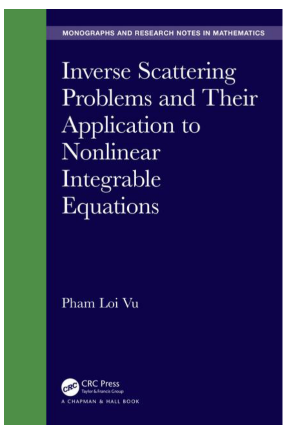
Vietnam Journal of Mechanics, VAST, Vol. 42, No. 2 (2020), pp. 206 – 208

BOOK ANNOUNCEMENT

INVERSE SCATTERING PROBLEMS AND THEIR APPLICATION TO NONLINEAR INTEGRABLE EQUATIONS by Pham Loi Vu

The monograph entitled "Inverse Scattering Problems and Their Application to Nonlinear Integrable Equations" has been published by CRC–Press Taylor Francis Group in November 2019 at the website: https://www.taylorfrancis.com/books/9780429328459. This monograph is devoted to inverse scattering problems for differential equations and their application to nonlinear evolution equations. It contains most of published papers of Prof. Pham Loi Vu, which are mentioned in the Introduction and at the beginning of every chapter of the book. Prof. Dr. Sci. Pham Loi Vu, born in 1934, is a professor of Institute of Mechanics, Vietnam Academy of Science and Technology.



The cover book of Prof. Pham Loi Vu

Prof. Dr. Sc. Pham Loi Vu

Contents

In this book, the following equations of inverse scattering problems (ISPs) are studied:

- The systems of two first-order ordinary differential equations (ODEs) on a half-line with a non-self-adjoint potential matrix and with a self-adjoint potential matrix in Chapters 1 and 3;

- The system of n ($n > 2$) first-order ODEs and the transport equation in Chapter 5. The transport equation is continual generalization from the system of $n > 2$ first-order ODEs on the whole line;

- The Schrödinger equations on a half-line in Chapters 6 and 7;

- The perturbed string equation in characteristic variables on the whole line in Chapter 8.

The results of considered ISPs are not only independently interesting by themselves but also are effective tools for solving initial value problems (IVPs) and initial-boundary value problems (IBVPs) for a class of distinguished nonlinear evolution equations (NLEEs), such as:

- The nonlinear attractive and repulsive Schrödinger equations in Chapter 2;
- Cubic NLEEs and modified Korteweg-de Vries (KdV) equations in Chapter 3;
- Sine-Gordon and sinh-Gordon equations in Chapter 4;
- The continual system of nonlinear interaction waves in Chapter 5;
- The KdV equation in Chapter 6;
- The KdV equation with dominant surface tension in Chapter 7;
- The two-dimensional generalization from the KdV equation in Chapter 8.

The listed NLEEs above are most widely studied nonlinear wave equations, because of its intrinsic mathematical properties and their wide applicability in physics, plasma physics and fluid mechanics for the description of stratified internal waves, hydrodynamic waves, ion-acoustic waves, lattice dynamic, . . .

Extending application of the Inverse Scattering Method (ISM) to NLEEs

There exist essential difficulties in extending the application of the ISM.

- The ISM makes possible a complete investigation of the Cauchy IVPs for integrable NLEEs. However, this method is difficult to transfer to IBVPs for the NLEEs with arbitrary initial and boundary conditions. The essential difficulty associated with these problems is that the time-evolution equations for the scattering data cannot be integrated into most of the cases, because their coefficients depend on unknown boundary values. There exists a special class of boundary conditions that are completely consistent with the integrability property. Under these conditions, the IBVPs for NLEEs are solved by the ISM in Chapters 2, 3, 4, 6, and 7.

- The difficulty arises from the strict restriction on coefficients of multidimensional operators. By using the generalized Lax equation, the two-dimensional generalization from the KdV is derived, and the IBVP for this two-dimensional KdV is solved by the ISM in Chapter 8.

- The transition matrix for the scattering problem for the system of $n > 2$ first-order ODEs on the whole line do not admit analytic factorization. This analytic property plays a basic role in solving the inverse problem. In Chapter 5, the intermediate matrix admitting the analytic factorization is introduced for the system of $n > 2$ first-order ODEs. The relations between the transition matrix and the intermediate matrix are established. Analogous results of the ISP for the transport equation are obtained. By using results of the ISP for the transport equation and by the ISM, we solve the Cauchy IVP problem for the continual system of nonlinear interaction waves;

- There exists a remarkable class of potentials, for which the ISP can be solved exactly. These potentials are non-scattering potentials. The non-scattering data of scattering problems with the non-scattering potentials are completely determined by the discrete spectrum of the scattering problem. In Chapters 2, 3, 4 and 6, the explicit solutions and soliton-solutions of NLEEs are found that are presented in explicit forms in the class of non-scattering potentials.

- In Chapter 9, some methods, such as Bäcklund transformations, Hirota's method, Fokas's global algebraic relation, a Riemann–Hilbert problem, are introduced for establishing the connections between these methods and the ISM used in this book.

Features

The scattering data (SD) set s of the considered ISPs is described completely. That is to establish the necessary and sufficient conditions for given quantities to be the SD for the problem generated by the considered differential equation with the known initial and boundary conditions (BCs). Due to this fact, the unique solvability of the ISP is proven. Further, the operators governing the time-evolution of eigenfunctions of the SP are shown. Using these governing operators, we derive the Volterra integral equations for unknown time-dependent boundary values. The solutions of derived equations are the found time-dependent boundary values in terms of the known initial and BCs. With the help of the Lax (generalized) equation, the application of obtained results of the ISP to solving the associated IVP or IBVP for the NLEE is carried out step by step. Namely, the NLEE can be written as the compatibility condition of two linear equations. Such NLEEs are called integrable. The time-dependent SD set $s(t)$ are constructed from the SD set s in terms of known initial and BCs. Due to the complete description of the SD, the fundamental equations in inverse problem have unique solutions. Then the solution of the NLEE is expressed through the solution of time-dependent fundamental equations. Hence, the ISM consists of two steps:

- Step 1: The transformation operators and special Volterra's integral equations are utilized for representations of solutions of the direct and ISP. Then the unique solvability of the ISP is proven. The schema for restoring the potential matrix $C(x)$ is shown as follows: The SP with given potential $C(x)$ and BCs $\Leftrightarrow$ SD set s. By this schema, the SP with given potential $C(x)$ and BCs maps into the SD set s. This map is uniquely reversible, i.e., the SP with potential $C(x)$ is recovered uniquely from SD set s.

- Step 2: The time-dependent SD set $s(t)$ is constructed from the SD set s and the calculated boundary values. Due to the complete description of the SD, the fundamental equations have unique solutions. Hence, the potential $C(x,t)$ is recovered uniquely, which is expressed through the solutions of fundamental equations in terms of the time-dependent SD $s(t)$. Then the solution of NLEEs is expressed uniquely through the recovered potential $C(x,t)$ of the ISP. The schema for finding the solution of the IBVP for NLEEs is shown as follows: The Lax equation with initial-BCs $\Rightarrow$ SD $s(t) \Leftrightarrow$ solution in ISP $C(x,t) \Leftrightarrow$ solution of IBVP.

Moreover, there exists a one-to-one correspondence between time-dependent SD $s(t)$ and the found solution of the IBVP for NLEEs. This means that $s(t) \Leftrightarrow$ solution of IBVP for NLEEs.

Concluding remarks

Since the considered ISPs are solved well, then the time-dependent SPs generated by the Lax pair of two linear equations constitute the ISM, that can be applied to finding the solution of the IBVP or IVP for NLEEs. The application of the ISM is implemented consistently and effectively embedded in the schema of the ISM.