2.5605
	HST	2295.5	26.472	8.8593	1314.8	6.1631	2.5665
	EST	2295.5	26.476	8.8612	1314.8	6.1542	2.5472
	FST	2295.5	26.262	8.7031	1314.4	5.8544	2.3699
	CST	2295.3	25.201	7.9099	1312.9	4.2161	1.2649

is observed from Table 1 that the central deflection increases as h/a and R/a ratios increase. The central deflection of the shell decreases with increase in the h/a ratio whereas increases with an increase in R/a ratio. For the same dimensions and loading conditions, orthotropic shells predict less deflection compared to isotropic shells. The FST and CST underestimate the transverse central deflections for all h/a and R/a ratios due to neglecting transverse shear deformation.

Table 2 shows a comparison of central deflections for laminated composite spherical shells subjected to sinusoidally distributed loadings. Central deflections are obtained for anti-symmetric two-ply $(0^\circ/90^\circ)$ laminated shells and symmetric three-ply $(0^\circ/90^\circ/0^\circ)$ laminated shells. Examination of Table 2 shows that the symmetric lamination scheme $(0^\circ/90^\circ/0^\circ)$ of laminated shells predicts low central deflection compared to anti-symmetric $(0^\circ/90^\circ)$ laminated shells. This is due to the presence of the bending-stretching coupling effect in the anti-symmetric laminated shells. The transverse central deflections predicted by the present refined theories are in close agreement with the exact 3-D elasticity solution. In both the laminated shells, classical theories (FST and CST) underestimate the transverse central deflections due to the absence of transverse shear deformation. Figs. 2–7 show through-the-thickness distributions of in-plane

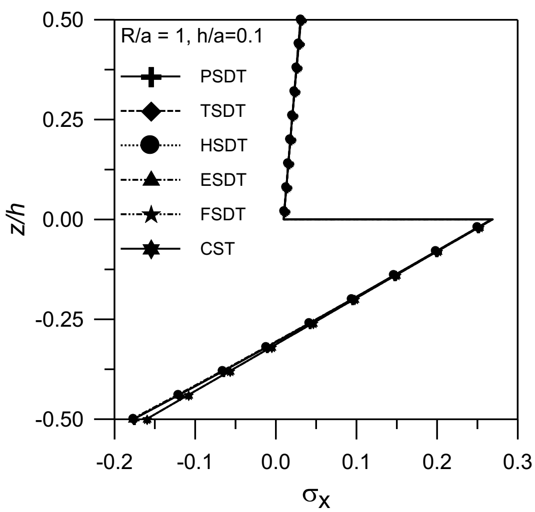

Fig. 2. Through-the-thickness distribution of in-plane normal stress $(\bar{\sigma}_x)$ for two-ply $(0^\circ/90^\circ)$ laminated spherical shell

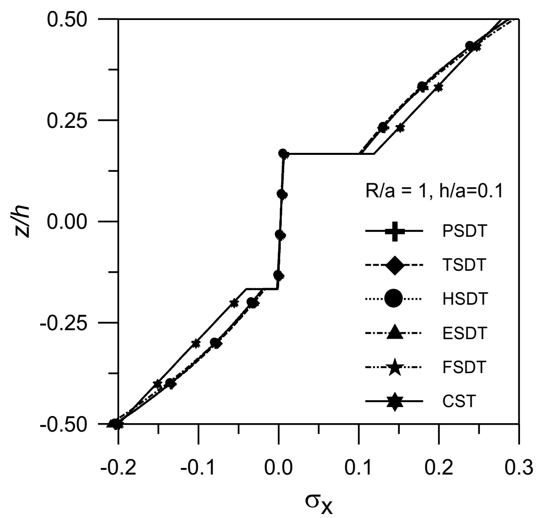

Fig. 3. Through-the-thickness distribution of in-plane normal stress $(\bar{\sigma}_x)$ for three-ply $(0^\circ/90^\circ/0^\circ)$ laminated spherical shell

Table 2. Comparison of central deflection ($\bar{w}$) for laminated spherical shells with different h/a and R/a ratios ($a/b = 1, R_1 = R_2 = R$)

R/a	Theory	Two-ply (0°/90°)			Three-ply (0°/90°/0°)		
		$h/a = 0.01$	$h/a = 0.1$	$h/a = 0.15$	$h/a = 0.01$	$h/a = 0.1$	$h/a = 0.15$
1	3-D	54.129	4.6920	2.7386	54.252	4.0811	2.4345
	PST	53.503	3.7683	1.9578	53.490	3.0770	1.6564
	TST	53.503	3.7674	1.9563	53.490	3.0872	1.6649
	HST	53.503	3.7656	1.9534	53.490	3.0760	1.6556
	EST	53.503	3.7535	1.9330	53.490	3.0948	1.6673
	FST	53.499	3.8167	2.0495	53.489	2.9135	1.5014
	CST	53.493	3.5718	1.6769	53.486	2.4008	0.9438
2	3-D	212.33	8.8092	3.8190	208.36	6.3134	3.0931
	PST	210.83	7.8108	3.2481	206.33	5.3616	2.5253
	TST	210.83	7.8054	3.2432	206.33	5.3927	2.5451
	HST	210.83	7.7989	3.2357	206.33	5.3587	2.5235
	EST	210.83	7.7360	3.1737	206.33	5.4159	2.5508
	FST	210.82	8.0897	3.5484	206.32	4.8842	2.1818
	CST	210.79	7.1163	2.5835	206.27	3.5965	1.1739
3	3-D	456.46	10.512	4.0856	441.81	6.9888	3.2228
	PST	462.92	9.7471	3.6996	438.22	6.2163	2.7970
	TST	462.92	9.7383	3.6930	438.22	6.2582	2.8213
	HST	462.92	9.7286	3.6834	438.22	6.2124	2.7948
	EST	462.92	9.6277	3.6019	438.22	6.2894	2.8283
	FST	462.90	10.205	4.1042	438.21	5.5835	2.3817
	CST	462.82	8.7192	2.8709	437.92	3.9619	1.2294
4	3-D	799.81	11.263	4.1758	727.62	7.7476	3.2605
	PST	796.05	10.673	3.8888	722.36	6.5837	2.9065
	TST	796.05	10.662	3.8815	722.36	6.6307	2.9327
	HST	796.05	10.651	3.8709	722.36	6.5793	2.9041
	EST	796.05	10.528	3.7803	722.36	6.6657	2.9403
	FST	796.04	11.233	4.3423	722.35	5.8781	2.4606
	CST	795.86	9.4655	2.9872	721.54	4.1080	1.2501
5	3-D	1198.7	11.639	4.2131	1039.0	7.3674	3.2736
	PST	1193.6	11.164	3.9830	1032.1	6.7688	2.9601
	TST	1193.6	11.152	3.9754	1032.1	6.8185	2.9873
	HST	1193.6	11.139	3.9643	1032.1	6.7642	2.9576
	EST	1193.6	11.005	3.8691	1032.1	6.8556	2.9952
	FST	1193.5	11.783	4.4621	1032.0	6.0252	2.4989
	CST	1193.3	9.8559	3.0443	1030.4	4.1794	1.2599

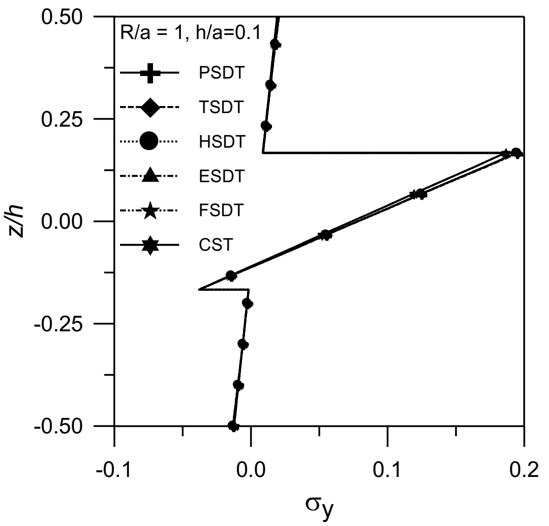

Fig. 4. Through-the-thickness distribution of in-plane normal stress ($\bar{\sigma}_y$) for three-ply ($0^\circ/90^\circ/0^\circ$) laminated spherical shell

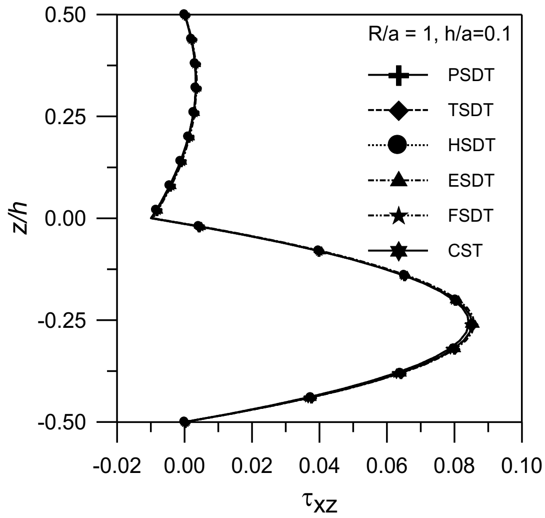

Fig. 5. Through-the-thickness distribution of transverse shear stress ($\bar{\tau}_{xz}$) for two-ply ($0^\circ/90^\circ$) laminated spherical shell

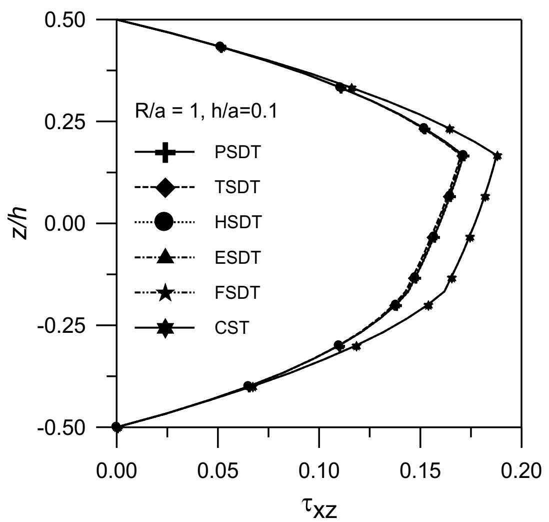

Fig. 6. Through-the-thickness distribution of transverse shear stress ($\bar{\tau}_{xz}$) for three-ply ($0^\circ/90^\circ/0^\circ$) laminated spherical shell

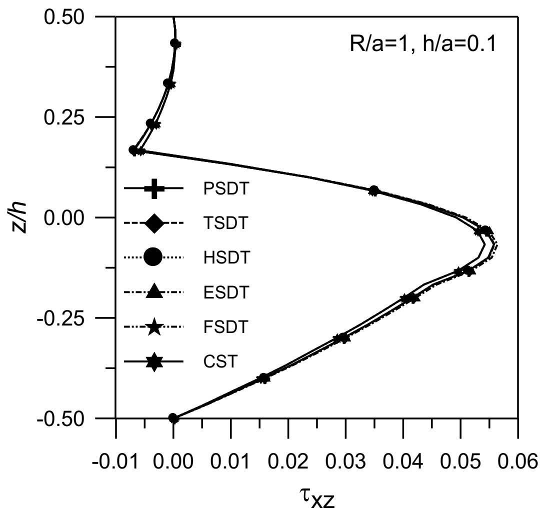

Fig. 7. Through-the-thickness distribution of transverse shear stress ($\bar{\tau}_{yz}$) for three-ply ($0^\circ/90^\circ/0^\circ$) laminated spherical shell

and transverse stresses of laminated composite spherical shells under sinusoidally distributed load. Maximum stresses are observed in 0° layers whereas minimum stresses are observed in 90° layers.

Table 3. Non-dimensional fundamental frequencies $(\bar{\omega})$ of laminated composite spherical shells with different h/a and R/a ratios $(h/a = 0.1, a/b = 1, R_1 = R_2 = R)$

R/a	Theory	Isotropic	Orthotropic	$0°/90°$	$0°/90°/0°$
1	PST	0.17764	16.0735	14.930	16.5745
	TST	0.17764	16.0704	14.930	16.5473
	HST	0.17765	16.0849	14.932	16.5769
	EST	0.17766	16.1080	14.942	16.5618
	FST	0.17782	16.3042	14.958	17.0261
	CST	0.17877	17.9837	15.268	18.7091
2	PST	0.12151	13.5405	10.988	13.3159
	TST	0.12151	13.5365	10.981	13.2776
	HST	0.12152	13.5552	10.983	13.3193
	EST	0.12154	13.5850	11.008	13.2982
	FST	0.12181	13.8371	11.007	13.9454
	CST	0.12336	15.9408	11.466	16.2058
3	PST	0.10681	12.9646	9.9408	12.5105
	TST	0.10681	12.9603	9.9445	12.4688
	HST	0.10682	12.9802	9.9496	12.5142
	EST	0.10685	13.0119	9.9713	12.4912
	FST	0.10715	13.2790	9.9651	13.1945
	CST	0.10896	15.4918	10.475	15.6192
4	PST	0.10107	12.7525	9.5370	12.2064
	TST	0.10106	12.7482	9.5418	12.1633
	HST	0.10108	12.7684	9.5464	12.2103
	EST	0.10111	12.8009	9.5696	12.1866
	FST	0.10143	13.0739	9.5629	12.9125
	CST	0.10336	15.3282	10.091	15.4017
5	PST	0.09828	12.6524	9.3420	12.0613
	TST	0.09827	12.6480	9.3472	12.0175
	HST	0.09828	12.6685	9.3529	12.0653
	EST	0.09829	12.7013	9.3754	12.0412
	FST	0.09868	12.9771	9.3670	12.7782
	CST	1.10064	15.2513	9.9097	15.2988

Table 3 shows a comparison of fundamental frequencies for isotropic, orthotropic and laminated composite spherical shells. The fundamental frequencies are presented in the following non-dimensional forms.

$$\text{Isotropic Shells: } \bar{\omega} = \omega h \sqrt{\frac{\rho}{G}},$$

$$\text{Laminated Shells: } \bar{\omega} = \omega \frac{a^2}{h} \sqrt{\frac{\rho}{E_2}}.$$

The numerical results obtained using the present refined theories. It is observed that CST overestimates the fundamental frequencies whereas refined theories predict the more or less same results.

5. CONCLUSIONS

The static and free vibration responses of orthotropic laminated composite shells are investigated in this study using various refined theories. Analytical solutions are obtained using Navier's technique for the simply supported boundary conditions. The present results are compared with the exact 3-D elasticity solution and found in good agreement with those. Based on the numerical results and discussion presented, the following conclusions are drawn:

- The non-dimensional central deflection values are higher for anti-symmetric lamination shells and lower for the symmetric laminated shells.

- The values of non-dimensional central deflection increases with increase in radii of curvature and decreases with increase in thickness to length (h/a) ratios.

- Through-the-thickness distributions of stresses show that maximum values of stresses are observed in 0° layers whereas minimum values of stresses are observed in 90° layers.

- Non-dimensional fundamental frequencies decrease with increase in the radii of curvature to length ratio (R/a).

The present refined theories can be applied for the static and free vibration analysis of other type of shells such as angle-ply laminated shells, sandwich shells, functionally graded shells, etc.

REFERENCES

[1] G. R. Kirchhoff. Über das Gleichgewicht und die Bewegung einer elastischen Scheibe. *Journal für die reine und angewandte Mathematik (Crelles Journal)*, **1850**, (1850), pp. 51–88. https://doi.org/10.1515/crll.1850.40.51.

[2] R. D. Mindlin. Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates. *Journal of Applied Mechanics*, **18**, (1951), pp. 31–38. https://doi.org/10.1115/1.4010217.

[3] A. S. Sayyad and Y. M. Ghugal. On the free vibration analysis of laminated composite and sandwich plates: A review of recent literature with some numerical results. *Composite Structures*, **129**, (2015), pp. 177–201. https://doi.org/10.1016/j.compstruct.2015.04.007.

[4] A. S. Sayyad and Y. M. Ghugal. Bending, buckling and free vibration of laminated composite and sandwich beams: A critical review of literature. *Composite Structures*, **171**, (2017), pp. 486–504. https://doi.org/10.1016/j.compstruct.2017.03.053.

[5] M. S. Qatu. Recent research advances in the dynamic behavior of shells: 1989-2000, Part 1: Laminated composite shells. *Applied Mechanics Reviews*, **55**, (2002), pp. 325–350. https://doi.org/10.1115/1.1483079.

[6] M. S. Qatu. Recent research advances in the dynamic behavior of shells: 1989–2000, Part 2: Homogeneous shells. *Applied Mechanics Reviews*, **55**, (2002), pp. 415–434. https://doi.org/10.1115/1.1483078.

[7] M. S. Qatu, E. Asadi, and W. Wang. Review of recent literature on static analyses of composite shells: 2000-2010. *Open Journal of Composite Materials*, **02**, (03), (2012), pp. 61–86. https://doi.org/10.4236/ojcm.2012.23009.

[8] A. Bhimaraddi. Three-dimensional elasticity solution for static response of orthotropic doubly curved shallow shells on rectangular planform. *Composite Structures*, **24**, (1993), pp. 67–77. https://doi.org/10.1016/0263-8223(93)90056-v.

[9] A. Bhimaraddi and K. Chandrashekhara. Three-dimensional elasticity solution for static response of simply supported orthotropic cylindrical shells. *Composite Structures*, **20**, (1992), pp. 227–235. https://doi.org/10.1016/0263-8223(92)90028-b.

[10] J. N. Reddy. Exact solutions of moderately thick laminated shells. *Journal of Engineering Mechanics*, **110**, (1984), pp. 794–809. https://doi.org/10.1061/(asce)0733-9399(1984)110:5(794).

[11] J. N. Reddy and C. F. Liu. A higher-order shear deformation theory of laminated elastic shells. *International Journal of Engineering Science*, **23**, (1985), pp. 319–330. https://doi.org/10.1016/0020-7225(85)90051-5.

[12] K. P. Soldatos and T. Timarci. A unified formulation of laminated composite, shear deformable, five-degrees-of-freedom cylindrical shell theories. *Composite Structures*, **25**, (1993), pp. 165–171. https://doi.org/10.1016/0263-8223(93)90162-j.

[13] T. Timarci and K. P. Soldatos. Comparative dynamic studies for symmetric cross-ply circular cylindrical shells on the basis of a unified shear deformable shell theory. *Journal of Sound and Vibration*, **187**, (1995), pp. 609–624. https://doi.org/10.1006/jsvi.1995.0548.

[14] S. Pradyumna and J. N. Bandyopadhyay. Static and free vibration analyses of laminated shells using a higher-order theory. *Journal of Reinforced Plastics and Composites*, **27**, (2007), pp. 167–186. https://doi.org/10.1177/0731684407081385.

[15] J. L. Mantari and C. G. Soares. Analysis of isotropic and multilayered plates and shells by using a generalized higher-order shear deformation theory. *Composite Structures*, **94**, (2012), pp. 2640–2656. https://doi.org/10.1016/j.compstruct.2012.03.018.

[16] J. L. Mantari and C. G. Soares. Optimized sinusoidal higher order shear deformation theory for the analysis of functionally graded plates and shells. *Composites Part B: Engineering*, **56**, (2014), pp. 126–136. https://doi.org/10.1016/j.compositesb.2013.07.027.

[17] J. L. Mantari, A. S. Oktem, and C. G. Soares. Static and dynamic analysis of laminated composite and sandwich plates and shells by using a new higher-order shear deformation theory. *Composite Structures*, **94**, (2011), pp. 37–49. https://doi.org/10.1016/j.compstruct.2011.07.020.

[18] J. L. Mantari, A. S. Oktem, and C. G. Soares. Bending and free vibration analysis of isotropic and multilayered plates and shells by using a new accurate higher-order shear deformation theory. *Composites Part B: Engineering*, **43**, (2012), pp. 3348–3360. https://doi.org/10.1016/j.compositesb.2012.01.062.

[19] R. K. Khare, V. Rode, A. K. Garg, and S. P. John. Higher-order closed-form solutions for thick laminated sandwich shells. *Journal of Sandwich Structures & Materials*, **7**, (2005), pp. 335–358. https://doi.org/10.1177/1099636205050260.

[20] A. K. Garg, R. K. Khare, and T. Kant. Higher-order closed-form solutions for free vibration of laminated composite and sandwich shells. *Journal of Sandwich Structures & Materials*, **8**, (2006), pp. 205–235. https://doi.org/10.1177/1099636206062569.

[21] A. S. Sayyad and Y. M. Ghugal. Effect of stress concentration on laminated plates. *Journal of Mechanics*, **29**, (2012), pp. 241–252. https://doi.org/10.1017/jmech.2012.131.

[22] A. S. Sayyad and Y. M. Ghugal. Flexure of cross-ply laminated plates using equivalent single layer trigonometric shear deformation theory. *Structural Engineering and Mechanics*, **51**, (5), (2014), pp. 867–891. https://doi.org/10.12989/sem.2014.51.5.867.

[23] A. S. Sayyad and Y. M. Ghugal. On the buckling of isotropic, transversely isotropic and laminated composite rectangular plates. *International Journal of Structural Stability and Dynamics*, **14**, (2014). https://doi.org/10.1142/s0219455414500205.

[24] A. S. Sayyad and Y. M. Ghugal. A new shear and normal deformation theory for isotropic, transversely isotropic, laminated composite and sandwich plates. *International Journal of Mechanics and Materials in Design*, **10**, (2014), pp. 247–267. https://doi.org/10.1007/s10999-014-9244-3.

[25] A. S. Sayyad and Y. M. Ghugal. A sinusoidal beam theory for functionally graded sandwich curved beams. *Composite Structures*, **226**, (2019). https://doi.org/10.1016/j.compstruct.2019.111246.

[26] E. Carrera and S. Brischetto. Analysis of thickness locking in classical, refined and mixed theories for layered shells. *Composite Structures*, **85**, (2008), pp. 83–90. https://doi.org/10.1016/j.compstruct.2007.10.009.

[27] E. Carrera and S. Brischetto. A comparison of various kinematic models for sandwich shell panels with soft core. *Journal of Composite Materials*, **43**, (2009), pp. 2201–2221. https://doi.org/10.1177/0021998309343716.

[28] E. Carrera, M. Cinefra, and F. A. Fazzolari. Some results on thermal stress of layered plates and shells by using unified formulation. *Journal of Thermal Stresses*, **36**, (2013), pp. 589–625. https://doi.org/10.1080/01495739.2013.784122.

[29] E. Carrera, M. Cinefra, A. Lamberti, and M. Petrolo. Results on best theories for metallic and laminated shells including Layer-Wise models. *Composite Structures*, **126**, (2015), pp. 285–298. https://doi.org/10.1016/j.compstruct.2015.02.027.

[30] F. Tornabene. Free vibrations of anisotropic doubly-curved shells and panels of revolution with a free-form meridian resting on Winkler–Pasternak elastic foundations. *Composite Structures*, **94**, (2011), pp. 186–206. https://doi.org/10.1016/j.compstruct.2011.07.002.

[31] F. Tornabene. RETRACTED: Free vibrations of laminated composite doubly-curved shells and panels of revolution via the GDQ method. *Computer Methods in Applied Mechanics and Engineering*, **200**, (2011), pp. 931–952. https://doi.org/10.1016/j.cma.2010.11.017.

[32] F. Tornabene, N. Fantuzzi, E. Viola, and E. Carrera. Static analysis of doubly-curved anisotropic shells and panels using CUF approach, differential geometry and differential quadrature method. *Composite Structures*, **107**, (2014), pp. 675–697. https://doi.org/10.1016/j.compstruct.2013.08.038.

[33] F. Tornabene, N. Fantuzzi, M. Bacciocchi, and E. Viola. Accurate inter-laminar recovery for plates and doubly-curved shells with variable radii of curvature using layer-wise theories. *Composite Structures*, **124**, (2015), pp. 368–393. https://doi.org/10.1016/j.compstruct.2014.12.062.

[34] F. Tornabene, N. Fantuzzi, M. Bacciocchi, and J. N. Reddy. A posteriori stress and strain recovery procedure for the static analysis of laminated shells resting on nonlinear elastic foundation. *Composites Part B: Engineering*, **126**, (2017), pp. 162–191. https://doi.org/10.1016/j.compositesb.2017.06.012.

[35] E. Viola, F. Tornabene, and N. Fantuzzi. Static analysis of completely doubly-curved laminated shells and panels using general higher-order shear deformation theories. *Composite Structures*, **101**, (2013), pp. 59–93. https://doi.org/10.1016/j.compstruct.2013.01.002.

[36] S. Brischetto and F. Tornabene. Advanced GDQ models and 3d stress recovery in multilayered plates, spherical and double-curved panels subjected to

transverse shear loads. *Composites Part B: Engineering*, **146**, (2018), pp. 244–269. https://doi.org/10.1016/j.compositesb.2018.04.019.

[37] A. S. Sayyad and Y. M. Ghugal. A unified five-degree-of-freedom theory for the bending analysis of softcore and hardcore functionally graded sandwich beams and plates. *Journal of Sandwich Structures & Materials*, **23**, (2), (2021), pp. 473–506. https://doi.org/10.1177/1099636219840980.

[38] A. S. Sayyad and N. S. Naik. New displacement model for accurate prediction of transverse shear stresses in laminated and sandwich rectangular plates. *Journal of Aerospace Engineering*, **32**, (2019). https://doi.org/10.1061/(asce)as.1943-5525.0001074.

[39] N. S. Naik and A. S. Sayyad. 2d analysis of laminated composite and sandwich plates using a new fifth-order plate theory. *Latin American Journal of Solids and Structures*, **15**, (2018). https://doi.org/10.1590/1679-78254834.

[40] B. M. Shinde and A. S. Sayyad. Analysis of laminated and sandwich spherical shells using a new higher-order theory. *Advances in Aircraft and Spacecraft Science*, **7**, (1), (2020), pp. 19–40. https://doi.org/10.12989/aas.2020.7.1.019.

[41] B. M. Shinde and A. S. Sayyad. Thermoelastic analysis of laminated composite and sandwich shells considering the effects of transverse shear and normal deformations. *Journal of Thermal Stresses*, **43**, (2020), pp. 1234–1257. https://doi.org/10.1080/01495739.2020.1786484.

[42] J. Monge, J. Mantari, J. Yarasca, and R. Arciniega. Bending response of doubly curved laminated composite shells using hybrid refined models. *IOP Conference Series: Materials Science and Engineering*, **473**, (2019). https://doi.org/10.1088/1757-899x/473/1/012006.

[43] J. N. Reddy. *Mechanics of laminated composite plates and shells: Theory and analysis*. CRC Press, second edition, (2003). https://doi.org/10.1201/b12409.

[44] J. N. Reddy. *Theory and analysis of elastic plates and shells*. CRC Press, second edition, (2006). https://doi.org/10.1201/9780849384165.

[45] J. N. Reddy. *An introduction to nonlinear finite element analysis*. Oxford University Press, second edition, (2014).